ON THE CHOICE OF GLASSES FOR CEMENTED ACHROMATIC APLANATIC DOUBLETS

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Summary

It is shown that the types of glass given by tables for thin, low aperture telescope objectives of the " crown leading " doublet form are also those most suitable for thick, high aperture systems. The maximum focal lengths and the tolerances permissible in glass properties are given for each aperture, and the variation of these properties is considered when the thickness or wavelength of correction is changed, or the objective used with prisms.

I. Introduction

Three variables are required to design a cemented doublet telescope objective of the Clairaut type, corrected simultaneously for longitudinal chromatic aberration, spherical aberration, and coma. If the focal length is fixed, two of these variables are provided by the relative powers of the crown and flint components and the bending of the lens as a whole, and the third must be chosen from the indices and dispersions of the two glasses used. Existing glass types do not provide a continuous variation of these properties, and hence some means is required of choosing suitable glass pairs.

Several investigators have given methods for choosing suitable glasses for thin, low aperture objectives. Harting (1898) gives tables for this purpose, and Moffitt (1925) and Moffitt and Kaspereit (1925) give a graphical method. Very complete tables based on third order aberration theory are given by Brown and Smith (1946).

This paper discusses the glass properties required for objectives of large relative apertures for which higher order aberrations become appreciable and thick components must be used. The " crown leading " case only is considered, as this has the smaller contact curvature.

A series of doublets was designed at angular semi-apertures from 0 up to 0·15 by trigonometrical methods : a pair of indices was chosen for the crown and flint components and the spherical aberration and coma corrected by ray traces. The most suitable form of the lens thus having been found for each aperture, the relative dispersions of the two glasses necessary for chromatic correction were calculated. The residual secondary aberrations were also calculated and expressed as the maximum scale (maximum focal length) to which the objective could be made without exceeding aberration tolerances.

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Two pairs of glasses were initially studied in detail, chosen to represent the two extremes of refractive index difference within the range of glasses commonly used for doublets. It was found that the variations of relative dispersion and maximum focal length with aperture were practically the same in both cases, and further calculations with an intermediate pair of glasses at an angular semi-aperture of $0 \cdot 15$ confirmed this observation. Hence it may be assumed that the results of this investigation should apply to any glass pair whose properties lie between those of the two pairs investigated.

As variations of the thickness of each component could disturb the results, the thicknesses were chosen to be a constant fraction of the lens aperture. However, it was found that quite considerable variations from this standard thickness had very little effect on the choice of glass. Other effects considered were a change of the wavelength at which the spherical aberration was corrected, the results of under- or over-correction of the aberrations, and the tolerable variations of the relative dispersions at each aperture.

II. Method of Calculation

The Brown and Smith (1946) tables give, in terms of the indices of the two glasses, the curvatures of the surfaces and the required relation between the glass dispersions for chromatic correction. This is given as $N=(\nu_a+\nu_b)/(\nu_a-\nu_b)$, where ν_a and ν_b are the Abbe values for the crown and flint components respectively. In this paper it is found more convenient to use the variable

$$\rho=\frac{\nu_a}{\nu_b}=\frac{N+1}{N-1}.$$

For lenses of finite aperture, the third order aberrations should not be corrected completely but should be used to balance the aberrations of higher orders. If these higher order aberrations are assumed to be predominantly of the fifth order the best image is obtained when the ray aberrations are corrected at the apertures given by Maréchal (1947), namely, the spherical aberration at the margin and the coma at $\sqrt{1 \cdot 2}$ times the marginal aperture. It was found to be more convenient to trace an extra ray at this $\sqrt{1 \cdot 2}$ aperture than to balance the marginal and primary coma using the rays traced to find the spherical aberration. For each pair of indices chosen, the coma was made zero by bending the lens, and then the relative powers of the two components were adjusted until the spherical aberration was also zero.

To correct the lens for longitudinal chromatic aberration, the ratio ρ of the two ν values was calculated by equating to zero Conrady's (1904) formula for chromatic path differences

$$\Sigma E=\Sigma(d'-D')\delta n, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where d' and D' are the thicknesses of each component along the axis and along the ray and δn is the index difference between the wavelengths at which the primary aberration is corrected. If the C and F rays are brought to the same focus, it follows that

$$\rho = -\frac{(n_d-1)_a(d'-D')_a}{(n_d-1)_b(d'-D')_b}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where the suffices a and b refer to the crown and flint components respectively.

