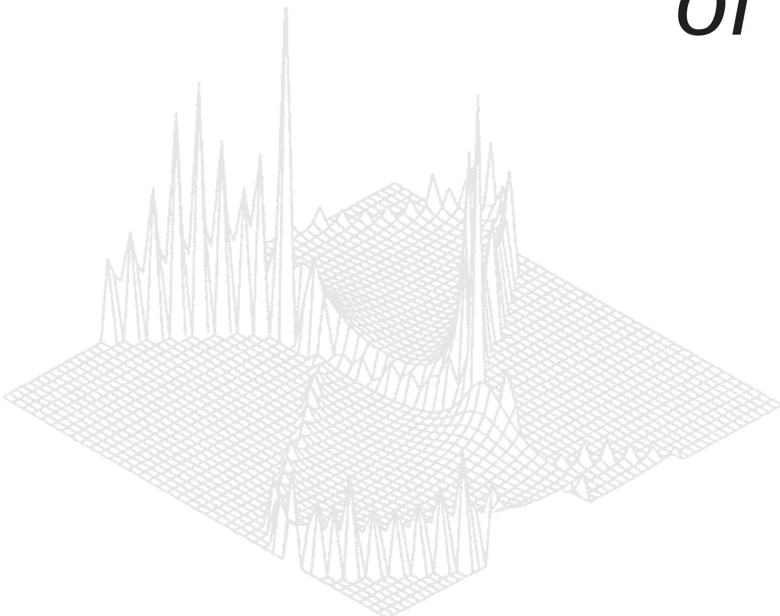
CSIRO PUBLISHING

Australian Journal of Physics



Volume 53, 2000

A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia

Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: ajp@publish.csiro.au

CSIRO AUSTRALIA

Published by **CSIRO** PUBLISHING
for CSIRO and
the Australian Academy of Science

Academy of Science

Aust. J. Phys., 2000, **53**, 653–664

Monopole in the Dilatonic Gauge Field Theory

D. Karczewska and R. Manka

Department of Astrophysics and Cosmology, Institute of Physics,
University of Silesia, Uniwersytecka 4, 40–007 Katowice, Poland.

Abstract

A numerical study of static, spherically symmetric monopole solutions coupled to the dilaton field, inspired by the Kaluza–Klein theory with large extra dimensions is presented. The generalised Prasad–Sommerfield solution is obtained. We show that the monopole may also have the dilaton cloud configurations.

1. Introduction

Recently there has been considerable interest in field theories with large extra spacetime dimensions. In comparison to the standard Kaluza–Klein theory these extra dimensions may be restricted only to the gravity sector of theory, while the Standard Model (SM) fields are assumed to be localised in four-dimensional spacetime (Antoniadis *et al.* 1998; Arkani-Hamed *et al.* 1999). It is a promising scenario from the phenomenological point of view because it shifts the energy scale of unification from 10^{19} GeV to 10–100 TeV.

The gauge field theory is extended by inclusion of the dilatonic field in such theories. These fields appear also in a natural way in Kaluza–Klein theories (Appelquist *et al.* 1987), superstring inspired theories (Witten 1985; Ferrata *et al.* 1989) and in theories based on the noncommutative geometry approach (Chamseddine and Fröhlich 1993).

As previous studies have already shown, the inclusion of a dilaton in a pure Yang–Mills theory has consequences already at the classical level. In particular the dilaton Yang–Mills theories possess ‘particle-like’ solutions with finite energy which are absent in the pure Yang–Mills case. Analogous equations have recently been obtained for the ’t Hooft–Polyakov monopole model coupled to the dilatonic field (Lavreashvili and Maison 1992*a*, 1992*b*, 1997).

2. The Dilatonic Gauge Field Theory

Dilatons appear in the higher dimensional theory after the process of spacetime compactification. The main idea of the theory with large extra dimensions is that gravity is realised in the more dimensional spacetime (the bulk), while matter is confined to four-dimensional spacetime (the brane). To be clear and simple, we consider six-dimensional gravity. Let us now consider the action integral of Einstein–Yang–Mills–Higgs theory in six-dimensional spacetime:

$$\mathcal{S} = \int d^6x\sqrt{-g_6}L, \tag{1}$$

 10.1071/PH99059 0004-9506/00/050653$05.00

where $g_6 = det(g_{MN})$ and $M = \{\mu, i\}$, $N = \{\nu, j\}$ with $x^M = \{x^\mu, y^i\}$, i =1, 2. The metrical tensor in the six-dimensional spacetime can be written:

$$g_{MN} = \begin{pmatrix} e^{-2\xi(x)/f_0}\overline{g}_{\mu\nu} & 0 \\ 0 & -r_2^2\delta_{ij}e^{+2\xi(x)/f_0} \end{pmatrix}. \tag{2}$$

