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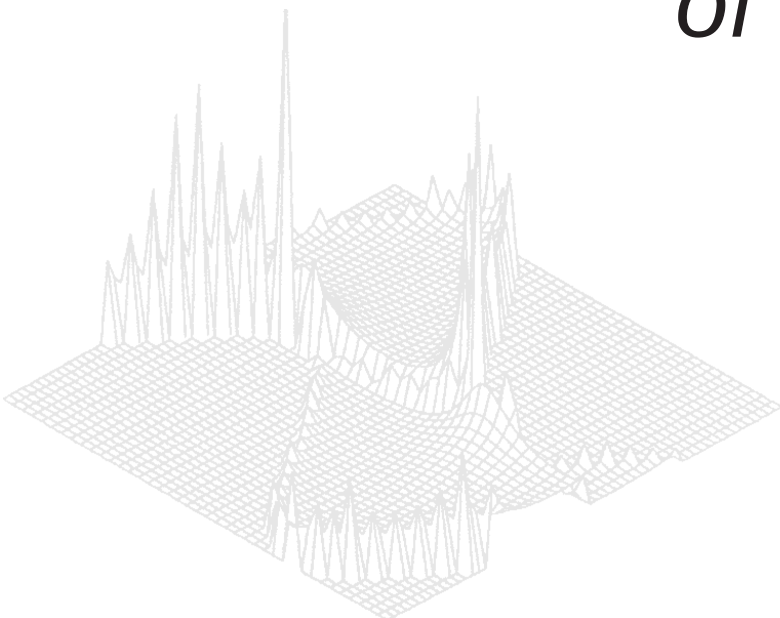

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Optical Multistability from a Squeezed Vacuum in the Presence of Quantum Interference

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Abstract

The effect of a squeezed vacuum on the optical bistable and multistable behaviour of a system of N three-level atoms is investigated, where the quantum interference between the atomic transition paths is considered. It is shown that the squeezed vacuum can profoundly affect the output field of this system. A strong squeezed vacuum can lead to multistable behaviour, even in the case where the cooperation parameter C is so small that the input–output relationship curve is monotone in the absence of the squeezed vacuum. The optical multistability is most significant when the quantum interference is perfect. The bistability and multistability can be controlled by the squeezing photon number and the strength of the two-photon correlation.

1. Introduction

Since Szöke *et al.* (1969) suggested that optical bistability can be realised by a nonlinear medium in a cavity, optical bistability and multistability have been extensively investigated (see e.g. Lugiato 1984 and references therein). In practice, these systems have great potential as devices because they can function as transistors, memory elements, pulse shapers, etc. Optical bistability and multistability, remarkable examples of cooperative behaviour (Haken 1977) in open systems far from thermal equilibrium, have also aroused wide theoretical interest.

As nonclassical states of electromagnetic fields, the significance of the squeezed state (Loudon and Knight 1987) has been discussed extensively in the literature, along with their generation, detection, and physical properties (see e.g. Walls 1983; Slusher *et al.* 1985; Wu *et al.* 1986; Orozco *et al.* 1987; Shapiro *et al.* 1986). A squeezed vacuum can be produced successfully in the laboratory. In practice, a degenerate parameter amplifier has been proved to be the most successful source of squeezed light [see e.g. the two special issues on the squeezed state published in 1987 in *J. Mod. Opt.* **34** (no. 6/7) and *J. Opt. Soc. Am.* B **4** (no. 10)]. As shown by Gardiner (1986), the squeezed vacuum leads to two different dipole decays constants, and affects the rate of the atomic population of atomic system as well. The effects of the squeezed vacuum on three-level atoms have also been examined (see e.g. Ficek and Drummond 1991, 1993; Ferguson *et al.*. 1995). Because the squeezed vacuum can have a strong effect on the atomic transition, optical bistability with squeezed vacuum input has its own distinctive features. The tunneling time, for example, with a squeezed vacuum input can be as long as 2 s, instead of around 0.1 s as in an ordinary vacuum (Savage and Walls 1986). Bergou and Zhao (1995) have investigated the effects of the squeezed vacuum input on the bistable behaviour of a system of two-level atoms and found that a squeezed (stretched) vacuum input field tends to increase (decrease) the range of optical bistability. The effects of squeezed state fields on optical