This equation was used as recommended by Conrady for a marginal ray. However, for angular semi-apertures greater than $0 \cdot 1$, the longitudinal chromatic aberration was found to be corrected somewhat inside the $1/\sqrt{2}$ zone. If equation (2) is applied instead to the ray at the $\sqrt{1 \cdot 2}$ aperture used to calculate the coma, the aperture at which the chromatic aberration is zero is very close to the $1/\sqrt{2}$ zone, but the change in ρ is unimportant.

III. Results

The first pair of glasses studied had refractive indices $n_a = 1 \cdot 5056$, $n_b = 1 \cdot 6557$, that is, a borosilicate or phosphate crown with extra dense flint; and the second the indices $n_a = 1 \cdot 5778$, $n_b = 1 \cdot 6267$, that is, medium barium crown and dense flint.

(*a*) *Dispersion Ratio*

The changes in ρ necessary to achieve achromatism at various apertures are shown in Figure 1 for these two glass pairs. Although the curvatures of the surfaces and the relative powers of the two components change considerably with aperture, the variation of ρ is very small and is almost the same in both cases.

These curves have been calculated for the standard thickness of one-tenth the aperture for the edge thickness of the crown and the centre thickness of the flint components, and for spherical aberration and coma corrected for the e ray ($\lambda = 5461$ Å). When other conditions were to be satisfied, the following effects were noted.

(i) *Thickness Change.*—If the standard thickness is increased to $0 \cdot 15$ times the aperture for both components, the change of ρ is negligible, being less than $0 \cdot 001$. However, a slightly larger change is found if the thickness of one component only is altered: the crown thickness remaining constant, an increase of the flint thickness from $0 \cdot 1$ to $0 \cdot 15$ of the aperture decreases ρ by $0 \cdot 003$.

(ii) *Change of Wavelength of Correction.*—If it is desired to correct the spherical aberration and coma for a wavelength x different from 5461 Å, the change required in ρ may be found with sufficient accuracy from the equation

$$\delta\rho \simeq P_{xe}\frac{\rho - 1}{\nu_b}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where P_{xe} is the relative partial dispersion between the e ray and the new x ray,

$$P_{xe} = \frac{n_x - n_e}{n_F - n_C}.$$

It is sufficient to take an average value of P_{xe} for all glasses.

If the aberrations are corrected for the d line,

$$P_{de} \simeq -0\cdot 238 \text{ and } \delta\rho = -0\cdot 007, \quad \text{pair I,}$$
$$= -0\cdot 004, \quad \text{pair II.}$$

When the Brown and Smith tables are used with refractive indices corresponding to a wavelength other than that of the d ray, a similar correction must be applied to ρ, this time taking the partial dispersion from the d line for which these tables are calculated.

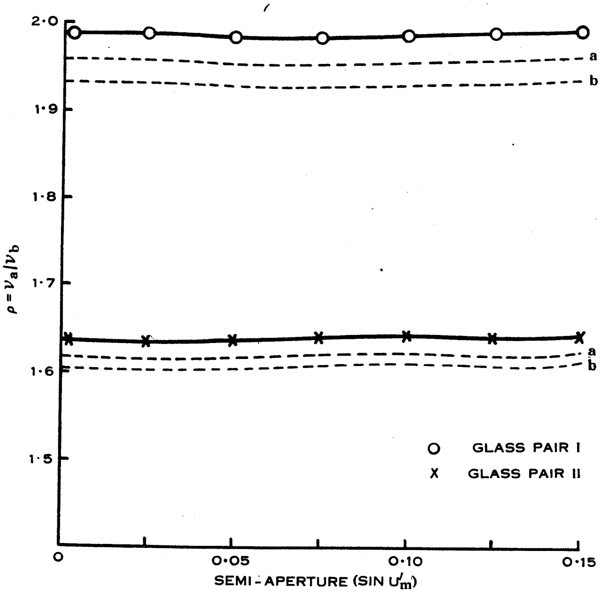

Fig. 1.—The variation of relative dispersion with aperture. Full curves—fully corrected objective; broken curves—objective corrected for use with a prism, (a) of HC glass, (b) of LF glass.