According to the above definition we can write:

$$\sqrt{-g_6} = \sqrt{-\overline{g}}r_2^2 e^{-2\xi(x)/f_0}. \tag{3}$$

In equation (2)

$$g_{\mu\nu} = e^{-2\xi(x)/f_0}\overline{g}_{\mu\nu} \tag{4}$$

represents the four-dimensional metric in the Jordan frame, while $\overline{g}_{\mu\nu}$ is in the Einstein frame. We consider the Lagrangian of the Einstein–Yang–Mills–Higgs field as follows:

$$L = L_g + L_{YMH}\delta(y), \tag{5}$$

$$L_g = -\frac{1}{2\kappa_6}(R-2\Lambda), \tag{6}$$

$$L_{YMH} = -\tfrac{1}{4}F^a_{\mu\nu}\,F^{a\,\mu\nu} + \tfrac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - U(\Phi), \tag{7}$$

where κ_6 is the six-dimensional gravitational coupling and L, L_g and L_{YMH} describe the total Lagrange function, gravity in six-dimensional spacetime and the Yang–Mills–Higgs field parts on the brane embedded in six-dimensional space respectively. In general a non-vanishing cosmological constant ($\Lambda \neq 0$) is possible. This case leads to the interesting monopole solution (Lugo and Shaposhnik 1999; Lugo *et al.* 2000). In our paper we shall focus our attention on the $\Lambda = 0$ case. All calculations should include g_{MN} (equation 2), so for example $D^\mu = g^{\mu\nu}D_\nu$. Let us compactify the six-dimensional spacetime to the four-dimensional Minkowski one on the torus ($M_6 \to M_4 \times S^1 \times S^1$). In this paper we assume that the extra dimensions are compactified to a two-dimensional torus with a single radius r_2. The six-dimensional action may be written as:

$$\mathcal{S} = \int d^4x \int d^2y \sqrt{-g_6}L = \int d^4x \sqrt{-\overline{g}}\mathcal{L}, \tag{8}$$

where $'d^2y = (2\pi r_2)^2$ and L is the effective Lagrange function in four-dimensional spacetime. The six-dimensional gravitational coupling $\kappa_6 = 8\pi G_6$ is conveniently defined as

$$G_6^{-1} = \frac{1}{(2\pi)^2}M^4,$$

where M is the energy scale of the compactification (~10 – 100 TeV). Compactification of the six-dimensional gravity on the torus gives the Lagrangian (8) for the four-dimensional gravity as

$$L = -\frac{1}{2\kappa}R_{(4)} \tag{9}$$

in the Einstein frame, where

$$\frac{1}{\kappa} = \frac{(2\pi r_2)^2}{\kappa_6} \tag{10}$$

is the four-dimensional coupling constant or $\kappa = 8\pi G_N = 8\pi M_{Pl}^{-2}$. From equation (10) we get

$$M_{Pl}^2 = 4\pi M^4 r_2^2. \tag{11}$$

Cosmological considerations (Hall and Smith 1999) give a bound of $M \sim 100$ TeV which corresponds to $r_2 \sim 5.1 \times 10^{-5}$ mm from equation (11). Compactification of gravity on five-dimensional spacetime is rather unphysical ($r_2 \sim 10$ km), however an interesting five-dimensional spacetime compactification has been proposed recently (Randal and Sundram 1999*a*, 1999*b*). The Planck mass M_{Pl} in (11) is no longer a fundamental constant, but may change during the evolution of the Universe (Flanagen *et al.* 1999). For a four-dimensional Minkowski spacetime ($\bar{g}_{\nu\mu} = \eta_{\mu\nu}$)

$$R_{(4)} = \frac{4}{f_0^2} e^{2\xi(x)/f_0} \{-\partial_\mu \xi\, \partial^\mu \xi + f_0 \partial_\mu \partial^\mu \xi\}. \tag{12}$$

The last term in (12) can be transformed into the first by differentiating by parts. The parameter f_0 [or re-scaling of the $\xi(x)$ field] is determined by the Planck mass (at the present time) as

$$f_0 = \frac{1}{\sqrt{2\pi}} M_{Pl} \sim 4.87\times 10^{18}\ \text{GeV}/c^2 \tag{13}$$

to produce the $\frac{1}{2}$ term in the dilaton field in (16). The f_0 parameter determines the dilaton scale f_0. At the present time f_0 is rather high, so the interaction with dilatons can be neglected. However, in the early Universe when the Planck mass M_{Pl} was smaller (for details see Flanagen *et al.* 1999), the value of f_0 was also smaller.

