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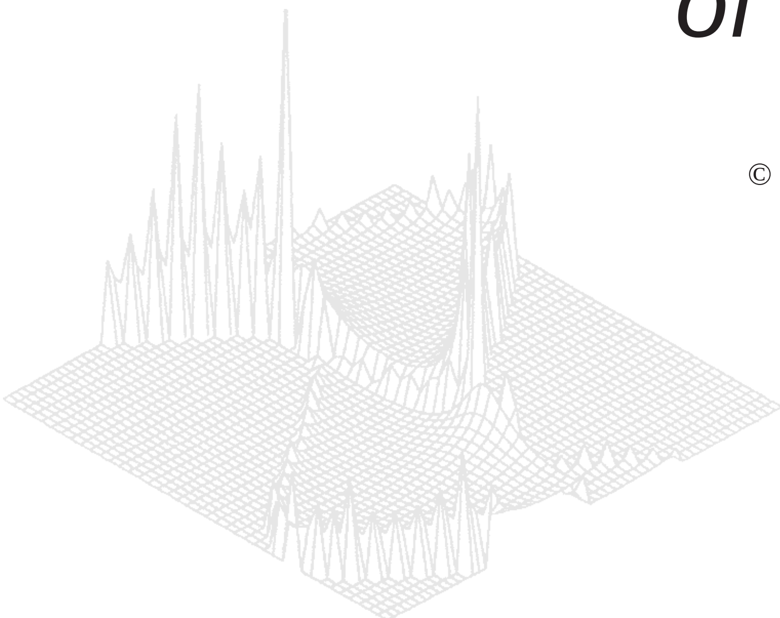

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An Atom-optical Maxwell Demon

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Abstract

The separation, by an optical standing wave, of a beam of two-level atoms prepared in a thermal mixture of ground and excited states, is considered as an example of a Maxwell demon. By including the momentum exchanged with the cavity, it is shown how no violation of the second law is possible. A classical and quantum analysis is given which illustrates this principle in some detail.

1. Introduction

Maxwell's demon is a hypothetical information gathering and processing device that can apparently extract work in a closed cycle from the thermal motion of gas molecules, in apparent contradiction of the second law of thermodynamics. The demon uses information of the velocity of a molecule to open or close a door between two gas chambers; fast (hot) molecules are allowed to pass through, slow (cold) molecules are not. The resulting temperature difference between the two chambers can then be used to extract useful work form the system. As Bennett (1987) has explained however, the demon cannot violate the second law as it must erase information in order to work in a cycle. The erasure of information necessarily requires that a minimum amount of heat be dissipated into the environment. This later result is the content of Landauer's (1961) principle, which states that a minimum of $k_B T \ln 2$ energy is dissipated into an environment at temperature T when a single bit of information is erased. A computer simulation of this process has been given by Skordas and Zurek (1992).

In this paper I consider another form of Maxwell's demon in which the thermal degrees of freedom are the internal electronic states of two-level atoms in an atomic beam. An atom in the excited state $|2\rangle$ is 'hot' and an atom in the ground state $|1\rangle$ is 'cold'. The measurement of which state the atom is in is carried out by the atom-optical Stern–Gerlach effect (Sleator *et al.* 1992). In this process the atom is passed transversely through a standing wave optical field. If the field is well detuned from the atom, virtual two-photon transitions transfer momentum from the field to the atom, while leaving the electronic state of the atom unchanged. The sign of the momentum transfer depends on what state the atom is in as it enters the beam. The result is a deflection of the atom, in opposite directions, depending on the electronic state. For a thermalised beam,

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this leads to a separation into a hot beam, in which all atoms are in the excited state, and a cold beam, in which all atoms are in the ground state. In this case the entropy of each beam is effectively zero.

If we assume that the state of the cavity field and the cavity itself are initially pure states and are unchanged by the interaction, then the process described above has apparently taken a state of non-zero entropy to a state of zero entropy. Can this device really work? The answer is provided by considering the other important dynamical system in this problem—the motion of the mirror forming the optical cavity. I will show that in order for the device to keep working, the mirror must be reset, and this entails an entropy cost sufficient to avoid any violation of the second law.

The analysis can be carried out at two levels. Firstly, one may treat the problem entirely classically, at least as far as the centre of mass motion of the atom and cavity mirrors are concerned. This is the appropriate level of description if one wishes to give a standard statistical thermodynamic explanation. On the other hand, the problem can be analysed quantum mechanically. This is the appropriate level of description for a system in which the bulk motion of the atoms and mirrors are completely isolated, except for the interaction with the standing wave.

