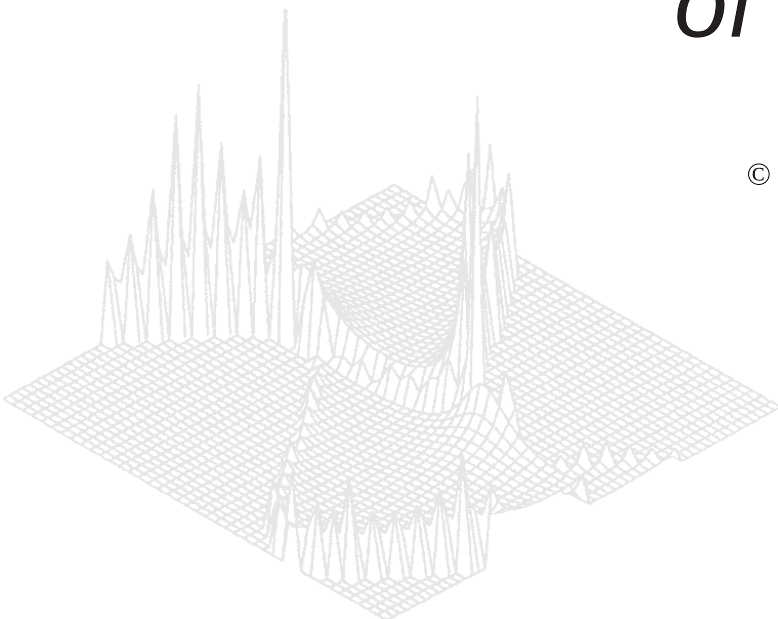
CSIRO PUBLISHING

Australian Journal of Physics

Volume 53, 2000



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia
Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: peter.robertson@publish.csiro.au

CSIRO AUSTRALIA

Published by **CSIRO** PUBLISHING
for CSIRO Australia and
the Australian Academy of Science

Academy of Science

Aust. J. Phys., 2000, **53**, 421–7

Preparation of Superpositions of SU(2) Coherent States of the Motion of a Trapped Ion

Shi-Biao Zheng

Department of Electronic Science and Applied Physics,
Fuzhou University, Fuzhou 350 002, P. R. China.

Abstract

A scheme is presented for the generation of superpositions of two two-mode SU(2) coherent states of the motion for a trapped ion. In the scheme an ion is trapped in a two-dimensional isotropic harmonic potential and driven by two resonant laser beams. Under certain conditions, the motional two-mode SU(2) cat state can be generated after a conditional measurement on the internal state following the ion–laser interaction.

1. Introduction

Over the past few years, much attention has been paid to the so-called Schrödinger cat states. In quantum optics, such states are described as superpositions of coherent states. These states are of particular interest since they may exhibit various nonclassical properties such as sub-Poissonian photon statistics and squeezing (Xia and Guo 1989; Janszky and Vinogradov 1990; Janszky *et al.* 1993, 1995), though formed by quantum states having closest classical analogs. In the context of cavity QED, a number of theoretical schemes have been proposed for generating such states (Brune *et al.* 1992; Garraway *et al.* 1994; Gerry 1996; Guo and Zheng 1996).

Recently, there have been multi-mode generalisations of the cat states. Entangled coherent states (Sanders 1992*a*, 1992*b*), also referred to as superpositions of two-mode coherent states (Chai 1992), have been intensively studied. These superposition states may exhibit various nonclassical properties, such as two-mode squeezing and violation of the Cauchy–Schwarz inequality. Multi-mode even and odd coherent states (Ansari and Man'ko 1994; Dodonov *et al.* 1995) have also been investigated. On the other hand, Gerry and Grobe (1995) have studied an alternative type of cat states formed by pair coherent states (Agarwal 1988), which are already highly nonclassical. More recently, Gerry and Grobe (1997) have also investigated different types of two-mode cat states defined as superpositions of SU(1,1) or SU(2) coherent states. In this paper we propose a method for the generation of the SU(2) cat states in the motion of a trapped ion.