As a result of compactification of the six-dimensional Lagrangian we get the Lagrange function for the Yang–Mills–Higgs fields. Fluctuations around the four-dimensional Minkowski

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x, y) \tag{14}$$

will produce an interaction with Kaluza–Klein dilatons of

$$h_{\mu\nu}(x, y) = \sum_{\mathbf{n}} h_{\mu\nu}^{\mathbf{n}}(x) \exp\left(i 2\pi n^i y^i / r_2\right) \tag{15}$$

with a typical mass scale M (for $n^i \neq 0$).

In this paper we shall apply this approach to the simplest $SO(3)$ gauge field theory. The $SO(3)$ gauge field theory has nice monopole solutions ('t Hooft 1976*a*, 1976*b*) which have produced difficulties in cosmology and were the reason for introducing the idea of inflation. The main idea of this paper is to examine how the monopole solution looks in the dilatonic gauge field theory inspired by Kaluza–Klein gravity on a TeV scale.

The dilatonic gauge field theory may be described by the Lagrangian [defined by equation (8) in the first approximation when $\bar{g}_{\nu\mu} = \eta_{\mu\nu}$ in the Minkowski spacetime]

$$\mathcal{L} = \frac{1}{2}\partial_\mu \xi(x)\partial^\mu \xi(x) - \frac{1}{4}e^{2\xi(x)/f_0}F^a_{\mu\nu}F^{a\mu\nu}$$
$$+ \frac{1}{2}(D_\mu \Phi^a)D^\mu \Phi^a - e^{-2\xi(x)/f_0}U(\Phi), \tag{16}$$

where for $SO(3)$ theory we have

$$U(\Phi) = \frac{\lambda}{4}(\Phi^a \Phi^a - \nu^2)^2 \tag{17}$$

with the $SO(3)$ field strength tensor $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon_{abc}W^b_\mu W^c_\mu$. The $SO(3)$ gauge symmetry rotates the Higgs field Φ^a. The covariant derivative is given by $D_\mu \Phi^a = \partial_\mu \Phi^a - g\varepsilon_{abc}W^b_\mu \Phi^c$. Now we have $D^\mu = \eta^{\mu\nu}D_\nu$. The Higgs potential has degenerate true vacuums forming the sphere S^2 ($\Phi^a\Phi^a = \nu^2$). The Euler–Lagrange equations for the Lagrangian (9) are scale-invariant:

$$x^\mu \to x'^\mu = e^{u/f_0}x^\mu, \tag{18}$$

$$\xi \to \xi' = \xi + u, \tag{19}$$

$$\Phi \to \Phi' = \Phi, \tag{20}$$

$$W^a_\mu \to W^a_\mu = e^{-u/f_0}W^a_\mu. \tag{21}$$

These transformations change the Lagrange function in the following way:

$$\mathcal{L} \to \mathcal{L}' = e^{-2u/f_0}\mathcal{L}. \tag{22}$$

This symmetry can be formulated equivalently as a scaling symmetry on the coordinates, and the dilaton is often denoted as a Goldstone boson for dilatations. The origin of the symmetry of the equations of motion is easily understood from the Kaluza–Klein origin of the action. The scale transformations are equivalent to a rescaling of the internal dimensions.