2. Classical Description

A single two-level atom passes transversely through an optical standing wave. One mirror of the cavity is so massive it cannot move, the other mirror of the cavity however can move, and take up momentum exchanges between the atom and the field. If the electronic resonance is well detuned from the cavity frequency, the atom experiences a conservative force proportional to the intensity gradient. This force is due to the electric dipole induced in the atom by the far off-resonance field. If the field is tuned below the resonance the force is directed toward regions of high intensity. If the field is detuned above the atomic resonance, the force is directed away from regions of high intensity. In both cases the electronic state of the atom does not change, as only virtual two-photon transitions are involved. The dipole force may be described in terms of an effective potential which for a standing wave is described by the Hamiltonian (Walls 1994)

$$H_K = \hbar g \sigma_z \cos[2k(q_A - q_M)]\,, \tag{1}$$

where q_A is the position of the atom as it enters the beam and q_M is the position of the cavity mirror, both measured along the cavity axis. The wave number of the field is k while the interaction strength g is given by

$$g = \frac{\Omega^2}{4\Delta}\,, \tag{2}$$

where Ω is the Rabi frequency of the electronic transition and Δ is the detuning between the atom and the field. The operator σ_z describes the inversion of the two-level atom

$$\sigma_z = \tfrac{1}{2}(|2\rangle\langle 2| - |1\rangle\langle 1|)\,, \tag{3}$$

where $|1\rangle$ is the ground state and $|2\rangle$ is the excited state.

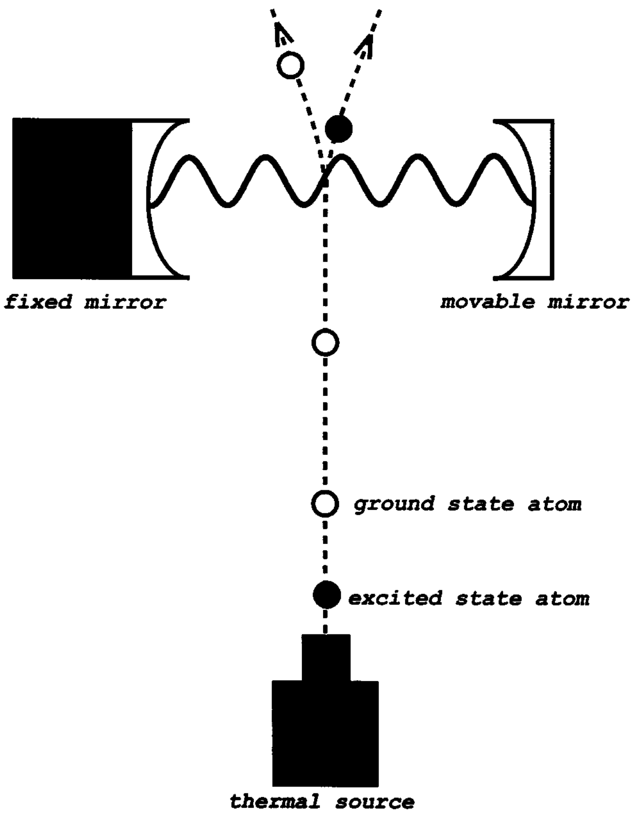

Fig. 1. A schematic representation of the atom-optical Maxwell demon. A beam of atoms prepared in a thermal distribution of two electronic states is deflected by a far-off resonant optical standing wave. Atoms in different electronic states are deflected in different directions.

In the Raman–Nath regime, the time of interaction of the atom with the field is small, so that the free motion of the centre-of-mass of the atom can be neglected while it traverses the cavity. Similarly we can neglect the free motion of the mirror during the passage of the atom through the cavity. In this case, the effect of the interaction is to give the atom and the mirror equal and opposite momentum kicks, the size of which depends on just where the atom enters the standing wave. The largest transfer of momentum occurs when the atom passes through a node of the standing wave, i.e. $2k(q_A - q_M) = \pi/2$. After the passage of the atom through the cavity the mirror moves freely, unless steps are taken to prevent it, until the next atom enters. The atom moves off on a deflected path, the direction of which indicates what the electronic state of the atom was as it entered the cavity (see Fig. 1). We will assume the electronic state of the incoming atom is

$$\rho_{el} = \lambda_1|1\rangle\langle 1| + \lambda_2|2\rangle\langle 2|\,. \tag{4}$$