The two-mode SU(2) coherent states are defined as

$$|\xi, N\rangle = S(r)\,|0, N\rangle \ , \tag{1}$$

 10.1071/PH99064 0004-9506/00/030421$05.00

where $S(r)$ is given by

$$S(r) = \exp(ra^+b - r^*ab^+)\,, \tag{2}$$

and $|0, N\rangle$ is a two-mode Fock state. Such states can be expanded in terms of two-mode Fock states

$$|\xi, N\rangle = (1 + |\xi|^2)^{-N/2} \sum_{n=0}^{N} \binom{N}{n}^{\frac{1}{2}} \xi^n \,|n, N-n\rangle \ , \tag{3}$$

where

$$\xi = \tanh(\rho/2)e^{-i\theta}\,, \tag{4}$$

with ρ and θ given by the equality $r = \rho e^{-i\theta}$. It has been shown (Buzek and Quang 1987) that these states are sub-Poissonian in both modes. Two-mode SU(2) Schrödinger cat states are defined by

$$|\xi, q, \varphi\rangle = N_\varphi[|\xi, q\rangle + e^{i\varphi}\,|-\xi, q\rangle]\,, \tag{5}$$

where N is a normalisation factor. They can be also rewritten as

$$|\xi, q, \varphi\rangle = N_\varphi[S(r) + e^{i\varphi}S(-r)]\,|q, 0\rangle \ , \tag{6}$$

where N_φ is a normalisation factor given by

$$N_\varphi = \sqrt{\tfrac{1}{2}}\left[1 + \cos\varphi\left(\frac{1 - |\xi|^2}{1 + |\xi|^2}\right)^N\right]^{-1/2} . \tag{7}$$

The nonclassical features may be enhanced in the superpositions compared with those of SU(2) coherent states. However, the problem remains of how to generate such cat states. We here discuss this problem in the context of an ion trap.

Recently, remarkable advances in laser cooling and ion trapping have made it possible to generate a variety of quantum states. When an ion is trapped in a harmonic potential and driven by laser fields, its internal degrees and external degrees are coupled via the momentum exchange between the ion and laser fields. Hence, by adjusting the configuration of laser fields one can manipulate the vibrational motion of the trapped ion. Based on this approach, a number of proposals have been made for generating various nonclassical vibrational states of a trapped ion such as Fock states (Cirac *et al.* 1993), even and odd coherent states (de Matos Filho and Vogel 1996; Poyatos *et al.* 1996; Gerry 1997; Zheng 1998), SU(2) states (Gou and Knight 1996), pair coherent states (Gou *et al.* 1996*a*), pair cat states (Gou *et al.* 1996*b*), and superpositions of squeezed states (Gou *et al.* 1997). To date, motional Fock states, squeezed states, coherent states (Meekhof *et al.* 1996), and Schrödinger cat states (Monroe *et al.* 1996) have been experimentally observed. In this paper we propose a scheme for the generation of two-mode SU(2) cat states for the motion of a trapped ion.

This paper is organised as follows. In Section 2 we present a scheme for preparing motional SU(2) cat states. In Section 3 we discuss the problem of distinguishing an SU(2) cat state from an incoherent mixture of two SU(2) coherent states. A summary appears in Section 4.

2. Motional SU(2) Cat States

We consider a two-level ion trapped in a two-dimensional isotropic harmonic potential excited by two lasers in the trap plane. These lasers are both tuned to the ion transition and propagating along the direction with an angle $\pi/4$ and $3\pi/4$ relative to the X axis, respectively. The Hamiltonian for this system is given by (assuming $\hbar = 1$)

$$H = \nu(a^{+}a + b^{+}b) + \omega_0 S_z + [\lambda E^{-}(x, y, t)S^{-} + \text{H.c.}] \,, \tag{8}$$

