

Abstract:

This paper presents a new mathematical model to predict steady-state external filter cake thickness distribution and velocity profile along wellbore during overbalanced drilling. Several models have been suggested for prediction of external cake thickness using force balance method. Yet, a comprehensive literature survey reveals that electrostatic forces and permeate force correction factor have been neglected; while both can significantly change the conditions of particle detachment from the cake surface. Torque balance of hydrodynamic (lifting, tangential and permeate drag), gravity and electrostatic (DLVO) forces along with Darcy's law and material balance is used to investigate the conditions of particle attachment/detachment on the cake surface. The results show strong effect of mud chemistry, particle size, cake permeability, tangential flow velocity, overbalance pressure and Young's modulus on external filter cake thickness and velocity profile. The mathematical model can be applied as a predictive tool for estimation of filter cake thickness. It allows calculation of external filter cake distribution using physiochemical properties of mud and particles.

1. Introduction

Several experimental studies have shown an initial stage of high fluid loss followed by constant mud invasion rate during dynamic filtration of drilling mud. Similar results have been reported during cross-flow filtration in membrane studies.

Deposition of suspended particles and consequent growth of external filter cake during dynamic circulation of drilling fluid is controlled by colloidal forces exerted on the particle at the cake surface.

Several studies have shown that the permeate drag force (F_p) on an approaching particle to the impermeable surface becomes infinite at small gaps i.e. very close to the cake surface. Consequently, the permeate force increases as an inverse function of the separation gap and must be modified by a correction factor.

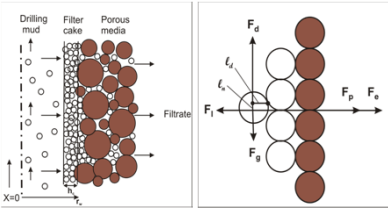


Fig 1. Dynamic filtration Fig 2. Forces and levers

Jiao and Sharma (1994) proposed a mathematical model using torque balance analysis. The proposed model ignores the electrostatic force and assumes permeate force correction factor equal to unity. Deposition-erosion model presented by Civan (1998) also ignores electrostatic force in calculation of critical shear stress.

Similar models and assumptions have been used to predict external filter cake profile in water injection wells. Electrostatic force is a major attaching force in high salinity and low pH solution conditions. Moreover, a correction factor for permeate force is also assumed to be unity. Neglecting the variation of permeate force correction factor results in underestimation of the permeate force. It significantly changes the condition of particle stability on the cake surface, which can encourage an incorrect prediction of cake thickness.

Despite significant roles of filter cake thickness in drilling operations, it has received very little attention in the literature. Prediction of cake thickness is important for cake permeability calculations, fluid loss estimation and formation damage analysis. A mathematical model based on torque balance of detaching and attaching forces is developed in this paper. The model includes all affecting forces and accounts for both electrostatic force and permeate force correction factor variations. Steady-state external filter cake profile along a vertical well is calculated and sensitivity analysis to hydrodynamic and physiochemical parameters are presented.

2. Forces and Torques

Figure 2 presents a schematic of all forces and corresponding lever arms exerting a single particle on the cake surface in a hydrodynamic flow field. Hydrodynamic tangential drag force (F_d) from tangential flow (cross-flow) of suspension fluid (shear stress), permeate (normal) force (F_p) from filtrate flux, hydrodynamic lift force (F_l), net gravitational force (F_g) and electrostatic force (F_e) are acting on a particle.

Tangential drag force

The tangential drag force (F_d) exerted on a spherical particle in contact with a plane wall is determined by local flow field of fluid from modified Stokes law and is given by :

$$F_d = 6(1.9)\pi\mu r_p u_t = 10.2\pi r_p^2 \tau$$

$$\tau = k' \gamma^n, \gamma = \left(\frac{4\bar{u}_t}{r_w - h_c} \right)$$

where μ is the carrier fluid viscosity, r_p is the particle radius, u_t is the fluid velocity at the distance r_p measured from the cake surface, τ is wall shear stress for non-Newtonian power law drilling mud, k' and n' are consistency constant and flow index respectively, γ is shear rate, r_w is the well radius and h_c is external filter cake thickness.

