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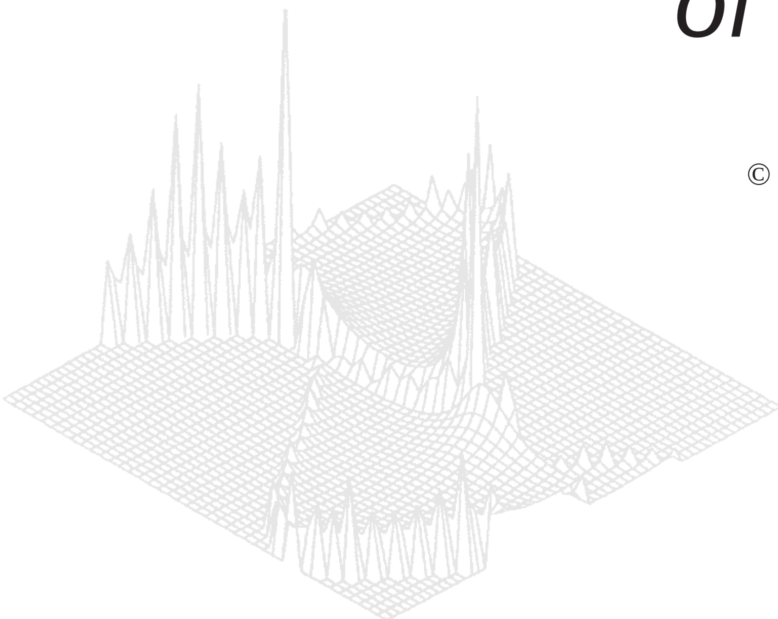

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Exact Causal Bulk Viscous Stiff Cosmologies

M. K. Mak and T. Harko

Department of Physics, University of Hong Kong,
Pokfulam, Hong Kong, People's Republic of China
mkmak@rtc.edu.hk, tcharko@hksua.hku.hk

Abstract

An exact solution of the gravitational field equations is presented for a homogeneous flat Friedmann–Robertson–Walker universe filled with a causal bulk viscous fluid obeying the Zeldovich stiff equation of state and having bulk viscosity coefficient proportional to the fourth root of the energy density.

Dissipative thermodynamic processes of bulk viscous type are supposed to play a crucial role in the dynamics and evolution of the early universe. Over thirty years ago Misner (1966) suggested that the observed large-scale isotropy of the universe is due to the action of the neutrino viscosity which was effective when the universe was about 1 second old. There are many processes capable of producing bulk viscous stresses in the early universe, such as interaction between matter and radiation, quark and gluon plasma viscosity, strings and superstrings, different components of dark matter or particle creation (Chimento and Jakubi 1996). Traditionally the theories of Eckart (1940) and Landau and Lifshitz (1987) were used for the description of these phenomena. However, the results of Israel (1976), Israel and Stewart (1976) and Hiscock and Lindblom (1989) showed that the Eckart-type theories suffer from serious drawbacks concerning causality and stability. Regardless of the choice of equation of state, all equilibrium states in these theories are unstable and in addition signals may be propagated through the fluid at velocities exceeding the speed of light (Israel 1976; Israel and Stewart 1976; Hiscock and Lindblom 1989; Hiscock and Salmonson 1991). These problems arise due to the first-order nature of the theory, i.e. it considers only first-order deviations from equilibrium. The neglected second-order terms are necessary to prevent non-causal and unstable behaviour. A relativistic second-order theory was found by Israel (1976) and developed by Israel and Stewart (1976) into what is called 'transient' or 'extended' irreversible thermodynamics. Due to the complicated nonlinear character of the evolution equations, very few exact cosmological solutions of the gravitational field equations in the framework of the full causal theory are known. For a homogeneous universe filled with a full causal viscous fluid source obeying the relation $\xi \sim \rho^{\frac{1}{2}}$, exact general solutions of the field equations have been obtained by Chimento and Jakubi (1997, 1998)

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and Mak and Harko (1998*a*, 1998*b*, 1999*a*, 1999*b*). It has also been proposed that causal bulk viscous thermodynamics can also model, on a phenomenological level, matter creation in the early universe (Mak and Harko 1998*a*, 1999*b*). Two exact classes of solutions of the field equations with $\xi \sim \rho^s$, $s \neq \frac{1}{2}$ and $1 \leq \gamma < 2$ have been obtained by Harko and Mak (1999).

