MICROWAVE AND METRE WAVE RADIATION FROM THE POSITIVE COLUMN OF A GAS DISCHARGE

By L. W. Davies* and E. Cowcher†

[*Manuscript received September 16, 1954*]

Summary

Experiments are described in which the intensity of radio-frequency "noise" radiation from the positive column of low pressure discharges in neon was measured at 3000 Mc/s and at 200 Mc/s. Measurements were made by inserting the positive column in a resonant cavity and comparing the cavity output with that from a noise generator of known intensity; corrections were made for contributions to the noise radiation from dissipative elements in the cavity at room temperature. The mean energy of the electrons in the discharge was calculated from measured values of the electric field.

As the discharge current was lowered the noise temperature T' at 3000 Mc/s increased above the calculated value of the electron temperature for an assumed Maxwellian distribution of electron velocities; this phenomenon indicated a change in the distribution function as the electron density was reduced. The thermal level of noise radiation from a plasma with Druyvesteyn distribution of electron velocities was calculated and compared with results for low values of the discharge current. For high values of the current there was good agreement between the values of T' at 3000 Mc/s and the electron temperature only when the pressure was sufficiently high; for lower pressures T' was less than the electron temperature.

In a limited range of discharge conditions the measured values of T' at 200 Mc/s were considerably greater than the thermal level. Experiments were carried out to ascertain the source of this enhanced radiation.

I. Introduction

An ionized medium in equilibrium at temperature T, and of sufficient optical depth, will emit radiation of all frequencies with an intensity equal to that of a black body at the same temperature. In other words, at radio frequencies, the available noise power from an ionized medium in equilibrium at temperature T amounts to kT per unit bandwidth. In general, the positive column of a gas discharge is not in equilibrium, and we specify the radio-frequency radiation from a discharge by means of an equivalent noise temperature T'; it is such that the available noise power per unit bandwidth is kT', where T' may be a function of frequency.

Microwave "noise" radiation from the positive column was first observed by Mumford (1949); it has been investigated in more detail by several writers (Easley and Mumford 1951; Johnson and DeRemer 1951; Knol 1951). In general, the equivalent noise temperature T' has been found to be comparable

* Division of Radiophysics, C.S.I.R.O., University Grounds, Sydney.

† Division of Electrotechnology, C.S.I.R.O., University Grounds, Sydney.

with the temperature T_e of the electrons in the discharge ; however, it was thought worth while to carry out a more extensive investigation than had been made previously, particularly at low values of the discharge current. Relationships between electron temperature, equivalent noise temperature, and collision frequency of the electrons in the discharge are important in connexion with the development of discharges as standard noise signal generators ; it is also of interest to study such relationships in the light of theories of the origin and transfer of radio-frequency radiation in an ionized medium (Smerd and Westfold 1949) with their particular application to radio-astronomical observations.

This paper describes experiments in which the noise radiation from the positive column of discharges in neon was observed at frequencies of 3000 and 200 Mc/s, over a wide range of discharge conditions.

Preliminary results were reported at the 10th General Assembly of the International Union of Scientific Radio (U.R.S.I.), Sydney, 1952.

II. Contribution of the Positive Column to the Observed Radiation

When the positive column is inserted in a resonant cavity, it may be considered as a load at a temperature equal to the equivalent noise temperature of the discharge. To determine the contribution of this load to the observed noise from the cavity, we consider the equivalent circuit of the cavity and impedance transformer as set down in Figure 1 (a), in which reactances introduced

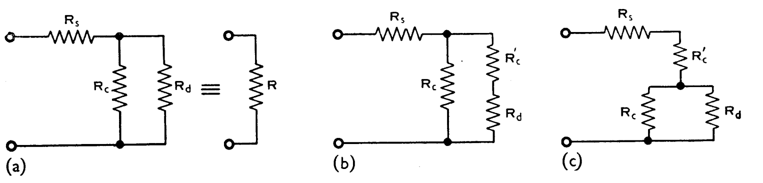

Fig. 1.—Equivalent circuits of resonant cavity containing discharge tube.

by the discharge have been neglected (see Section IV) ; the resistances R_s, R_c, and R_d correspond respectively to losses in the transformer, cavity, and discharge. Regarded as generators of noise, the resistances R_s and R_c are at room temperature T_r, and the resistance R_d at the effective noise temperature of the discharge, T'. We denote by T_n the observed noise temperature of the cavity when matched to the transmission line.