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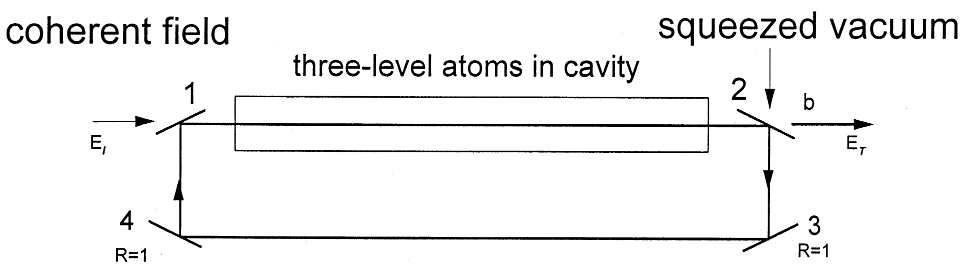

Fig. 1. Representation of the system of the cavity containing N three-level atoms with coherent pumping and squeezed vacuum input and the boundary condition for the traveling-wave cavity.

bistability have also been investigated by Singh *et al.* (1992) and Galatola *et al.* (1991). It should be noted that they have only analysed the case of two-level atoms. For multilevel systems, however, there must be some interesting new features due to their complexity.

In this paper we discuss the input–output relationship from the system of N homogeneously broadened three-level atoms in the Λ configuration which is pumped simultaneously by a coherent input field and a squeezed vacuum field (see Fig. 1). For simplicity, we assume that mirror 3 and 4 have 100% reflectivity, and call R and T (with $R + T = 1$) the reflection and transmission coefficients of mirrors 1 and 2. On one hand, our three-level system is similar to the two-level system discussed by Bergou and Zhao (1995); on the other hand, there are interesting differences between the previous study and the present one–for example, in the present paper the effect of the quantum interference between atomic transition paths on the input–output relationship is discussed.

The three-level model may be better connected with the real word than the two-level model. In our travelling-wave cavity system the coherent driving field and the squeezed vacuum field are injected via different ports, rendering the scheme more feasible experimentally than that presented by Galatola *et al.* (1991). We neglect the coupling between the coherent pumping field and the squeezed vacuum because it does not contribute to the equation of motion for the atomic variables. Making use of numerical methods, we find that the squeezed vacuum input tends to significantly increase the range of optical bistability, and even leads to optical multistability. We also find that the quantum interference effect is crucial to the multistability.

2. Maxwell–Bloch Equations

We consider a three-level atom (see Fig. 2) with one excited state $|3\rangle$, and two lower states $|1\rangle$ and $|2\rangle$. The excited state $|3\rangle$ decays to the lower-lying state $|1\rangle$ ($|2\rangle$) by spontaneous emission with rate γ_1 (γ_2). The transition frequencies are ω_1 and ω_2. These transitions are associated with electric dipole matrix elements μ and μ' (assumed real for simplicity) respectively, whereas the transition between the lower states is forbidden in the electric dipole approximation ($\mu_{12} = 0$). The coherence transfer rates γ_{12} and γ_{21} are associated with the quantum correlation between the $|1\rangle - |3\rangle$ and $|2\rangle - |3\rangle$ transition pathways, and are strongly dependent upon the orientations of the atomic dipole polarisations. In this paper we take $\gamma_{12} = \gamma_{21} = r\sqrt{\gamma_1 \gamma_2}$, where r measures the strength of quantum interference, and $0 \le r \le 1$. Here $r = 1$ corresponds to the case where the interference strength is perfect and $r = 0$ to the case where the interference is absent. The frequency of the coherent input field is ν, and the carrier frequency of the squeezed input field is ω_s. For simplicity, we shall assume that ω_s is equal to ν. The slowly varying amplitude of the coherent field is $E = |E| exp(i\phi_L)$.

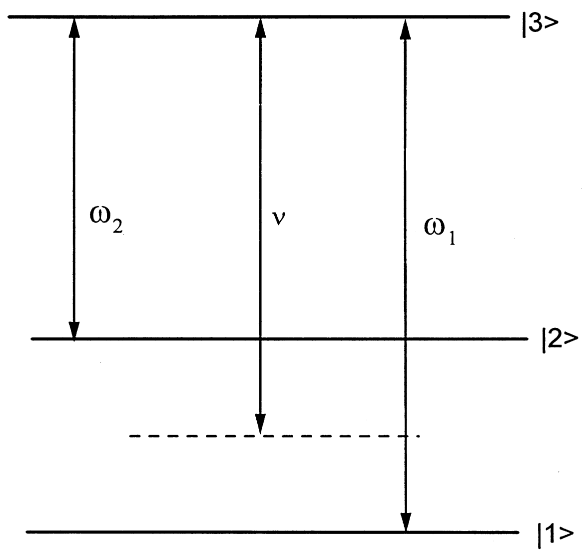

Fig. 2. Atomic level diagram.