The energy–momentum tensor of a bulk viscous cosmological fluid filling a flat FRW universe with a line element

$$ds^2 = dt^2 - a^2(t)(dx_1^2 + dx_2^2 + dx_3^2) \tag{1}$$

is given by Maartens (1995):

$$T_i^k = (\rho + p + \Pi)u_i u^k - (p + \Pi)\delta_i^k \,, \tag{2}$$

where ρ is the energy density, p is the thermodynamic pressure, Π is the bulk viscous pressure and u_i is the four-velocity satisfying the condition $u_i u^i = 1$. We shall use units such that $8\pi G = c = 1$. The gravitational field equations together with the continuity equation $T_{i,k}^k = 0$ imply

$$3H^2 = \rho, \quad 2\dot{H} + 3H^2 = -p - \Pi, \quad \dot{\rho} + 3(\rho + p)H = -3\Pi H \,. \tag{3}$$

The causal evolution equation for the bulk viscous pressure is given by (Maartens 1995)

$$\tau\dot{\Pi} + \Pi = -3\xi H - \tfrac{1}{2}\tau\Pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right), \tag{4}$$

where T is the temperature, ξ is the bulk viscosity coefficient and τ is the relaxation time, and we have denoted $H = \dot{a}/a$ (Hubble factor). Equation (4) arises as the simplest way (linear in Π) to satisfy the H-theorem (Israel and Stewart 1976; Hiscock and Lindblom 1989); i.e. for the entropy production to be non-negative,

$$S_{;i}^i = \frac{\Pi^2}{\xi T} \geq 0 \,,$$

where

$$S^i = sN^i - \frac{\tau\Pi^2}{2\xi T}u^i$$

is the entropy flow vector, s is the entropy per particle and $N^i = nu^i$ is the particle flow vector. In order to close the system of equations (3) and (4), we have to give the equation of state for p and specify T, τ and ξ. We shall assume the following phenomenological laws (given in Maartens 1995):

$$p = (\gamma - 1)\rho, \quad \xi = \alpha\rho^s, \quad T = \beta\rho^r, \quad \tau = \frac{\xi}{\rho} = \alpha\rho^{s-1} \,, \tag{5}$$

where $\alpha \geq 0$, $\beta > 0$, $s \geq 0$, $r \geq 0$ and $1 \leq \gamma \leq 2$ are constants. Equations (5) are standard in the analysis of bulk viscous cosmological models. The requirement that the entropy be a state function imposes the constraint $(\gamma - 1/\gamma)$ so that $0 \leq r \leq \frac{1}{2}$ for $0 \leq \gamma \leq 2$ (Chimento and Jakubi 1997, 1998).

The growth of the total comoving entropy over a proper time interval (t_0, t) is given by (Maartens 1995)

$$\Sigma(t) - \Sigma(t_0) = -\frac{3}{k}\int \frac{\Pi H a^3}{T} dt\,, \tag{6}$$

where k is Boltzmann's constant. The Israel–Stewart theory is derived under the assumption that the thermodynamical state of the fluid is close to equilibrium, that is, the non-equilibrium bulk viscous pressure should be small when compared to the local equilibrium pressure, $|\Pi| \ll p$ (Maartens 1995; Zimdahl 1996; Maartens and Mendez 1997). If this condition is violated, then one is effectively assuming that the linear theory holds also in the nonlinear regime far from equilibrium (Maartens and Mendez 1997). To see if a cosmological model inflates or not, it is convenient to introduce the deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{\rho + 3p + 3\Pi}{2\rho}.$$

The positive sign of the deceleration parameter corresponds to standard decelerating models, whereas the negative sign indicates inflation. By introducing a set of dimensionless variables h and θ by means of the transformations

$$H = \alpha_0 h, \qquad t = \frac{2}{3\alpha_0}\theta, \qquad \alpha_0 = \frac{\sqrt{3\alpha^2}}{4}\,,$$

we obtain the following evolution equation for H (Harko and Mak 1999; Maartens 1995):

$$\frac{d^2h}{d\theta^2} + (2h + h^{2(1-s)})\frac{dh}{d\theta} - \frac{1+r}{h}\left(\frac{dh}{d\theta}\right)^2 + \frac{2r-1}{1-r}h^3 + \frac{1}{1-r}h^{2(2-s)} = 0\,. \tag{7}$$