Lift force

The lifting force results from a gradient in the shear flow and acts normal to and away from the cake surface.

$$F_l = \chi \gamma r_p^3 (\rho_f \tau)^{1/2}$$

χ is lifting coefficient and ρ_f is fluid density.

Permeate drag force

The hydrodynamic permeate drag force can also be presented by the modified Stokes law in terms of particle Reynolds number.

Several studies have shown that the drag force on an approaching particle to the impermeable surface becomes infinite at small gaps. Consequently, the drag force must be modified by a correction factor. The modified form of permeate drag force is expressed by:

$$F_p = \pi r_p^2 \mu_p \frac{1}{2} \rho_f \frac{24}{(\text{Re})_p} \Phi_H$$

$$(\text{Re})_p = \frac{2n' r_p^{n'} u_p^{2-n'} \rho_f}{3^{n'-1} k'}$$

$$F_p = 12\pi r_p^{2-n'} \frac{3^{n'-1}}{2n'} u_p^{n'} \Phi_H, \Phi_H = 0.36 \left(\frac{k_c}{r_p^2} \right)^{\left(\frac{2}{3} \right)}$$

where u_p is the permeate velocity, $(\text{Re})_p$ is the particle Reynolds number and Φ_H is the correction factor to permeate force and k_c is the external filter cake permeability.

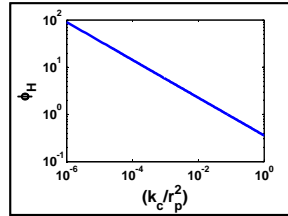


Fig 3. variation of permeate correction factor

Net gravitational force

The net gravitational force exerted on the particle is determined by:

$$F_g = \frac{4}{3} \pi r_p^3 (\rho_p - \rho_f) g$$

Electrostatic force

Well-known DLVO theory is commonly used to describe and estimate net interaction energy and electrostatic force of colloid-surface interaction. The electrostatic force is derivative of the net total potential energy

$$F_e = - \frac{\partial V}{\partial h}$$

where the total energy (V) is the sum of the London-van-der-Waals, double electric layer and Born potentials, given by so-called DLVO (Derjagin-Landau-Verwey-Overbeek) theory.

$$V_{DLV} = \frac{A_{32}}{6} \left[\frac{2(1+Z)}{Z(2+Z)} + \ln \left(\frac{Z}{2+Z} \right) \right]; Z = \frac{h}{r_p}$$

$$V_{BR} = \frac{e_0 D r_p}{4} \left[2\psi_{01} \psi_{02} \ln \left(\frac{1 + \exp(-\lambda h)}{1 - \exp(-\lambda h)} \right) - (\psi_{01}^2 + \psi_{02}^2) \ln(1 - \exp(-2\lambda h)) \right]$$

$$V_{BR} = \frac{A_{32}}{7560} \left(\frac{\sigma_{DL}}{r_p} \right)^6 \left[\frac{8+Z}{(2+Z)^7} + \frac{6-Z}{Z^7} \right]$$

$$V = V_{DLV} + V_{DLR} + V_{BR}$$

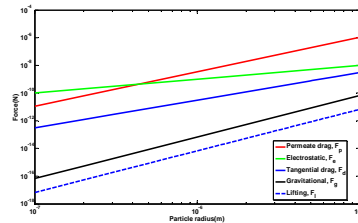


Fig 4. Comparison of forces vs particle size (typical hydrodynamic and chemical conditions)

3. Detachment mechanisms

Lifting

$$F_l \geq (F_p + F_g)$$

Sliding

$$(F_d - F_g) = \mu_f (F_p + F_e - F_l)$$

μ_f is static friction coefficient.

Rolling

$$(F_d - F_g) l_d = (F_p + F_e - F_l) l_n$$

l_d and l_n are detaching and normal levers respectively.