(b) *Maximum Focal Length*

At each aperture, the scale to which the objective may be made and hence the permissible focal length is limited by the tolerances for the residual aberrations. The aberrations that are important and their tolerances are as follows.

(i) *Zonal Spherical Aberration*.—This was found by calculating ΣW_p, the fourth order path difference for the marginal aperture. Its tolerance is 6λ.

(ii) *Zonal Coma*.—This was calculated as the "offence against the sine condition" for the marginal ray. The tolerance for this is $0\cdot 53\lambda/h' \sin U'_m$ (Maréchal 1947), the field size h' being chosen so that the astigmatism and field curvature at the edge of the field were equal to one-half their combined tolerance. It was found that zonal coma was negligible in comparison with the other aberrations and it has therefore not been included in Table 1.

(iii) *Secondary Chromatic Aberration.*—This was calculated as the difference between the marginal optical path for e rays and that for C or F rays, these latter rays having the same path length if the primary aberration is corrected. Conrady (1929) gives the tolerance for secondary chromatic aberration as $1\frac{1}{4}$ times the focal range tolerance, that is, $5\lambda/8$ in terms of path difference.

(iv) *Spherochromatic Aberration.*—This was calculated as $\Sigma E_m - \Sigma E_p$ the difference between the marginal and paraxial path differences for C and F rays. Conrady (1923) gives a tolerance of 2λ for this difference.

When each of these aberrations is equal to its tolerance, the maximum focal lengths obtained are given in Table 1, the case of coma being omitted. The value in column (v) is the maximum focal length when all aberrations are taken

TABLE 1

FOCAL LENGTHS AT WHICH SECONDARY ABERRATIONS ARE EQUAL TO TOLERANCE

(i) Zonal spherical aberration, (iii) secondary colour, (iv) spherochromatic aberration, (v) all aberrations considered

Semi-aperture	Maximum Focal Length (mm)							
Sin U'_m	Glass Pair I				Glass Pair II			
	(i)	(iii)	(iv)	(v)	(i)	(iii)	(iv)	(v)
0·025	10^7	2200	10^5	2200	10^7	2300	10^5	2300
0·050	10^5	540	6000	540	10^5	590	4000	590
0·075	10^4	240	1200	230	10^4	260	910	250
0·100	1500	140	410	130	1500	150	290	130
0·125	420	88	170	77	430	95	120	71
0·150	150	60	86	47	150	65	62	45

into account, all being assumed independent. At the focal length given in column (v) the maximum angular semi-field is limited by astigmatism and curvature, and, in all cases, a semi-field of about 0·012 radian is found to give the mean focus at the edge of the field at a distance of half the focal range tolerance from the best focus on the axis.

It is seen that there are only small differences between the maximum permissible focal lengths for the two glass pairs. The zonal spherical aberrations are practically identical; this agrees with Moffitt and Kaspereit's (1925) findings that, provided the index difference between the crown and the flint is greater than 0·04, a further increase in index difference has little effect. With regard to secondary colour, the glasses of pair I have typical relative partial dispersions while pair II was chosen to give the maximum practical improvement of this aberration that is obtainable from tabulated glass lists, yet the improvement obtained is very small. The spherochromatic aberration is decreased by an increase of index difference but this aberration is generally less serious than the secondary colour.

(c) Tolerances for Dispersion

The effects of variations of ν_a or ν_b and hence of ρ from the ideal value given in Figure 1 can be considered in two ways.

(i) If the doublet is kept exactly aplanatic, the chromatic aberration is given by

$$\left.\begin{array}{l} \Sigma E_p \simeq y^2 A/2 \\ \text{or} \qquad Lch'_p \simeq l'^2 A, \end{array}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

where l' is the image distance and

$$A = \frac{(n_d-1)_a c_a}{\nu_a} + \frac{(n_d-1)_b c_b}{\nu_b},$$

c_a and c_b being the total curvatures of the two components. For small changes in ρ from the ideal value,

$$\delta A \simeq -\frac{\delta\rho}{\nu_a f'(\rho-1)}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

and, from this, tolerances for ρ can be calculated from the tolerance for the chromatic aberration.