We shall assume that the causal bulk viscous cosmological fluid filling the universe obeys the Zeldovich stiff equation of state of the form $p = \rho$. Hence we suppose that $\gamma = 2$ and $r = \frac{1}{2}$. We shall suppose also that $s = \frac{1}{4}$ leading to a relation between the bulk viscosity coefficient and the energy density of the form $\xi \sim \rho^{\frac{1}{4}}$. With this choice of physical parameters, equation (7) becomes

$$\frac{d^2h}{d\theta^2} + \left(2h + h^{\frac{3}{2}}\right)\frac{dh}{d\theta} - \frac{3}{2h}\left(\frac{dh}{d\theta}\right)^2 + 2h^{\frac{7}{2}} = 0\,. \tag{8}$$

We find that equation (8) has a solution of the form

$$h = (\tfrac{2}{3})^{\frac{2}{3}}\theta^{-\frac{2}{3}}\,. \tag{9}$$

Hence we obtain the following new exact solution of the gravitational field equations for a Zeldovich-type bulk viscous fluid-filled FRW universe:

$$a = a_0 e^{3mt^{\frac{1}{3}}}, \qquad \rho = p = 3m^2 t^{-\frac{4}{3}}, \qquad \xi = (3)^{\frac{1}{4}} \alpha m^{\frac{1}{2}} t^{-\frac{1}{3}}\,, \tag{10}$$

$$T = 3^{\frac{1}{2}} \beta m t^{-\frac{2}{3}}, \qquad q = \frac{2}{3m} t^{-\frac{1}{3}} - 1,$$

$$\Sigma(t) - \Sigma(t_0) = 2\frac{\sqrt{3} m a_0^3}{k\beta}\left(\frac{e^{9mt^{\frac{1}{3}}}}{t^{\frac{2}{3}}}\right), \tag{11}$$

where a_0 and $\Sigma(t_0)$ are constants of integration and we denote $m = (\frac{4\sqrt{3}}{81}\alpha^2)^{\frac{1}{3}}$. By using the deceleration parameter, the condition of smallness of bulk viscous pressure can be expressed alternatively as $|q-2| \ll \frac{3}{2}$. The singular or non-singular character of the solution can also be obtained from a study of the singular points of the scalar invariants $R_{ij}R^{ij}$ and $R_{ijkl}R^{ijkl}$, given by

$$R_{ij}R^{ij} = \frac{12m^2}{t^{\frac{8}{3}}}\left(3m^2 - \frac{2m}{t^{\frac{1}{3}}} + \frac{4}{9t^{\frac{2}{3}}}\right),$$

$$R_{ijkl}R^{ijkl} = \frac{24m^2}{t^{\frac{8}{3}}}\left(m^2 - \frac{2m}{3t^{\frac{1}{3}}} + \frac{2}{9t^{\frac{2}{3}}}\right). \tag{12}$$

The bulk viscous universe described by the solution given above starts its evolution in the infinite past, with $t \to -\infty$. In this limit the scale factor, energy density, bulk viscosity coefficient, temperature and scalar invariants tend to zero. The universe starts to expand and there is a rapid increase in the energy density of its matter content, in the temperature and in the bulk viscosity coefficient. During this period the evolution of the Zeldovich bulk viscous fluid-filled universe is inflationary, with $q < 0$, $\forall t \in (-\infty,\ 0)$. For $t \to 0$ the universe reaches a singular state with $\rho(0) \to \infty$, $\xi(0) \to \infty$ and $T(0) \to \infty$, but with a finite scale factor $a(0) = a_0 = \text{constant}$. The invariants (12) also tend to infinity in the limit $t \to 0$. For $t > 0$ the Zeldovich fluid-filled universe continues to expand and the temperature, energy density and bulk viscosity coefficient decrease. The evolution of the universe is non-inflationary ($q > 0$) for all times $0 \leq t < t_c = (2/3m)^3$, but for times $t > t_c$, the deceleration parameter $q < 0$, hence the universe ends in an inflationary phase. During the second evolutionary era the expansion is associated with a rapid decrease in the energy density and temperature but a large amount of comoving entropy is produced.

The possibility that bulk causal thermodynamics can provide a phenomenological description of particle production processes in the very early universe has been developed in a systematic manner by Mak and Harko (1998*a*, 1999*b*). The basic features of the present solution also strongly support this (possible) physical interpretation. In the first phase of evolution a vacuum state is connected with a singular one, with infinite energy density. Hence this period corresponds to matter creation from the vacuum ('nothing'), probably as a result of some quantum processes. The matter creation process ends in a singular state ('big bang'). Thus we have obtained the possibility of a phenomenological pre-big-bang description of the evolution of the matter content of the universe modelled as a Zeldovich-type bulk viscous fluid.

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