3. The Dilatonic Monopole

The monopole scalar field configuration

$$\Phi^a = vh(r)n^a = vH(r)\frac{n^a}{gr}, \tag{23}$$

where

$$n^a = \frac{x^a}{r}, \tag{24}$$

describes the 'hedgehog' structure n^a and scalar spherically symmetric field $H(r)$. The $SO(3)$ gauge field is described by the $K(r)$ field:

$$A^a_i = -\varepsilon_{aij}\frac{1}{gr}n^j(1 - K(r)). \tag{25}$$

The dilaton field is described by the $S(x)$ function:

$$\xi(x) = f_0 S(x) \tag{26}$$

(if we introduce the dimensionless variable $x = gvr$). The field Euler–Lagrange equations generated by the Lagrange function (16) for $H(x)$, $K(x)$ and $S(x)$ are

$$H''(x) - \frac{2}{x^2}H(x)K(x)^2 + \frac{\varepsilon}{x^2}e^{-2S(r)}(x^2 - H^2(x))H(x) = 0, \tag{27}$$

$$K''(x) + 2K'(x)S'(x) - \frac{1}{x^2}e^{-2S(x)}H(x)^2K(x) - \frac{K(x)}{x^2}(K(x)^2 - 1) = 0, \tag{28}$$

$$S''(x) + \frac{2}{x}S'(x) - \alpha^{-1}\frac{1}{x^4}e^{2S(x)}\{(1-K(x)^2)^2 + 2x^2K'(x)^2\} + \frac{1}{2}\varepsilon\alpha^{-1}e^{-2S(r)}\left(1 - \frac{H^2(x)}{x^2}\right)^2 = 0. \tag{29}$$

In the dilatonic monopole we have two independent dimensionless constants:

$$\varepsilon = \frac{\lambda}{g^2}, \tag{30}$$

$$\alpha = \left(\frac{f_0}{v}\right)^2. \tag{31}$$

The mass (or the lowest energy) of the monopole in the rest frame is

$$M_{mon} = \frac{4\pi\nu}{g}\int \rho(x)x^2dx, \tag{32}$$

with the energy density given by

$$\rho(x) = \frac{1}{2}\alpha S'(x)^2 + \frac{1}{2x^4}e^{2S(x)}\{(1-K(x)^2)^2 + 2x^2K'(x)^2\} + \frac{1}{2x^4}\{2H(x)^2K(x)^2 + (xH'(x) - H(x))^2\} + \frac{1}{4}\varepsilon e^{-2S(x)}\left(1 - \frac{H^2(x)}{x^2}\right)^2. \tag{33}$$

Inside the monopole [according to equations (27)–(29)] the asymptotical behaviour when $x \to 0$ is given as

$$K(x) = 1 - tx^2 + O(3), \tag{34}$$

$$H(x) = ux^2 + O(3), \tag{35}$$

$$S(x) = a + bx^2 + O(3), \tag{36}$$

where u, t, a are local parameters and b must be determined as

$$b = \frac{24e^{2a}t^2 - \epsilon e^{-2a}}{12\alpha}. \tag{37}$$

Far from the monopole core, if $r \to \infty$ ($x \to \infty$), both functions $H(x)$ and $K(x)$ should describe the normal vacuum ($\Phi^a\Phi^a = v^2$) with $H \to x$ and $K \to 0$ according to the H, K function definitions (23) and (25), remembering that $x = gvr$. In this limit the energy density (33) has a simple limit

$$\rho(x) = \tfrac{1}{2}\alpha S'(x)^2 + \frac{1}{2x^4}e^{2S(x)}.$$

The monopole mass in this limit may be rewritten as

$$\begin{aligned} M_{mon} &= \frac{4\pi v}{g}\int \rho(x)x^2 dx \\ &= \frac{4\pi v}{g}\int\left(\sqrt{\alpha}xS' + \frac{1}{x}e^S\right)^2 dx + \frac{4\pi v}{g}\sqrt{\alpha}(e^{S(0)} - e^{S(\infty)}). \end{aligned}$$

The first term vanishes if the dilaton field obeys the Bogomolny equation

$$\sqrt{\alpha}xS' + \frac{1}{x}e^S = 0.$$

This equation has a nice solution in the uniform normal vacuum (Bizon 1993)

$$S_b = -ln[(\sqrt{\alpha}e^{-S(\infty)} - 1/x)/\sqrt{\alpha}]. \tag{38}$$

When $r \to \infty$ the dilaton field should disappear in the true vacuum. This demand gives $S(\infty) = 0$. So, when $r \to \infty$ we have the asymptotic behaviour of the solutions

$$H(x) = x\,(1 - we^{-\sqrt{2\varepsilon}x}) + O(1/x), \tag{39}$$

$$K(x) = ze^{-x} + O(1/x), \tag{40}$$

$$S(x) = -ln\left(\sqrt{\alpha}e^{-S(\infty)} - \frac{1}{\sqrt{\alpha}x}\right) + O(1/x). \tag{41}$$