where a and b are the annihilation operators for the vibrational motions along the X and Y axes, respectively, and S_z and $S^{\pm}$ are the electronic flip operators for the two-level ion with transition frequency ω_0 and dipole matrix element λ. Here ν is the trap frequency in the XY plane. The position operators x and y are given by $x = \sqrt{1/(2\nu M)}(a + a^{+})$ and $y = \sqrt{1/(2\nu M)}(b + b^{+})$, with M being the mass of the trapped ion. Here $E^{-}(x, y, t)$ is the negative frequency part of the driving fields,

$$E^{-}(x, y, t) = E_1 e^{i(\omega_0 - k_1 x' + \phi_1)} + E_2 e^{i(\omega_0 t - k_2 y' + \phi_2)} \,, \tag{9}$$

where E_j and ϕ_j $(j = 1, 2)$ are the amplitudes and phases of the driving fields respectively. The position operators x' and y' are related to x and y by a $\pi/4$ rotation in the XY plane.

In the resolved sideband limit, the ion–laser interaction can be described by the nonlinear Jaynes–Cummings model (Vogel and de Matos Filho 1995; de Matos Filho and Vogel 1996). Then, in the interaction picture the Hamiltonian can be described by

$$H_i = e^{-\eta^2/2}\left\{\sum_{n=0}^{\infty}\frac{(i\eta)^{2n}}{(n!)^2}\left[\Omega_1 e^{i\phi_1} a'^{+n} a'^{n} + \Omega_2 e^{i\phi_2} b'^{+n} b'^{n}\right]\right\} S^{-} + \text{H.c.} \,, \tag{10}$$

where a' and b' are the annihilation operators for the trap motion in the x' and y' directions, respectively, relative to a and b with the transformation (Gou and Knight 1996; Gou *et al.* 1996*a*, 1996*b*)

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \sqrt{\tfrac{1}{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} . \tag{11}$$

In equation (10) $\eta = \sqrt{k^2/2M\nu}$ is the Lamb–Dicke parameter, and $\Omega_j = \lambda E_j$ are the Rabi frequencies of the laser fields. In the Lamb–Dicke limit, i.e. $\eta \ll 1$, the Hamiltonian (10) can be well approximated by an expansion to second order in η. Thus we have

$$H_i = e^{-\eta^2/2}[\Omega_1 e^{i\phi_1}(1 - \eta^2 a'^{+} a') + \Omega_2 e^{i\phi_2}(1 - \eta^2 b'^{+} b')]S^{-} + \text{H.c.} \tag{12}$$

Setting $\Omega_1 = \Omega_2 = \Omega$, $\phi_1 = \pi$ and $\phi_2 = 0$, we have

$$H_i = g(a'^{+}a' - b'^{+}b')(S^{+} + S^{-})\,, \tag{13}$$

where

$$g = \Omega\eta^2 e^{-\eta^2/2}\,. \tag{14}$$

Using equation (11), we get

$$H_i = g(a^{+}b + ab^{+})(S^{+} + S^{-})\,. \tag{15}$$

We define the Hermitian operator

$$O = (a^{+}b + ab^{+})\,. \tag{16}$$

Then, the time evolution operator of the system can be expressed in the form of a 2×2 matrix with respect to the atomic basis:

$$U(\tau) = \begin{pmatrix} \cos(g\tau O) & -i\sin(g\tau O) \\ -i\sin(g\tau O) & \cos(g\tau O) \end{pmatrix}. \tag{17}$$

We suppose that the internal degree is initially in the ground state $|g\rangle$, and the external degrees in the two-mode Fock state $|0, N\rangle$, which can be prepared with high efficiency (Meekhof *et al.* 1996). Then, after an interaction time τ the system evolves to

$$|\psi(\tau)\rangle = \cos(g\tau O)\,|0, N\rangle\,|g\rangle - i\sin(g\tau O)\,|0, N\rangle\,|e\rangle\ . \tag{18}$$