4. Equilibrium Cake Thickness

Darcy's law in radial geometry

$$q_p(x) = \frac{2\pi k_o (P_w - P_r)}{\mu \ln \left(\frac{r_w}{r_c} \right)} \left[\frac{k_o}{k_c} \frac{\ln \left(\frac{r_w}{r_w - h_c(x)} \right)}{\ln \left(\frac{r_c}{r_w} \right)} \right]^{-1}$$

$$u_p(x) = \frac{q_p(x)}{2\pi(r_w - h_c(x))}$$

$$\frac{\partial Q_t}{\partial x} = -2\pi(r_w - h_c(x))u_p(x)$$

k_o =reservoir permeability, q_p =filtration flux per unit of reservoir thickness, Q_t =tangential flow rate, P_r =reservoir pressure, P_w =well pressure

Lever arms determination (Fig. 5)

$$l_n = \left(\frac{(F_e + F_p - F_l) r_p}{K} \right)^{\left(\frac{1}{3} \right)}, l_d = r_p$$

$$K = \frac{4}{3\pi \left(\frac{1-\sigma_p^2}{\pi E_p} + \frac{1-\sigma_c^2}{\pi E_c} \right)}$$

E and σ are particle Young modulus and Poisson's ratio respectively.

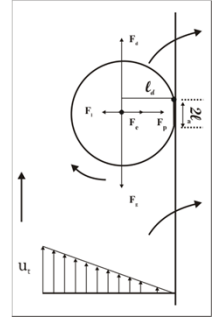


Fig 5. Lever arms

Equilibrium cake thickness:

$$\left(10.2\pi r_p^2 k' \left(\frac{4\bar{u}_t}{(r_w - h_{cr}(x))} \right)^n - \frac{4}{3} \pi r_p^3 (\rho_p - \rho_f) g \right) l = \left(12\pi r_p^{2-n'} \frac{3^{n'-1}}{2n'} u_p^{n'}(x) \Phi_H + F_e \right) l = \frac{l_d}{l_n}$$

5. Results

cake thickness profile along the well

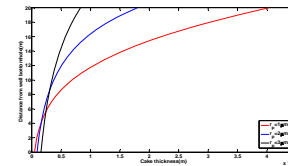


Fig 6. Effect of particle size

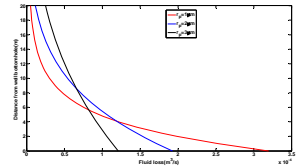


Fig 7. Variation of fluid loss along the well

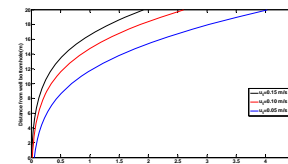


Fig 8. Effect of tangential velocity

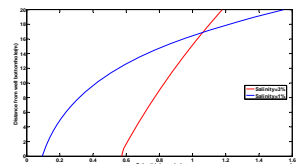


Fig 9. Effect of salinity

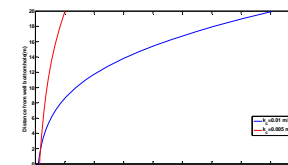


Fig 10. Effect of cake permeability

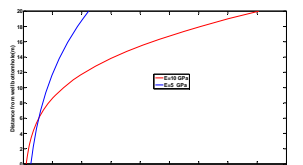


Fig 11. Effect of particle Young modulus

6. Conclusions

Ignoring the permeate force correction factor results on underestimating permeate force and consequently incorrect cake profile prediction. Cake thickness increases from bottom towards the top of the formation yielding in high fluid loss close to the bottom of the drilled formation. Using small particle sizes in the drilling mud results in creation of thicker external cake except along small distance close to the bottom of formation. Increase of mud flow velocity inside the wellbore decreases the external cake thickness and consequently increase of mud loss into the drilled formation. Low permeable cake creates high hydraulic resistance against mud flow into the drilled formation. Hence, the thickness of low permeable cake is smaller in comparison to that for high permeable cake.

The detaching torque is a function of particle deformation on the cake surface which depends on Young modulus of particle. Deformation of particles with small Young modulus yields in lower lever arm ratio comparing to that for particles with large Young modulus. Therefore, at the same flow velocity particles with small Young modulus are detached harder than particles with large Young modulus.

7. Selected References

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