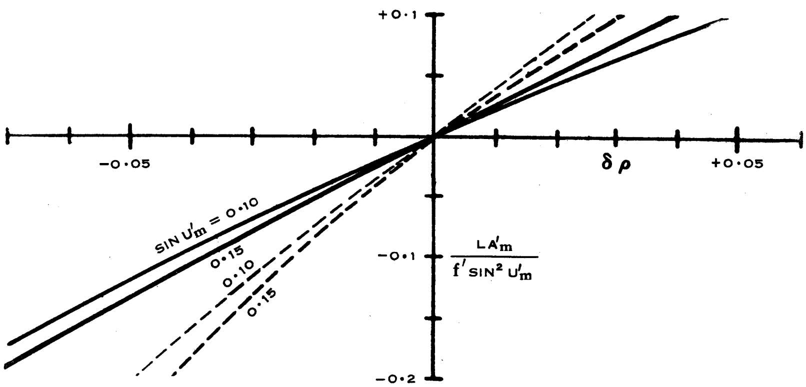

Fig. 2.—The change in spherical aberration due to a change in ρ : Full curves—glass pair I ; broken curves—glass pair II.

(ii) Alternatively, the lens can be designed to be chromatically corrected and the spherical correction allowed to change within the tolerance. The variation of spherical aberration with ρ cannot be expressed simply but Figure 2 gives the curves found for this relation during the trigonometrical correction of the objectives, $LA'_m/(f' \sin^2 U'_m)$ being plotted against $\delta\rho$. From these curves the tolerance for ρ can be calculated from the spherical aberration tolerance.

As the latter procedure allows the larger variation of ρ, it is the method that should be used in practice. In Table 2 the permissible variation $\delta\rho$ is given for each glass pair at its maximum focal length (column (v) of Table 1). For other focal lengths $\delta\rho/f'$ can be taken as constant. In this table it is seen that $\delta\rho$ varies approximately as the inverse square of the aperture.

The variation of ρ in Figure 1 from zero aperture to the highest aperture at which a doublet would be used is about 0·005, so it is seen that the same pair of glasses could be used at all apertures.

The strictest tolerance for ρ of 0·01 corresponds to tolerances $\delta\nu_a=0\cdot4$ or $\delta\nu_b=0\cdot2$ for the dispersions of the two glasses. As most optical glasses are supplied to the tolerance $\delta\nu=\pm0\cdot5$, it is only when sin $U'_m>0\cdot125$ that a closer specification of dispersion would be necessary.

Table 2

Tolerances for the relative dispersion $\nu_a/\nu_b=\rho$ for objectives of maximum focal length

Sin U'_m	0·025	0·050	0·075	0·100	0·125	0·150
$\delta\rho$, pair I	0·5	0·12	0·07	0·04	0·03	0·02
pair II	0·3	0·07	0·04	0·03	0·02	0·01

IV. Partial Correction

In many applications the doublet objective is not completely corrected but is used to correct the aberrations of other components such as eyepieces or prisms. In this case a different pair of glasses may be required.

(i) *Chromatic Correction.*—If the longitudinal chromatic aberration is not corrected completely, the value of ρ required can be found from equation (**4**), or by equation (**5**) for small variations from the corrected value. For under-corrected chromatic aberrations, where Lch'_p or ΣE_p is positive, a decrease of ρ is required.

(ii) *Spherical Correction.*—The change required in ρ for a given spherical aberration is given in Figure 2. It is seen that under-corrected (positive) spherical aberration demands an increase in ρ.

(iii) *Prisms.*—The most common case of incomplete correction is the use of the objective with a prism. This demands an objective under-corrected for both spherical and chromatic aberration; the chromatic effect predominates and a decrease in ρ is required.

In Figure 1 the broken curves represent the values of ρ required for hard crown and light flint prisms of thickness one-fifth the focal length of the objective, using the glasses HC519604 and LF578407. ρ is found to vary linearly with prism thickness.

