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Supplementary Material

A quantitative method for evaluating ecological risks associated with long-term degradation of deep-sea plastic-containing infrastructure

Alexander N. Testoff[A,*], *Nicholas A. Nelson*[A] and *Joseph P. Nicolette*[A]

[A]Montrose Environmental Solutions, Sandy Springs, GA 30350, USA

*Correspondence to: Email: atestoff@montrose-env.com

Supporting Information. Degradation Rate Model Derivation

Plastic mass remaining at time *t* based on integration and rearrangement of differential equation.

$$-\frac{dm}{dt} = k_d \rho S_A$$

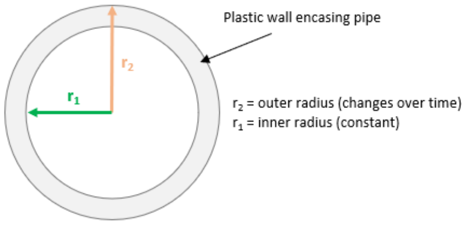
(Chamas *et. al.*, 2020)

$$-\frac{dm}{dt} = k_d \left(\frac{m}{V}\right) S_A$$

$$-\frac{dm}{dt} = k_d m \left(\frac{2r_2}{r_2^2 - r_1^2}\right)$$

$$-\frac{dm}{dt} = k_d m \left[\frac{2\left\{(\frac{m}{\rho\pi l} + r_1^2)^{1/2}\right\}}{\left(\frac{m}{\rho\pi l} + r_1^2\right) - r_1^2}\right]$$

Cross Section



Surface Area: Volume Ratio

$$\frac{S_A}{V} = \frac{2\pi r_2 l}{\pi l\,(r_2^2 - r_1^2)} = \frac{2r_2}{(r_2^2 - r_1^2)}$$

**Note: Degradation examined along some length (l) of flowline exposed to environment, not at the ends of the flowline.* *

r_2 in Terms of Mass:

$$V = \pi l\,(r_2^2 - r_1^2)$$

$$\frac{m}{\rho} = \pi l\,(r_2^2 - r_1^2)$$

$$r_2^2 = \frac{m}{\rho\pi l} + r_1^2 \;\rightarrow\; r_2 = (\frac{m}{\rho\pi l} + r_1^2)^{1/2}$$

$$-\frac{dm}{dt} = 2k_d\rho\pi l\left(\frac{m}{\rho\pi l} + r_1^2\right)^{1/2}$$

$$\int -\left(\frac{m}{\rho\pi l} + r_1^2\right)^{-\frac{1}{2}} dm = \int 2k_d\rho\pi l\; dt$$

$$-\int_{m_i}^{m_t}\left(\frac{m}{\rho\pi l} + r_1^2\right)^{-\frac{1}{2}} dm = \int_0^t 2k_d\rho\pi l\; dt$$

(U-substitution, see calculation on right)

U-Substitution

$$u = \frac{m}{\rho\pi l} + C\;;\; \frac{du}{dm} = \frac{1}{\rho\pi l} \rightarrow dm = \rho\pi l\; du$$

$$-\int\left(\frac{m}{\rho\pi l} + r_1^2\right)^{-\frac{1}{2}} dm$$

$$-\rho\pi l \int (u)^{-\frac{1}{2}}\, du = -2\rho\pi l\, u^{\frac{1}{2}} =$$

$$-2\,\rho\pi l\left(\frac{m}{\rho\pi l} + r_1^2\right)^{1/2}$$

$$-2\,\rho\pi l\left\{\left(\frac{m_t}{\rho\pi l} + r_1^2\right)^{\frac{1}{2}} - \left(\frac{m_i}{\rho\pi l} + r_1^2\right)^{\frac{1}{2}}\right\} = (2k_d\rho\pi l)t$$

(cont. next page)

$$\left(\frac{m_t}{\rho\pi l} + r_1^2\right)^{\frac{1}{2}} - \left(\frac{m_i}{\rho\pi l} + r_1^2\right)^{\frac{1}{2}} = -k_d t$$

$\left(\frac{m_t}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}=\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t$	
$\left(\frac{m_t}{\rho\pi l}+r_1^2\right)=\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t\right\}^2$	
$\frac{m_t}{\rho\pi l}=\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t\right\}^2-r_1^2$	
$\boldsymbol{m_t=\rho\pi l\left[\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t\right\}^2-r_1^2\right]}$	
Time for complete degradation (t_d) of plastic.	
$m_t=\rho\pi l\left[\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t\right\}^2-r_1^2\right]$	$m_t=0\ at\ t_d$
$0=\rho\pi l\left[\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t_d\right\}^2-r_1^2\right]$	
$0=\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t_d\right\}^2-r_1^2$	
$r_1^2=\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t_d\right\}^2$	
$r_1=\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-k_d t_d$	
$k_d t_d=\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-r_1$	
$\boldsymbol{t_d=\frac{1}{k_d}\left\{\left(\frac{m_i}{\rho\pi l}+r_1^2\right)^{\frac{1}{2}}-r_1\right\}}$	