Even when $S(\infty) \neq 0$ it may be removed by the dilaton transformation (19). We may solve the differential equations (27)–(29) by the iteration method, expanding them with respect to ε:

$$H(x) = \sum_{n=0}^{\infty} \varepsilon^n H_n(x), \tag{42}$$

$$K(x) = \sum_{n=0}^{\infty} \varepsilon^n K_n(x), \tag{43}$$

$$S(x) = \alpha^{-1}\sum_{n=0}^{\infty} \varepsilon^n S_n(x). \tag{44}$$

In the first step ($n = 0$) we obtain the equations

$$H_0''(x) - \frac{2}{x^2}H_0(x)K_0(x)^2 = 0, \tag{45}$$

$$K_0''(x) + \frac{2}{\alpha}K_0'(x)S_0'(x) - \frac{1}{x^2}e^{-2S_0(x)/\alpha}H_0(x)^2K_0(x)$$
$$-\frac{K_0(x)}{x^2}(K_0(x)^2 - 1) = 0, \tag{46}$$

$$S_0''(x) + \frac{2}{x}S_0'(x) - \frac{e^{2S_0(x)/\alpha}}{x^4}\{(1 - K_0(x)^2)^2 + 2x^2K_0'(x)^2\} = 0, \tag{47}$$

leading to the Prasad–Sommerfield (1975) solution [without the dilaton field $S(x)$]. When $\alpha \to \infty$ we have the Prasad–Sommerfield solution and we can easily find for $H_0(x)$ and $K_0(x)$

$$H_0(x) = x/\tanh(x) - 1, \tag{48}$$

$$K_0(x) = x/\sinh(x). \tag{49}$$

Finding a nice analytical solution for the dilaton field (see the dotted line in Fig. 3)

$$S_0(x) = a + \frac{Q_D}{x} + \frac{1}{4\sinh(x)^2x^2}(-1 + 2x^2 + 2xQ_D + \cosh(2x)$$
$$-2xQ_D\cosh(2x) - 2x\sinh(2x)) \tag{50}$$

is a crucial point of this paper. The leading term for the dilaton field at infinity will be the Coulomb one

$$S_0(x) = \frac{Q_D}{x} + O(1/x),$$

where Q_D is the dilatonic charge which originates from the global scale transformation (18)–(21). The similarity is striking, but we should remember that an electric charge comes from the $\exp(ie\alpha) \in U(1)$ gauge symmetry. The global scale transformation (18)–(21) is generated by the exponential transformation $\exp(u/f_0)$. However, the asymptotic behaviour at $x \to 0$ in (36) admits $Q_D = 0$.

We present the numerical solutions of the coupled set of differential equations (27)–(29) in the next section.

4. Numerical Solutions

To solve the monopole equations numerically, we need a starting point (Press *et al.* 1992). To find the starting conditions we can use the solutions found from the variational procedures or from the Prasad–Sommerfield approximation (48)–(50). The trial solutions depending on the variational parameters must be postulated in such a way as to fulfill the boundary conditions close to the centre (34)–(36) of the monopole and far outside (39)–(40). We postulate the trial solutions

$$H_v(x) = x\frac{[ux + x^2(1 - e^{-\sqrt{2\varepsilon x}})]}{(1 + x^2)} \sim ux^2 + O(3), \tag{51}$$

$$K_v(x) = \frac{(1 - tx^2 + zx^4)}{(1 + x^4e^x)} \sim 1 - tx^2 + O(3), \tag{52}$$

Table 1. Dependence of the monopole mass and parameters *u*, *t* and *z* on the parameter α