The probabilities λ_i can be defined in terms of an effective electronic temperature T_{el} by

$$\lambda_1 = (1 + \mathrm{e}^{-\epsilon/k_B T_{el}})^{-1} \tag{5}$$

and $\lambda_2 = 1 - \lambda_1$ where ϵ is the energy difference between the two electronic states. As ρ_{el} commutes with the interaction, equation (1), we can regard σ_z as a classical random variable which takes the values $\{\frac{1}{2}, -\frac{1}{2}\}$ with probabilities $P(\frac{1}{2}) = \lambda_2$ and $P(-\frac{1}{2}) = \lambda_1$.

If the first atom passes through a node of the standing wave, the momentum transferred to that atom Δp_A is given by

$$\Delta p_A = \pm\theta\,, \tag{6}$$

where $\theta = \hbar g k \tau$ and τ is the time it takes an atom to traverse the standing wave. The plus-sign corresponds to $\sigma_z = \frac{1}{2}$ and occurs with probability λ_2, while the minus-sign corresponds to $\sigma_z = -\frac{1}{2}$ and occurs with probability λ_1. An equal and opposite momentum is given to the cavity mirror, by conservation of momentum. In the time it takes the next atom to enter the cavity this transfer of momentum to the mirror will cause it to move a small distance and thus the next atom will not enter precisely at a node and thus will experience a different momentum kick to the first. The problem is now apparent. Unless steps are taken at each step to restore the cavity mirror to the same state, before an atom enters the cavity, it will become impossible to deduce the atomic state from the deflection of the atomic beam. In effect the mirror comes to thermal equilibrium with the electronic degrees of freedom of the atomic beam and beam separation is no longer possible. The demon's hand begins to shake so that it is incapable of extracting useful information about the electronic state of the atoms.

The simplest way to restore the mirror to the initial position is to provide a restoring force together with enough friction to allow the mirror to suffer over-damped motion. The average energy transferred to the mirror by the interaction is $\theta^2/2M$, where M is the mass of the mirror. If this is dissipated into a heat bath at temperature T_M, the entropy cost is

$$S = \frac{\theta^2}{2MT_M}\,. \tag{7}$$

There is an additional entropy cost. Frictional damping of the mirror must be accompanied by fluctuations in momentum (Gardiner 1991). Such fluctuations cause the position of the mirror to fluctuate as well, and thus each atom cannot enter, with certainty, at a nodal position. The result is a fluctuation in the momentum transferred to the atom and a resulting entropy penalty, the consequence of which is a difficulty in distinguishing the atomic state by beam deflection. Typical values for the parameters in the model are (Dyrting 1993) $g = 5 \times 10^8$ Hz and $k = 10^{-7}\ \mathrm{m}^{-1}$, while the interaction time is $\tau = 10^{-7}$. This gives the momentum transfer per atom as $\theta = 10^{-26}\ \mathrm{kg\,m^{-1}}$, rather small! Thus at room temperature $S \approx 10^{-50}\ \mathrm{J\,K^{-1}}$.

It is interesting to consider in a little more detail the classical description of the exchange of momenta in this model. The description breaks down into two steps; an impulsive change in the momenta of the atoms and mirror as each atom passes through the cavity and free evolution of the mirror until the next atom enters. The atom once past the cavity continues on its deviated path and the next atom enters the cavity at the same place as the first. Thus we really don't need to consider the centre-of-mass motion of the atoms. The dynamics is most easily described in terms of a map, from just after one atomic injection to just after the next, for the relative position and momentum variables (along the cavity axis),

$$X_1 = \sqrt{\tfrac{1}{2}}(q_A - q_M)\,, \tag{8}$$

$$X_2 = \sqrt{\tfrac{1}{2}}(p_A - p_M)\,. \tag{9}$$

The factor of $\sqrt{\frac{1}{2}}$ is included to ensure that this is a canonical transformation. If we assume the initial total momentum is zero the map may be written as

$$X_1^{n+1} = X_1^n + \frac{t}{2M}X_2^n\,, \tag{10}$$

$$X_2^{n+1} = X_2^n + 2\sqrt{2}\theta\sigma_z \sin(2\sqrt{2}kX_1^{n+1})\,, \tag{11}$$