We now detect the internal state of the ion. If we find the ion in the ground state $|g\rangle$, the vibrational motion collapses to

$$\begin{aligned} |\psi_v\rangle_g &= N_g \cos(g\tau O)\,|q, 0\rangle \\ &= \frac{N_g}{2}\,[e^{i(g\tau O)}\,|0, N\rangle + e^{-i(g\tau O)}\,|0, N\rangle]\,, \end{aligned} \tag{19}$$

where N_g is a normalisation factor. Substituting equation(15) into (18) we obtain

$$|\psi_v\rangle_g = N_0[S(r)\,|0, N\rangle + S(-r)\,|0, N\rangle]\,, \tag{20}$$

where the operator $S(r)$ is given by equation (2) with $r = ig\tau$. Thus the ion motion is prepared in an even SU(2) coherent state. On the other hand, the detection of the ion in the excited state $|e\rangle$ collapses the motion to an odd SU(2) coherent state.

3. Superposition States versus Incoherent Mixture

In this section we suggest a method to distinguish the superposition state of equation (20) from a classical mixture of the form

$$\tfrac{1}{2}[S(r)\,|0,N\rangle\,\langle 0,N|\,S^{+}(r)+S(-r)\,|0,N\rangle\,\langle 0,N|\,S^{+}(-r)]\,. \tag{21}$$

In order to do so we drive the ion again with the above-mentioned two laser beams. We suppose that the motional state is given by equation (20) and the electronic state is $|g\rangle$. Then after an interaction time τ the state of the whole system is

$$\begin{aligned}|\psi'(\tau)\rangle = \frac{N_0}{2}\,&\{[S(2r)\,|0,N\rangle + S(-2r)\,|0,N\rangle + 2\,|0,N\rangle]\,|g\rangle \\ &- [S(2r)\,|0,N\rangle - S(-2r)\,|0,N\rangle]\,|e\rangle\}\,. \end{aligned}\tag{22}$$

Now the probability of finding the ion in the ground electronic state $|g\rangle$ is given by

$$\begin{aligned}P_{g,s}^{N} = \tfrac{1}{4}&\left[1+\left(\frac{1-|\xi|^2}{1+|\xi|^2}\right)^N\right]^{-1} \\ &\times\left\{3+\left[\frac{\left(1-|\xi|^2\right)^2-4\,|\xi|^2}{\left(1+|\xi|^2\right)^2}\right]^N+4\left(\frac{1-|\xi|^2}{1+|\xi|^2}\right)^N\right\}\,,\end{aligned}\tag{23}$$

where ξ is given by equation (4). On the other hand, if the motional state is initially a classical mixture of equation (21) with the electronic state again being $|g\rangle$, the laser excitation leads to a probability of finding the ion in the state $|g\rangle$ of

$$P_{g,i}^{N} = \tfrac{1}{2}\left[1+\left(\frac{1-|\xi|^2}{1+|\xi|^2}\right)^N\right]\,. \tag{24}$$

Therefore, by measuring the probability of the ion in the ground electronic state after the laser excitation, we can determine whether the ion motion is in a superposition or an incoherent mixture of two SU(2) coherent states.

In order to obtain a SU(2) cat state we have assumed that the ion motion is initially prepared in a pure state $|0,N\rangle$, which is experimentally difficult. We suppose that the initial phonon number distribution is $P(M)$. Then after the second laser excitation the probability of finding the ion in the state $|g\rangle$ is

$$P_g = \sum_M P(M)P_{g,s}^{M}\,, \tag{25}$$

where

$$P_{g,s}^{M} = \frac{1}{4}\left[1 + \left(\frac{1-|\xi|^2}{1+|\xi|^2}\right)^M\right]^{-1}$$
$$\times \left\{3 + \left[\frac{\left(1-|\xi|^2\right)^2 - -4\,|\xi|^2}{\left(1+|\xi|^2\right)^2}\right]^M + 4\left(\frac{1-|\xi|^2}{1+|\xi|^2}\right)^M\right\}. \qquad (26)$$

Since $P_{g,s}^M$ depends upon M, P_g depends upon the distribution function $P(M)$.