α	u	t	z	M_{dil} (10^{15} GeV)
2.37×10^5	0.2396	0.6021	2.46903	16.4125
2.37×10^3	0.2396	0.6021	2.46886	16.4107
2.37	0.2591	0.5769	2.2637	15.0284
1	0.2879	0.5389	1.92221	14.4402
0.9	0.2937	0.5327	1.8099	14.4913
0.8	0.3011	0.52609	1.7035	14.6313

$$S_v(x) = \frac{(a + bx^2 + Q_D x^5)}{(1 + x^6)} \sim a + bx^2 + O(3), \tag{53}$$

where $O(3)$ are corrections of the third order. For these functions the monopole mass was calculated and the trial function with minimal energy was found. For the monopole without dilatons we get the M_{mon} mass of monopole [if $S(x) = 0$]:

$$M_{mon} = 15.886 \times 10^{15} \text{ GeV}.$$

The trial functions give the dilaton configuration close to the monopole case without dilatons. The minimal variational configurations for such a dilatonic monopole for $Q_D = 0$ and $v = 10^{16}$ GeV are presented in Table 1. This shows that the mass of the dilatonic monopole depends on the parameter α (f_0) and reaches the local minimum (for $\alpha \sim 1$) lower than for the monopole without dilatons.

For the dilatonic monopole the numerical method was independently verified using the Chebyshev polynomial expansion (Michaila 1999). The monopole solutions for $H(x)$ and $K(x)$ are known very well so our attention is focused on the dilaton solution $S(x)$ especially. The behaviour of the $H(x)$ and $K(x)$ solutions determined by the boundary conditions is the same as it is in the presence of the dilaton field.

The Chebyshev method allows us to calculate the exact solution of the differential equations for the discrete set of points. The trial function provides the starting data for the numerical solution of the ordinary differential equation (ODE) (the shooting method, see Press *et al.* 1992) or the Chebyshev functions method. After this preliminary numerical calculation the method based on the Chebyshev polynomial was used.

Clenshaw and Norton (1963) proposed almost forty years ago an integration method based on Chebyshev polynomials of the first kind of degree j:

$$t_j(x) = \cos(j * \arccos(x)). \tag{54}$$

Since then, these methods have become standard. Since the Chebyshev polynomials are orthogonal this allows us to rewrite the function $f(x)$ as

$$f(x) = \sum_{j=0} \alpha_j t_j(x), \tag{55}$$

where (for $j = 0$)

$$\alpha_0 = \frac{1}{n} \sum_{k=1}^{n} f(x_k) t_0(x_k), \tag{56}$$

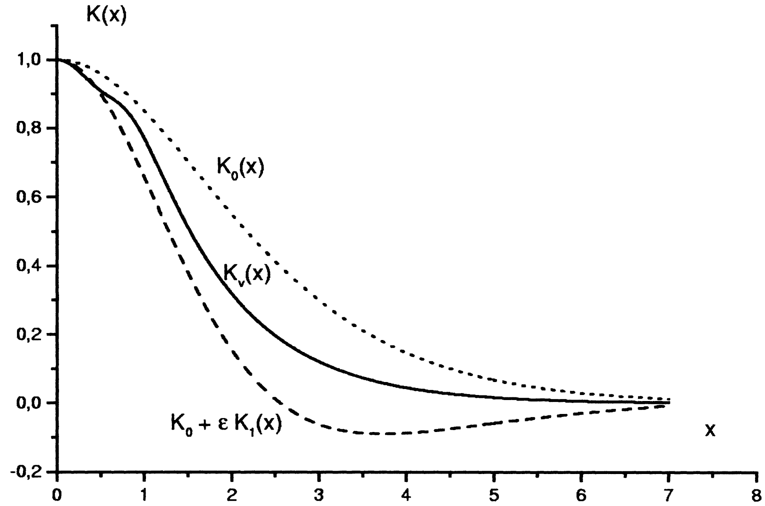


Fig. 1. Dependence of the gauge field $K(x)$ on the x parameter. The dotted line corresponds to the Prasad–Sommerfield solution (49), the solid one to the variational solution (52), and the dashed line presents the solution of the first iteration.