where t is the time between atomic injections. Defining the dimensionless variables

$$x_1 = 2\sqrt{2}kX_1\,, \tag{12}$$

$$x_2 = \frac{\sqrt{2}tk}{M}X_2\,, \tag{13}$$

we have the classical map

$$x_1^{n+1} = x_1^n + x_2^n\,, \tag{14}$$

$$x_2^{n+1} = x_2^n + 2\beta\sigma_z \sin x_1^{n+1}\,, \tag{15}$$

where $\beta = 2kt\theta/M$. This map is related to the 'standard-map' (Lichtenberg 1979), a well-known prototype of chaotic dynamics. The difference however is that here the control parameter $\beta\sigma_z$ is a random variable, and furthermore it is highly unlikely that the system could be operated in the chaotic region due to the very small exchanges of momenta involved. However, the map does enable us to simulate the thermalisation of the demon if no attempt is made to restore the mirror to a standard state after each atomic injection.

To monitor the gradual decay in the effectiveness of the demon we consider the momentum transferred to each atom. In dimensionless units this is simply proportional to $x_2^{n+1} - x_2^n$ which, from equation (15), is given by $2\beta\sigma_z \sin x_1^{n+1}$. If the mirror were restored to its standard position at each step, $x_2^{n+1} - x_2^n$ would always be given by $2\beta\sigma_z$. Thus, a measure of the performance of the demon can be given by considering the quantity

$$\Delta p = \left|\frac{x_2^{n+1} - x_2^n}{2\beta\sigma_z}\right| \tag{16}$$

$$= |\sin x_1^{n+1}|\,. \tag{17}$$

This quantity would remain at unity if the mirror was reset each time. In Fig. 2 we plot this quantity versus n for a particular sample path of σ_z chosen to have the required statistics. The decay of this quantity to zero is evidence that the uncertainty in the electronic state is disordering the position of the mirror at each step. In Fig. 3 we show the behaviour of this quantity averaged over one thousand sample trajectories with $\beta = 0\cdot 01$.

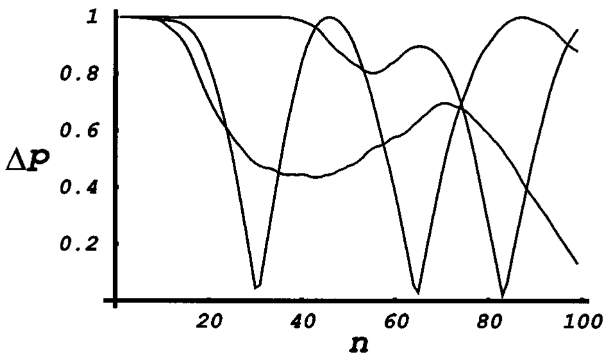

Fig. 2. Three typical simulations of the scaled momentum transferred at each atomic injection. Here $\Delta p = \left|\sin x_1^{n+1}\right|$ is plotted versus the number of atoms injected, where $\beta = 0\cdot 01$ and $\lambda_1 = \lambda_2 = 0\cdot 5$.

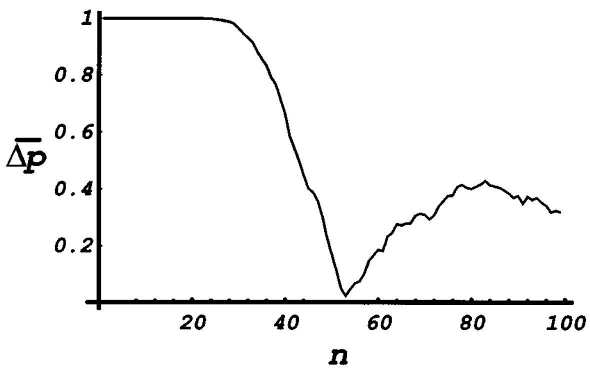

Fig. 3. Average of 1000 simulations of Δp versus the number of atomic injections, where $\beta = 0\cdot 01$ and $\lambda_1 = \lambda_2 = 0\cdot 5$.