The properties of the squeezed vacuum field are described by the expressions

$$\langle b^+(t)b(t')\rangle = n\delta(t-t'),$$
$$\langle b(t)b(t')\rangle = me^{i\phi}\delta(t-t'), \tag{1}$$

in which the parameter n is the squeezing photon number, m measures the strength of the two photon correlation, and ϕ_s is the phase of the squeezed vacuum. Here m and n obey the relation $m = \eta\sqrt{n(n+1)}$, where the quantity η measures the strength of the two-photon correlation in the squeezed vacuum ($0 \leq \eta \leq 1$). The case $\eta = 1$ corresponds to perfect correlation and $\eta < 1$ to imperfect correlation.

Using the rotating wave and Born–Markoff approximations and the Scully–Lamb technique to sum over all the atoms, the Maxwell–Bloch equations can be derived as follows:

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= (n+1)\gamma_1\rho_{33} - n\gamma_1\rho_{11} - \frac{n}{2}\gamma_{12}(\rho_{12}+\rho_{21}) - \frac{i}{\hbar}(\mu\rho_{31}E^* - \text{c.c.}),\\
\frac{\partial}{\partial t}\rho_{22} &= (n+1)\gamma_2\rho_{33} - n\gamma_2\rho_{22} - \frac{n}{2}\gamma_{12}(\rho_{12}+\rho_{21}) - \frac{i}{\hbar}(\mu'\rho_{32}E^* - \text{c.c.}),\\
\frac{\partial}{\partial t}\rho_{31} &= -\left[\frac{(2n+1)\gamma_1+(n+1)\gamma_2}{2} + i\delta_1\right]\rho_{31} - \frac{n}{2}\gamma_{12}\rho_{32} - \gamma_1 m\rho_{13}\\
&\quad -\frac{i}{\hbar}\mu(\rho_{11}-\rho_{33})E - \frac{i}{\hbar}\mu'\rho_{21}E - \gamma_{12}m\rho_{23},\\
\frac{\partial}{\partial t}\rho_{32} &= -\left[\frac{(2n+1)\gamma_2+(n+1)\gamma_1}{2} + i\delta_2\right]\rho_{32} - \frac{n}{2}\gamma_{12}\rho_{31} - \gamma_2 m\rho_{23}\\
&\quad -\frac{i}{\hbar}\mu'(\rho_{22}-\rho_{33})E - \frac{i}{\hbar}\mu\rho_{12}E - \gamma_{12}m\rho_{13},\\
\frac{\partial}{\partial t}\rho_{12} &= -\left[\frac{n}{2}(\gamma_1+\gamma_2) - i\omega_{21}\right]\rho_{12} + (n+1)\gamma_{12}\rho_{33} - \frac{n}{2}\gamma_{12}(\rho_{11}+\rho_{22})\\
&\quad -\frac{i}{\hbar}(\mu\rho_{32}E^* - \mu'\rho_{13}E),
\end{aligned} \tag{2}$$

$$\frac{\partial E}{\partial t} + c\frac{\partial E}{\partial z} = 2\pi i \nu N(\mu^2 \rho_{31} + \mu'^2 \rho_{32}). \tag{3}$$

Here $\omega_{21} \equiv \omega_2 - \omega_1$, and the frequency detuning is $\delta_1 \equiv \omega_1 - \nu$, $\delta_2 \equiv \omega_2 - \nu$. For perfect resonance and steady state, taking into account $\langle b \rangle = 0$, we can obtain the boundary conditions as follows:

$$\begin{aligned} E(L) &= E_T/\sqrt{T}, \\ E(0) &= \sqrt{T}E_I + RE(L), \end{aligned} \tag{4}$$

where E_I and E_T are the input field and the output field respectively and L is the length of the atomic sample.