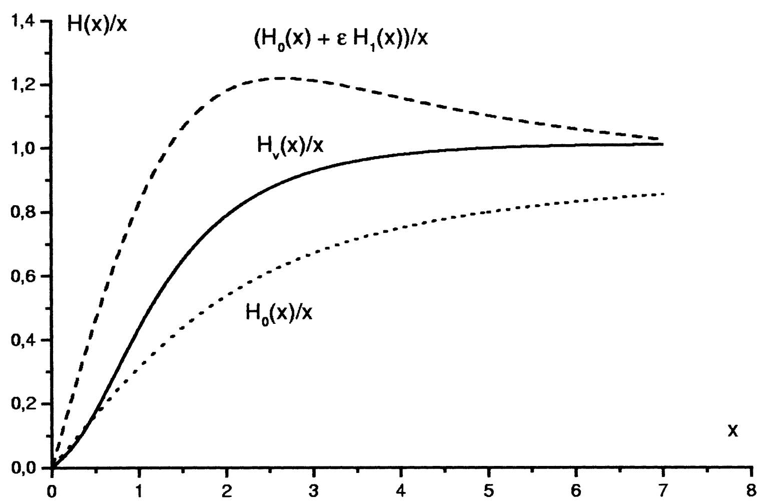


Fig. 2. Dependence of the Higgs field $H(x)/x$ on the x parameter. The dotted line corresponds to the Prasad–Sommerfield solution (48), the solid one to the variational solution (51), and the dashed line presents the solution of the first iteration.

and (for $j \neq 0$)

$$\alpha_j = \frac{2}{n} \sum_{k=1}^{n} f(x_k) t_j(x_k). \tag{57}$$

The grid of n points x_k are zeros of the Chebyshev polynomial $t_j(x)$. This decomposition allows us to present the derivative of the function $f(x)$ as

$$f'(x_i) = \sum_k D_{ik} f(x_k), \tag{58}$$

where the matrix

$$D_{ik} = \sum_{j=0}^{n-1} \frac{1}{c_j} t_j(x_k) t'_j(x_i), \tag{59}$$

and $c_0 = n$, $c_j = n/2$ (at $j \neq 0$). This fact transforms the ordinary differentional equation:

$$\frac{d^2 f}{dx^2} + p(x)\frac{df}{dx} + q(x) = r(x) \tag{60}$$

into an appropriate linear equation:

$$\sum_k A_{ik} f(x_k) = u_i. \tag{61}$$

So, we can have an exact solution for a discrete number of points. This method may be also used for the nonlinear equation

$$m\frac{d^2 f}{dx^2} + p(x)\frac{df}{dx} + F(f, x) = 0. \tag{62}$$

If we have a starting function f_0 then we expand f around f_0:

$$f(x) = f_0(x) + \varepsilon(x), \tag{63}$$

and approximate (62) with (60) and then solve numerically. The solution may be treated now as a starting function for the next iteration, and so on. The iteration can continue until an arbitrary precision is reached.

A perturbation around ε produces the series of differential equations

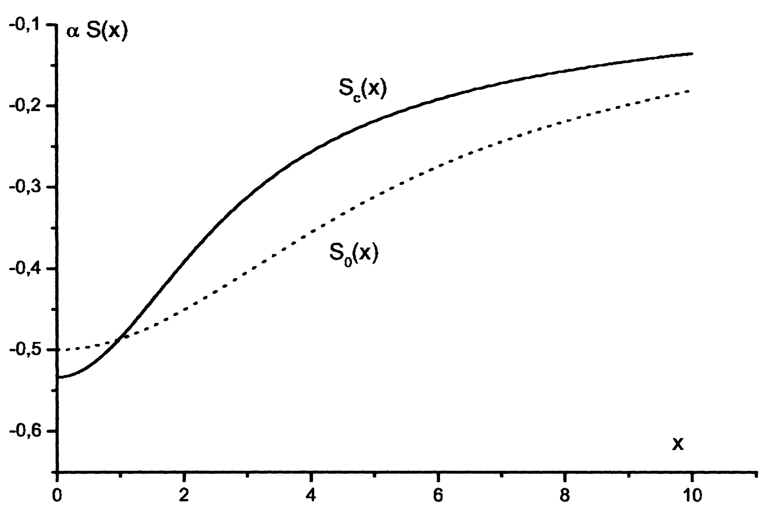


Fig. 3. Profile of the dilaton cloud $S(x)$. The solid line presents the Chebyshev numerical solution $S_c(x)$ and the dotted line is the Prasad–Sommerfield solution $S_0(x)$ in equation (50).

$$f''_{a,b} + \sum_b p_{ab,n}(x) f'_{b,n}(x) + \sum_b q_{ab,n}(x) f_{b,n}(x) = r_{a,n}(x), \tag{64}$$

where the vector is

$$f_n = \{H_n(x), K_n(x), S_n(x)\}. \tag{65}$$

For example, when $n = 1$ the first equation ($a = 1$) corresponds to $p_{1b} = 0$, $q_{11} = 2K_0^2(x)/x^2$, $q_{12} = 2H_0(x)K_0(x)/x^2$, $r_1 = \exp(-2S_0(x)/\alpha)(H_0^2(x) - x^2)H_0(x)/x^2$, and so on.