3. Quantum Description

As in the classical case, the quantum description of this system is given as a map. This is most easily done in the Schrödinger picture. The change in the state of the atomic centre-of-mass/mirror system, due to each atomic injection, with no account taken of which way the atom was deflected, is given by

$$\rho' = tr_{el}(U_K \rho_{el} \otimes \rho U_K^\dagger)\,, \tag{18}$$

where

$$U_K = \exp[-ig\tau\sigma_z \cos(2\sqrt{2}kX_1)] \tag{19}$$

and X_1 is the relative position variable defined by equation (8). Here ρ_{el} is the state of the electronic degrees of freedom, ρ is the state of the atomic centre-of-mass/mirror just before an atomic injection and ρ' is the atomic centre-of-mass/mirror just after an atomic injection. The partial trace tr_{el} is taken over electronic degrees-of-freedom. If the electronic state is as given in equation (4), then

$$\rho' = \lambda_1 D^\dagger \rho D + \lambda_2 D\rho D^\dagger \,, \tag{20}$$

where

$$D = \exp\bigg(- i\frac{g\tau}{2}\cos(2\sqrt{2}kX_1)\bigg) \,. \tag{21}$$

The state of the relative position is taken to be a pure state $|\psi\rangle$ with coordinate representation

$$\langle x_1|\psi\rangle = (2\pi\Delta)^{-1/4}\exp\bigg(- \frac{(x_1-\bar{x})^2}{4\Delta}\bigg) \,, \tag{22}$$

where $\bar{x} = \pi/4\sqrt{2}k$.

The statistical entropy is not a straightforward quantity to calculate. Instead we use the linear entropy defined by

$$S_L = 1 - tr(\rho^2) \,. \tag{23}$$

For a pure state this is zero and for a general mixed state is less than unity. Before a kick, the linear entropy for the atomic centre-of-mass/mirror system is zero, while the linear entropy for the electronic degrees-of-freedom is

$$S_L^i = 1 - (\lambda_1^2 + \lambda_2^2) \,, \tag{24}$$

which is a maximum of $\frac{1}{2}$ for $\lambda_1 = \lambda_2$. After an atomic injection, the linear entropy is given by

$$S_L^f = 1 - (\lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2|\langle\psi|D^2|\psi\rangle|^2) \,. \tag{25}$$

If we assume that the relative variable X_1 is much better defined than an optical wavelength, that is $\sqrt{\Delta} \ll \lambda$, then

$$S_L^f = 1 - \bigg(\lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2 \exp\bigg\{ - 2\frac{\Delta p_A^2}{\Delta p_i^2}\bigg\}\bigg) \,, \tag{26}$$

where Δp_A is the momentum transferred to the mirror and

$$\Delta p_i^2 = \frac{\hbar^2}{4\Delta} \tag{27}$$

is the initial uncertainty in the relative momentum. If any beam splitting is to occur at all it is necessary to arrange things so that $\Delta p_A \gg \Delta p_i$, in which case $S_L^f \approx S_L^i$. In general, however, the final entropy is greater than the initial entropy as the initial quantum fluctuations in the atomic centre-of-mass are transferred to the mirror.

4. Conclusion

The deflection of an atom by a standing wave in the atom-optical Stern–Gerlach effect provides a simple model system for a Maxwell demon. The entropy exchanges may be followed in some detail. If the atomic electronic states are thermal we have shown above that, unless steps are taken to restore the cavity mirror to its initial state for each atomic injection, the device eventually fails to separate atoms into an excited state beam and a ground state beam. The fluctuating exchanges of momentum between one atomic injection and the next are sufficient to cause the mirror to come to thermal equilibrium with the electronic degrees-of-freedom. The entropy of the mirrors increases by an amount sufficient to ensure the enforcement of the second law.

A number of questions remain to be answered. In this paper the electronic state is taken as a thermal state, with no coherences between atomic levels. How is the analysis changed if the electronic state is a pure superposition of the ground and excited states, with the same populations as the thermal mixed state? This would make for a true quantum Maxwell demon. As discussed above, the mirror must be restored to its initial state each time an atom passes through the cavity. This is a kind of feedback. The deflection of the atomic beam may be regarded as a measurement of the electronic state of the injected atom. We thus need to adjust the state of the mirror momentum by an amount depending on the results of the measurement. This is a particular case of quantum limited feedback (Wiseman and Milburn 1993). It is known that this kind of feedback necessarily introduces a certain level of noise into the system. It would be interesting to analyse this from the point of view of the second law. In practice the atoms arrive in the cavity at Poisson distributed times. In that case a master equation can be derived for the atomic centre-of-mass/mirror system (Milburn 1987) and entropy exchanges followed in some detail.

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