V. Choice of Glass

As the variations of ρ over the range of apertures and thicknesses considered are less than the tolerance for ρ, the methods of choosing glass for low aperture objectives referred to in the introduction can be used directly for any aperture. We shall represent the possible glass pairs graphically, using the Brown and Smith (1946) tables.

In Figure 3 the crown region of interest is plotted in terms of n_e and ν and superimposed are the curves for glasses that would form doublets with four typical flints, which are taken as Chance LF578407, DF605380, DF623360, and EDF651336. Each curve is shaded to one side to represent the small increase in ρ with aperture and the curves corresponding to light flint prisms of thickness one-fifth the objective focal length are shown. The curves are terminated at an index difference of 0·04 between crown and flint to avoid high zonal spherical aberrations.

It is seen that there is no suitable glass for use with light flints. Two glasses lie near the dense flint curves, Schott PSK3 and Chance MBC576594, while there is a considerable selection of glasses that can be used with the extra dense flint.

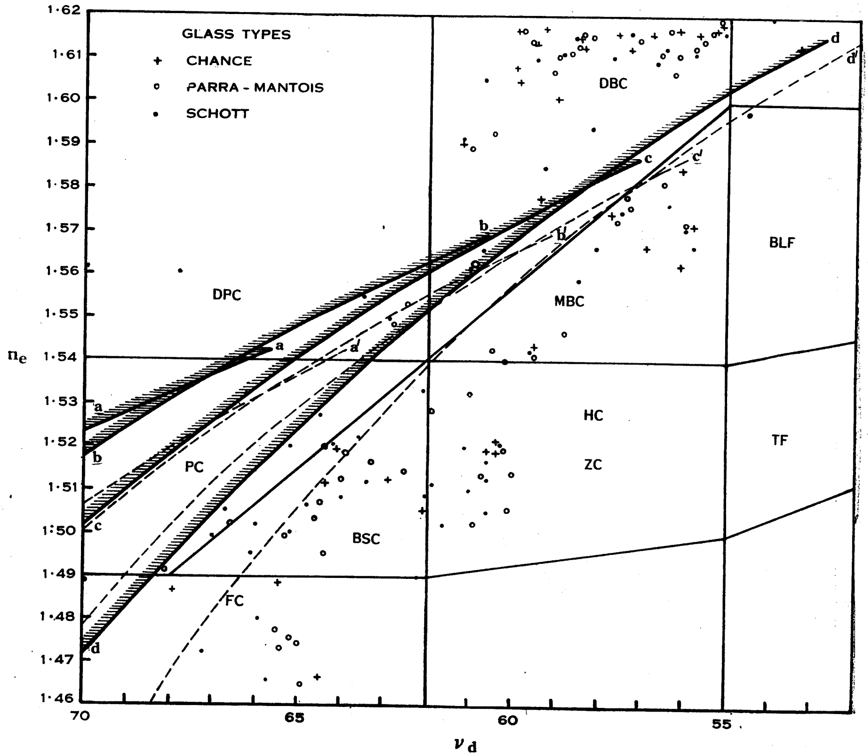

Fig. 3.—Plot of crown glass types suitable for use with the flint glasses: (*a*) LF578407, (*b*) DF605380, (*c*) DF623360, (*d*) EDF651336. Broken curves show change in glass type when the objective is used with a prism of LF glass.

In Figure 4 the curves for PSK3 and MBC576594 are shown in the region of flint glasses. Both these glasses give close fits to several flints, the former throughout the dense flint range and the latter for the top of the dense flints and the lower extra dense flints.

The curves for the two glasses BSC510644 and MBC572577 which are commonly used for doublets in combination with extra dense flint and dense flint respectively, are shown also in Figure 4. It is seen that these glasses only give perfect correction with a high index extra dense flint, $n \simeq 1 \cdot 69$. But they can be combined with the usual flints when the objective is used with thick prisms.

VI. Conclusions

The representative cases studied have shown that the glass properties required for cemented doublets which are simultaneously aplanatic and achromatic are the same for lenses of any aperture and that the desirable glasses may be found from tables calculated for thin objectives of small aperture.