In the monopole case the starting functions are those obtained by the variational method. After expanding around trial functions (51)–(52) we obtain a system of differential equations of the type (60). After that the numerical solution may be obtained on the grid of the x_k. The numerical solution for the dilaton field found by the Chebyshev numerical method is presented in Fig. 3 (solid line).

5. Conclusions

The aim of this paper was to present a numerical study of the classical monopole solutions of the $SO(3)$ theory coupled to the dilaton fields. We have shown that a monopole is surrounded by the dilaton cloud $S(x)$. In field theories with large extra spacetime dimensions the Planck mass is no longer a fundamental constant and may have changed during the evolution of the Universe. As a consequence, the parameter f_0 changes too. We have shown that the dilatonic monopole reaches the minimal mass when $f_0 \sim v$ with the mass slightly lower than for a monopole without dilatons.

There is an analytical solution in the Prasad–Sommerfield limit.

The spherically symmetric dilaton solutions coupled to the gauge field or gravity are interesting in their own right and may moreover influence the monopole catalysis.

However, in the theory inspired by the Kaluza–Klein theory with large extra dimensions a new interaction with massive ($\sim M$) Kaluza–Klein gravitons also takes place. In four-dimensional spacetime the monopole solution is stable due to the monopole topological charge. Now the interaction with Kaluza–Klein gravitons $h_{\mu\nu}^{\mathbf{n}}(x)$ may cause disintegration of the monopole.

References

Antoniadis, I., Arkani-Hamed, N., Dimopoulos, S., and Dvali, G. (1998). *Phys. Lett.* B **436**, 257.

Appelquist, T., Chodos, A., and Freund, P. G. O. (1987). 'Modern Kaluza–Klein Theories' (Addison–Wesley: Menlo Park).

Arkani-Hamed, N., Dimopoulos, S., and Dvali, G. (1998). *Phys. Lett.* B **429**, 263.

Arkani-Hamed, N., Dimopoulos, S., and Dvali, G. (1999). *Phys. Rev.* D **59**, 086004.

Bizon, P. (1993). *Phys. Rev.* D **47**, 1656.

Chamseddine, A. H., and Fröhlich, J. (1993). *Phys. Lett.* B **314**, 308.

Clenshaw, C. W., and Norton, H. J. (1963). *Computer J. (UK)* **6**, 88.

Ferrata, S., Lüst, P., and Teisen, S. (1989). *Phys. Lett.* **233**, 147.

Flanagen, E. E., Tye, S.-H., and Wasserman I. (1999). A cosmology of the brane world. Preprint hep-ph/9909373.

Hall, L. J., and Smith, D. (1999). *Phys. Rev.* D **60**, 085008.

Lavreashvili, G., and Maison, D. (1992*a*). *Phys. Lett.* **295**, 67.

Lavreashvili, G., and Maison, D. (1992*b*). Regular and black hole solutions of Einstein–Yang–Mills dilaton theory. Preprint MPI-ph/92-115.

Lavreashvili, G., and Maison, D. (1997). Preprint SLAC-PUB-7479.

Lugo, A. R., and Shaposhnik, F. A. (1999). *Phys Lett.* B **467**, 43.

Lugo, A. R., Moreno, E. F., and Shaposhnik, F. A. (2000). *Phys. Lett.* B **473**, 35.
Michaila, B. (1999). Numerical approximation using Chebyshev polynomial expansion. Preprint physics/9901005.
Prasad, M. K., and Sommerfield, C. M. (1975). *Phys. Rev. Lett.* **35**, 760.
Press, W. H., Teukolsky, S. A., and Vetterlino, W. T. (1992). 'Numerical Recipes: The Art of Scientific Computing' (Cambridge Univ. Press).
Randal, L., and Sundram, R. (1999*a*). *Phys. Rev. Lett.* **83**, 3370.
Randal, L., and Sundram, R. (1999*b*). *Phys. Rev. Lett.* **83**, 4660.
't Hooft, G. (1976*a*). *Phys. Rev. Lett.* **37**, 11.
't Hooft, G. (1976*b*). *Phys. Rev.* D **14**, 3432.
Witten, E. (1985). *Phys. Lett.* B **245**, 561.

Manuscript received 21 June 1999, accepted 16 May 2000