In order to investigate the bistable behaviour for this ring cavity, similar to Harshawardhan and Agarwal (1996), we give the absorption coefficient α for the Λ medium as

$$\alpha = \frac{4\pi N \nu \mu^2}{\hbar c \gamma}, \tag{5}$$

where

$$\gamma = \gamma_1 + \gamma_2. \tag{6}$$

Furthermore, the usual cooperation parameter C is defined as $C \equiv \alpha L/2T$.

Setting all these derivatives with respect to time in equations (2) equal to zero for the steady state, and taking $\mu' = \mu$ for simplicity in the following, we can obtain the field E as

$$\frac{\partial E}{\partial z} = 2\pi i \nu N \mu^2 (\rho_{31} + \rho_{32})/C, \tag{7}$$

where $\rho_{31} + \rho_{32}$ can be obtained from the steady-state solution of equations (2) and depends on the atomic decay rates, the Rabi frequency of the coherent field and the squeezed vacuum field, as well as the frequency detunings.

In the mean-field limit (Gong *et al.* 1997), using the boundary conditions (4), and normalising the fields by letting $y = \mu E_I/\gamma\sqrt{T}$, and $x = \mu E_T/\gamma\sqrt{T}$ we can get the input–output relationship

$$y = x + C\gamma(\rho_{31} + \rho_{32}). \tag{8}$$

In the following section, we will use numerical and graphical methods to investigate the effect of the squeezed vacuum on the bistable behaviour according to the steady-state solution of equations (2) and the input–output relationship (8).

3. Numerical Results

Numerical analyses show that the squeezed vacuum input can dramatically change the output field from the three-level atomic system. It can significantly increase the range of the optical bistability and cause optical multistability.

Fig. 3 presents the effect of a perfect squeezed vacuum input on the input–output relationship with a small cooperation parameter $C = 5$. It is easy to see that in the case of the usual vacuum input $n = 0$ there is no bistable or multistable behaviour (see curve 1 in Fig.

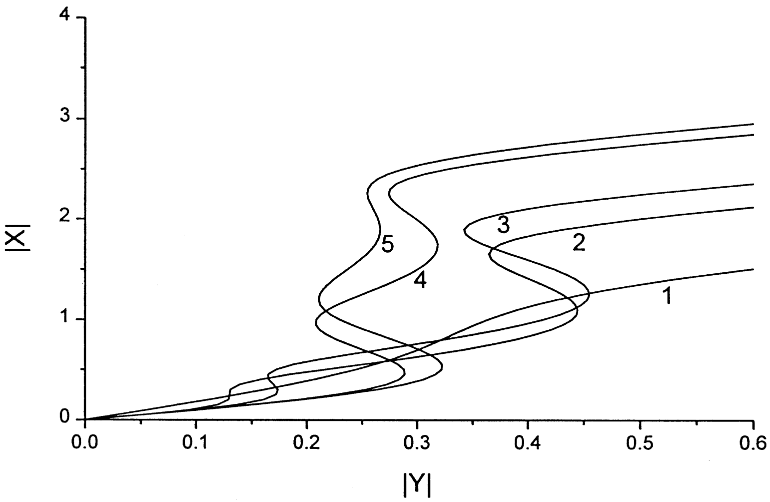

Fig. 3. Input–output relationship with normal vacuum (n = 0) and perfect correlation squeezed vacuum input for $C = 5$, $\gamma_1 = 0.2\gamma$, $\gamma_2 = 0.8\gamma$, $r = 1$, $\delta_1 = 2.2\gamma$ and $\delta_2 = -2.2\gamma$. Curve 1 corresponds to $n = 0$, curve 2 to $n = 0.05$, curve 3 to $n = 0.1$, curve 4 to $n = 0.3$ and curve 5 to $n = 0.37$.

