SHORT COMMUNICATIONS

TORTUOSITY CONCEPTS*

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It is well known that the application of the Kozeny (1927) equation to the flow of fluids through porous media involves the use of the Kozeny constant k which can be determined from flow measurements. Then since the Kozeny equation as normally derived involves a factor $(L_e/L)^2$, it has been customary to split this constant into two parts, shape factor k_0, and tortuosity $T=(L_e/L)^2$ (Rose and Bruce 1949).

Thus

$$k=k_0T=k_0(L_e/L)^2, \qquad (1)$$

where L_e is the effective fluid flow path length and L the length of the plug packing. T can be obtained from electrical resistivity measurements provided the electrical and fluid flow paths are identical.

There are two concepts concerning the area available for flow, Wyllie and Rose (1950) assuming that it is $A_1=\varphi A$ (A is the cross-sectional area of the plug and φ is the porosity), whilst Cornell and Katz (1953) and O'Connor, Street, and Buchanan (1954), following the Slawinski (1926) concept,‡ assume that it is $A_2=\varphi AL/L_e$.

From Wyllie and Rose's approach, the resistance of a porous medium is $\rho L_e/\varphi A$, and since the resistance of a volume of fluid of the same overall dimensions is $\rho L/A$, hence

$$F=(\rho L_e/\varphi A).(A/\rho L)$$
$$=(1/\varphi).(L_e/L),$$

or

$$(F\varphi)^2=(L_e/L)^2. \qquad (2)$$

From the Cornell and Katz approach, because the resistance of the porous medium is $\rho L_e^2/\varphi AL$ it follows that,

$$F\varphi=(L_e/L)^2, \qquad (3)$$

where F is the "formation factor", that is, the ratio of the resistivity of a porous

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‡ Slawinski's approach is, the volume of pore space can be given either by multiplying the volume of the plug by its porosity (i.e. $LA.\varphi$) or by multiplying a hypothetical pore area A_2, by a hypothetical pore length L_e, thus, pore volume is

$$LA.\varphi=L_eA_2,$$

that is,

$$A_2=\varphi AL/L_e.$$

medium saturated with an electrolyte solution, to the resistivity ρ of the electrolyte solution.

Wyllie and Gregory (1955) concluded after extensive experimental work that $(F\varphi)^2$ is preferable to $F\varphi$ as a measure of tortuosity since less variation then occurs in their calculated shape factor.

However, their approach is inconsistent since the comparison should not be between tortuosity as calculated by $F\varphi$ and tortuosity as calculated by $(F\varphi)^2$, but rather between the concepts of $A_1=\varphi A$ and $A_2=\varphi AL/L_e$. Now if either of these concepts is used in determining tortuosity from electrical measurements then the same concept should be used in the derivation of the Kozeny equation.

Thus in the derivation given by Wyllie and Spangler (1952 ; see also Carman 1948) the fluid velocity through a porous medium u_e is greater than the approach velocity u, because the actual path length is greater than the plug length in the ratio L_e/L, and the pore area is less than the cross-sectional area either in the ratio A_1/A or A_2/A.

If the area is A_1 then

$$u_e=u(A/\varphi A)(L_e/L)=(u/\varphi)(L_e/L), \qquad \ldots\ldots\ldots\ldots \quad (4)$$

whilst if the area is A_2 then

$$u_e=u(AL_e/L\varphi A)(L_e/L)=(u/\varphi)(L_e/L)^2. \qquad \ldots\ldots\ldots\ldots \quad (5)$$

From equations (4) and (2), Wyllie and Spangler show that the equation for the approach velocity becomes

$$u=(\varphi m^2/k_0)(P/\eta L)(L/L_e)^2. \qquad \ldots\ldots\ldots\ldots \quad (6)$$

On the other hand, using the concept that $A_2=\varphi AL/L_e$ in the Wyllie and Spangler derivation gives as the final equation,

$$u=(\varphi m^2/k_0)(P/\eta L)(L/L_e)^3, \qquad \ldots\ldots\ldots\ldots \quad (7)$$

where m is the hydraulic radius, η is the fluid viscosity, and P is the pressure differential causing flow.

We can compare experimental flow results with equations (6) and (7) using electrical measurements to calculate the $(L/L_e)^n$ term by either (2) or (3). Wyllie and Gregory in doing this have used firstly, equation (2) as a measure of $(L_e/L)^2$ and then compared it with (3) as a measure of $(L_e/L)^2$ without in the second case also using the Slawinski concept in the derivation of (5) and (7).

Actually, comparison between (6) and (7) gives a direct comparison between the two concepts $A_1=\varphi A$ and $A_2=\varphi AL/L_e$ thus using the appropriate equation (2) in (6) the $(L_e/L)^2$ can be replaced by $(F\varphi)^2$, on the other hand, use of the appropriate equation (3) in (7) shows that the term $(L_e/L)^3$ should be replaced by $(F\varphi)^{1\cdot 5}$.

Hence the true comparison, which must be between the two concepts applied to both derivations, is between $(F\varphi)^2$ and $(F\varphi)^{1\cdot 5}$, not between $(F\varphi)^2$ and $F\varphi$.

It is not presently possible to decide finally between the relative merits of the two concepts in the absence of some independent method of determining the shape factor.

An attempt was made by Faris *et al.* (1954) and Faris (1955) to compare measured permeabilities with those calculated from pore-size distributions determined by a mercury injection method; then

$$\text{permeability} = \frac{126 \cdot 6 \varphi \overline{r^2}}{(F\varphi)^n},$$

where $\overline{r^2}$ is a function determined by the distribution of pore radii (Burdine, Gournay, and Reichertz 1950).

This comparison indicated that the best agreement between measured and calculated values occurs when the exponent n is between 1·4 and 1·5, thus lending some support to the Slawinski concept.

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