4. Summary

In conclusion, we have proposed a scheme for preparing superpositions of two SU(2) coherent states of the motion of a trapped ion. In the scheme the ion is trapped in a two-dimensional harmonic potential and resonantly driven by two laser beams in the trap plane. With appropriate choice of the amplitudes and phases of the lasers, we can obtain the motional SU(2) cat states after the measurement of the electronic state provided the ion motion is initially prepared in a Fock state. We have also suggested a method to distinguish a superposition from incoherent mixture of two SU(2) coherent states. Finally, we have discussed the effects of the initial phonon number distribution on the observable effects resulting from the superposition state.

Acknowledgments

This work was supported by the Science Research Foundation of the Education Committee of Fujian Province and funds from Fuzhou University.

References

Agarwal, G. S. (1988). *J. Opt. Soc. Am.* B **5**, 1940.
Ansari, N. A., and Man'ko, V. I. (1994). *Phys. Rev.* A **50**, 1942.
Brune, M., Haroche, S., Raimond, J. M., Davidovich, L., and Zagury, N. (1992). *Phys. Rev.* A **45**, 5193.
Buzek, V., and Quang, T. (1987). *J. Opt. Soc. Am.* B **6**, 2447.
Chai, C. (1992). *Phys. Rev.* A **46**, 7187.
Cirac, J. I., Blatt, R., Parkins, A. S., and Zoller, P. (1993). *Phys. Rev. Lett.* **70**, 762.
de Matos Filho, R. L., and Vogel, W. (1996). *Phys. Rev. Lett.* **76**, 608.
Dodonov, V. V., Man'ko, V. I., and Nikonov, D. E. (1995). *Phys. Rev.* A **51**, 3328.
Garraway, B. M., Sherman, B., Moya-Cessa, H., Knight, P. L., and Kurizki, G. (1994). *Phys. Rev.* A **49**, 535.
Gerry, C. C. (1996). *Phys. Rev.* A **53**, 3818.
Gerry, C. C. (1997). *Phys. Rev.* A **55**, 2478.
Gerry, C. C., and Grobe, R. (1995). *Phys. Rev.* A **51**, 1698.
Gerry, C. C., and Grobe, R. (1997). *J. Mod. Opt.* **44**, 41.
Gou, S. C., and Knight, P. L. (1996). *Phys. Rev.* A **54**, 1682.
Gou, S. C., Steinbach, J., and Knight, P. L. (1996*a*). *Phys. Rev.* A **54**, R1014.
Gou, S. C., Steinbach, J., and Knight, P. L. (1996*b*). *Phys. Rev.* A **54**, 4315.
Gou, S. C., Steinbach, J., and Knight, P. L. (1997). *Phys. Rev.* A **55**, 719.
Guo, G. C., and Zheng, S. B. (1996). *Phys. Lett.* A **223**, 332.

Janszky, J., and Vinogradov, An. V. (1990). *Phys. Rev. Lett.* **64**, 2771.
Janszky, J., Domokos, P., and Adam, P. (1993). *Phys. Rev.* A **48**, 2213.
Janszky, J., Domokos, P., Szabo, S., and Adam, P. (1995). *Phys. Rev.* A **51**, 4191.
Meekhof, D. M., Monroe, C., King, B. E., Itano, W. M., and Wineland, D. J. (1996). *Phys. Rev. Lett.* **76**, 796.
Monroe, C., Meekhof, D. M., King, B. E., and Wineland, D. J. (1996). *Science* **272**, 1131.
Poyatos, J. F., Cirac, J. I., Blatt, R., and Zoller, P. (1996). *Phys. Rev.* A **54**, 1532.
Sanders, B. C. (1992*a*). *Phys. Rev.* A **45**, 6811.
Sanders, B. C. (1992*b*). *Phys. Rev.* A **46**, 2966.
Vogel, W., and de Matos Filho, R. L. (1995). *Phys. Rev.* A **52**, 4214.
Xia, Y., and Guo, G. (1989). *Phys. Lett.* A **136**, 281.
Zheng, S. B. (1998). *Phys. Rev.* A **58**, 761.

Manuscript received 19 July, accepted 28 October 1999