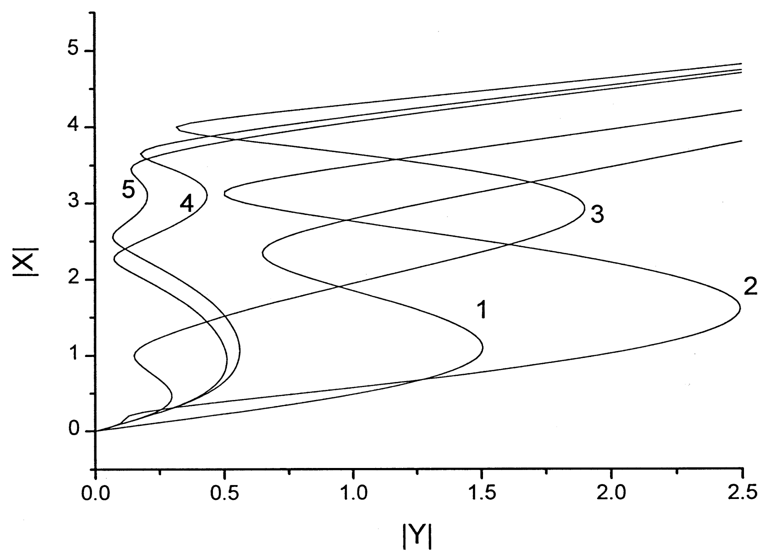

Fig. 4. Input–output relationship with normal vacuum (n = 0) and perfect correlation squeezed vacuum input for $C = 9$, $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.9\gamma$, $r = 1$ $\delta_1 = 2.2\gamma$ and $\delta_2 = -2.2\gamma$. Curve 1 corresponds to $n = 0$, curve 2 to $n = 0.1$ curve 3 to $n = 1$, curve 4 to $n = 2$ and curve 5 to $n = 2.2$.

3). In the case of the squeezed vacuum input $n \neq 0$, however, bistable or multistable behaviour appears. When the squeezed vacuum field is weak (i.e. n is small), only bistability appears, and in this case the hysteresis cycle increases with n. When the squeezed vacuum is strong enough, multistability appears. The feature of the optical multistability is dependent on the intensity of the squeezed vacuum. This is because the squeezed vacuum input field has a strong effect on the atomic transitions due to the two-photon correlation of the field. This can also be seen from Figs 4 and 6.

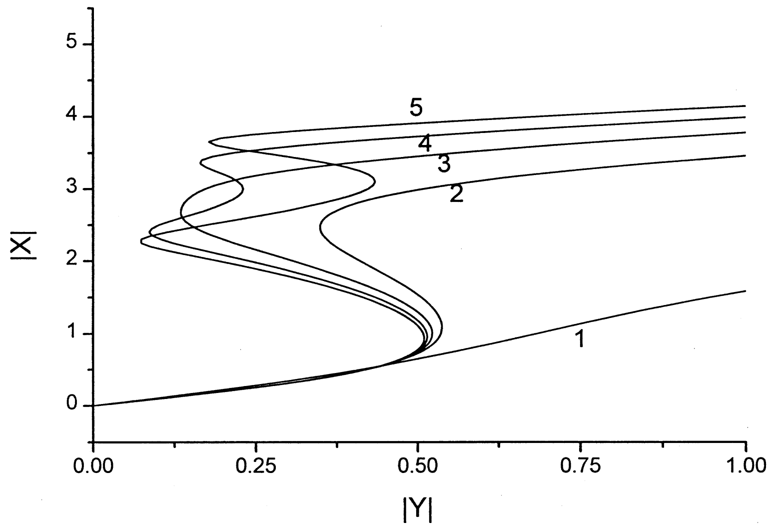

Fig. 5. Effect of quanum interference on multistability in the presence of a perfect squeezed vacuum input. Curve 1 corresponds to $r = 0$, curve 2 to $r = 0.9$, curve 3 to $r = 0.95$, curve 4 to $r = 0.98$ and curve 5 to $r = 1$, where $C = 9$, $n = 2$, $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.9\gamma$, $\delta_1 = 2.2\gamma$ and $\_{2} = \delta 2.2\gamma$.

Fig. 4 presents the strong effect of the squeezed vacuum input on the optical bistability and the multistability in the case of larger C, where optical bistability appears even in the case of the usual vacuum input ($n = 0$). It is easy to see that in the case of small n, with an increase in the squeezing parameter n, the hysteresis cycle becomes wider. In the case where n is large enough, significant optical multistability appears. This is also because the squeezed vacuum input has a strong effect on the atomic transitions due to two-photon correlation of the field.

Figs 3 and 4 show that a strong squeezed vacuum input can lead to multistability. These figures also show that one can control the optical multistability and optical bistability by the squeezing photon number n.