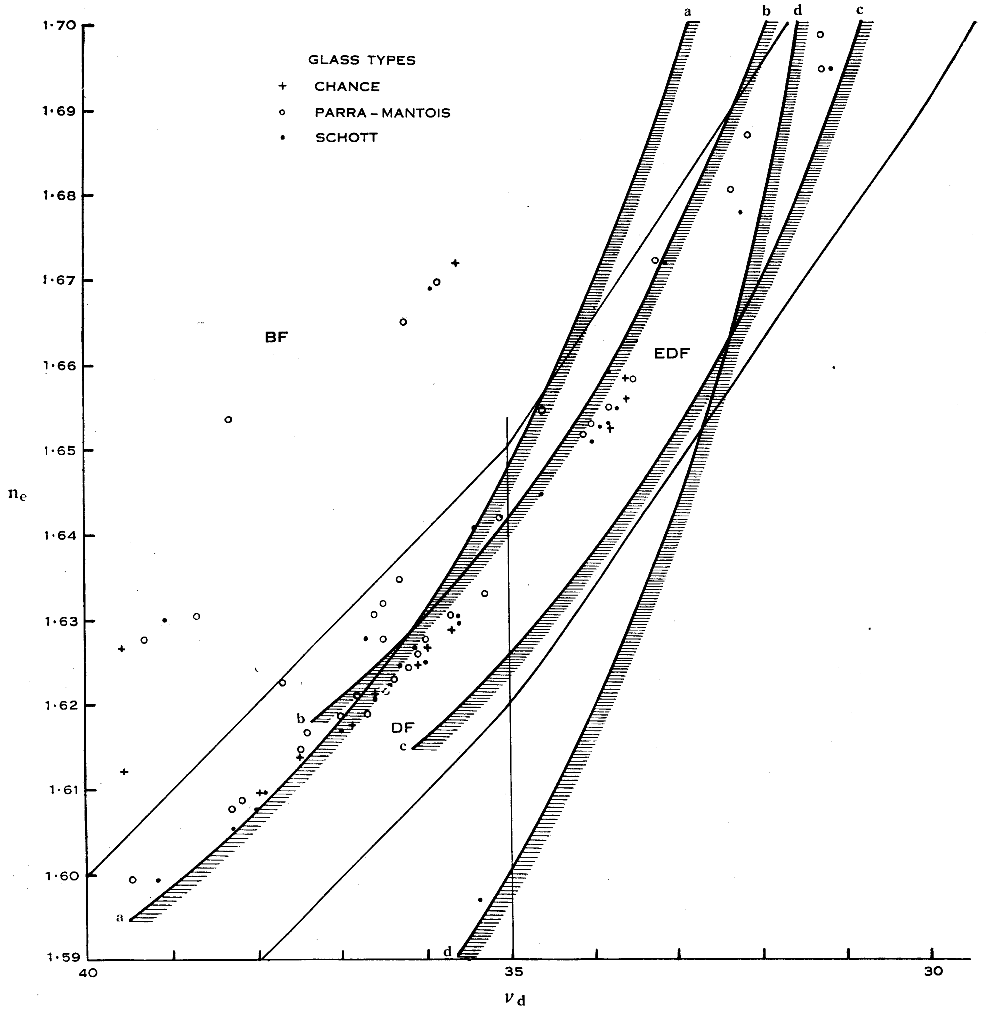

Fig. 4.—Plot of flint glass types suitable for use with the crown glasses : (*a*) PSK3, (*b*) MBC576594, (*c*) MBC572577, (*d*) BSC510644.

However, the curvatures of the lens surfaces cannot be found from these tables as they vary considerably as the aperture is increased. The greatest change occurs in the curvature of the contact surface which decreases at high apertures.

Tolerances on the glass dispersions have been calculated and at low apertures they are very broad. They are more severe, however, at high apertures and the more commonly used pairs of glasses do not lie sufficiently close to the ideal values to be suitable. More satisfactory glass types have been indicated; the high order aberrations, however, vary so little between different possible combinations that a final choice would probably depend on questions of stability and transparency of the glasses.

If the objective is used with a prism, or if the "flint in front" form is employed, the difference between the ν values of the two components should be reduced, and the common types of crown become the most suitable.

VII. Acknowledgment

The ray traces for this work were performed by Miss J. Ward to whom the author expresses his thanks.

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