Fig. 5 shows the effect of quantum interference on the multistability in the presence of a perfect squeezed vacuum input. It is easy to see from this figure that the quantum interference in this three-level system has a strong effect on the input–output relationship. In the case of $n = 2$, $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.9\gamma$, $\delta_1 = 2.2\gamma$ and $\delta_2 = -2.2\gamma$, there is no bistability and multistability in the absence of quantum interference $r = 0$. In the case of $r = 0.9$, obvious bistability appears and the hysteresis cycle becomes wider when the quantum interference strength increases. When r approaches its maximal value 1 (see e.g. $r = 0.98$), obvious multistability appears. This figure shows that the quantum interference between different atomic transition paths is crucial to the optical multistability.

Fig. 6 represents the input–output relationship with a perfect and imperfect correlation squeezed vacuum input, where $C = 9$, $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.9\gamma$, $r = 1$, $\delta_1 = 2.2\gamma$ and $\delta_2 = -2.2\gamma$. It is easy to see that optical bistability and multistability is sensitive to the strength of two-photon correlation of the squeezed vacuum η. Significant multistability appears in the case where the squeezed vacuum is perfect ($\eta = 1$) or nearly perfect ($\eta \simeq 1$).

With a decrease in η, the multistability and bistability disappear. Fig. 6 shows that the two-photon correlation of the squeezed vacuum has a strong effect on the multistability

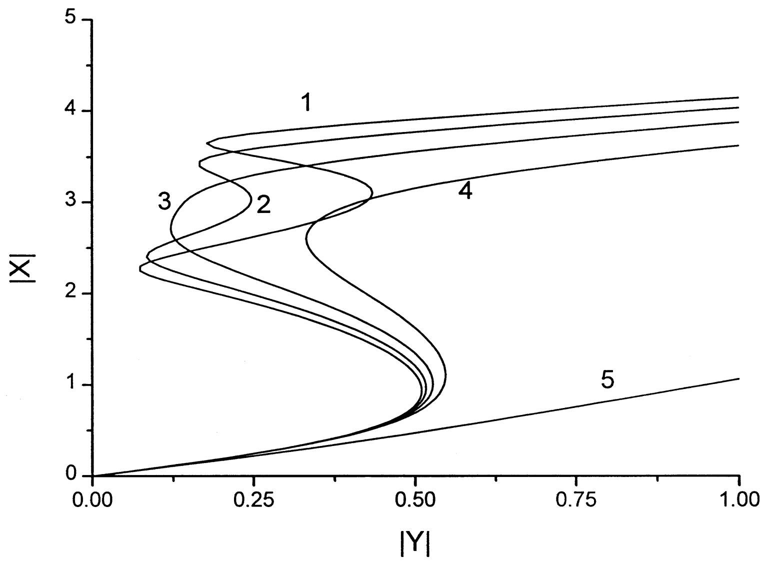

Fig. 6. Input–output relationship with perfect ($\eta = 1$) and imperfect ($\eta < 1$) correlation squeezed vacuum input for $n = 2$. Curve 1 corresponds to $\eta = 1$, curve 2 to $\eta = 0.998$, curve 3 to $\eta = 0.995$, curve 4 to $\eta = 0.990$ and curve 5 to $\eta = 0$, where $C = 9$, $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.9\gamma$, $r = 1$, $\delta_1 = 2.2\gamma$ and $\delta_2 = -2.2\gamma$.

and the bistability. Thus, optical multistability and bistability can be controlled by the intensity of the two-photon correlation of the squeezed vacuum. In addition, all numerical results show that the multistability appears only in the case of a nearly perfect squeezed vacuum, which shows that the multistability is a nonclassical effect.

4. Summary

We have investigated the optical bistable and multistable behaviour of a system of N homogeneously broadened Λ-type three-level atoms pumped by a coherent input field and coupled to a squeezed vacuum field. Using numerical calculations, we find that the squeezed vacuum input can not only significantly increase the range of the optical bistability, but can also cause optical multistability. The squeezed vacuum can cause optical multistability even in the case of a small cooperation parameter C, where neither optical bistable nor multistable behaviour appears in the absence of a squeezed vacuum input. We also found that the quantum interference between atomic transition paths is crucial to optical multistability. Significant multistability appears only in the case of perfect quantum interference. Properties of the bistable and multistable behaviour of such a system can be controlled by the squeezing number n and two-photon correlation m of the squeezed vacuum. Our model is suitable for controlling optical bistability and multistability.

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