From a consideration of the relative losses in the resistances of the circuit given in Figure 1 (a), it is easily shown that, where R is the resistance of the circuit of Figure 1 (a),

$$T_n = T'\left[\frac{1}{R}\cdot\frac{R_c^2 R_d}{(R_c+R_d)^2}\right] + T_r\left[\frac{1}{R}\left\{R_s+\frac{R_c R_d^2}{(R_c+R_d)^2}\right\}\right]. \quad \ldots . \quad (1)$$

In order to determine T' from observed values of the available noise power from the cavity, standing-wave measurements are made of the impedance presented by the cavity with discharge off, i.e. with resistance R_d removed from the equivalent circuit ; these are carried out at the resonant frequency of the cavity (voltage standing-wave ratio, V.S.W.R. $=\sigma_r$) and at a frequency well removed from resonance (V.S.W.R. $=\sigma_0$). Then we have (Slater 1950):

$$\left.\begin{aligned} 1&=r_s+(r_d^{-1}+r_c^{-1})^{-1},\\ \sigma_r&=r_s+r_c\geqslant 1,\\ \sigma_0&=r_s^{-1}, \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2})$$

where $r_s=R_s/R$, $r_c=R_c/R$, $r_d=R_d/R$, and we assume the cavity to be matched to unit standing wave ratio with discharge running. From (**1**) and (**2**) it follows that

$$T'=\frac{\sigma_r\sigma_0-1}{(\sigma_r-1)(\sigma_0-1)}T_n-\frac{\sigma_r+\sigma_0-2}{(\sigma_r-1)(\sigma_0-1)}T_r.$$

In general, the second term may be neglected, since $T_n\gg T_r$ and $\sigma_0\gg 1$, giving

$$T'=\{(\sigma_r-\sigma_0^{-1})(\sigma_r-1)^{-1}(1-\sigma_0^{-1})^{-1}\}T_n. \qquad \ldots\ldots\ldots\ldots \quad (\mathbf{3})$$

If the resistive load corresponding to the discharge losses is not completely in shunt with the cavity losses R_c, the equivalent circuit of the impedance presented by the cavity and matching transformer may take the forms shown in Figures 1 (*b*) and 1 (*c*), where R_c' represents losses in the cavity at room temperature which are not shunted by R_d. In either of these cases it may be shown that

$$T'\geqslant\{(\sigma_r-\sigma_0^{-1})(\sigma_r-1)^{-1}(1-\sigma_0^{-1})^{-1}\}T_n. \qquad \ldots\ldots\ldots \quad (\mathbf{4})$$

Such a case may arise, for example, when losses occur in the glass walls of the cylindrical discharge tube and the axis of the tube is not parallel to the electric field vector in the cavity (cf. Section III (*a*)).

III. Experimental Technique

(*a*) *Measurement of* T′ *at* 3000 *Mc/s*

A block diagram of the circuit used is shown in Figure 2. The resonant cavity consisted of a section of waveguide, short-circuited at one end and provided with two sliding stubs of variable depth at the other, with the discharge tube mounted in the E-plane of the TE_{01} mode (H_{01}) and inclined at an angle of 15° to the wider face of the waveguide. Lengths of cylindrical copper tubing, of diameter 2·5 cm, surrounded that portion of the tube extending outside the guide; they acted as a waveguide beyond cut-off for the 10-cm radiation, and were calculated to provide approximately 100 db attenuation for radiation entering the guide from the electrode regions of the discharge tube.

The noise temperature of the cavity, T_n, was determined by comparing its output with that of an argon gas discharge tube ($T=10{,}250$ °K), which in turn had been calibrated by direct reference to a hot, dissipative waveguide load (cf. Section III (*b*)). The procedure was to connect the "cavity" section of guide to the impedance measuring equipment and match the cavity to the waveguide (V.S.W.R. $\leqslant 1\cdot 03$) by means of the sliding-stub transformer. The noise-measuring receiver and image-frequency rejection cavity were accurately aligned with the signal generator, and thus with the frequency at which the cavity was matched, and the cavity section of the waveguide transferred to the noise-measurement gear. The image rejection cavity was necessary because the different bandwidths of the two noise sources gave rise to different degrees of mismatch, and hence of noise energy transfer, at the image frequency. The transmission of the cavity at the image frequency was 42 db down on that at the signal frequency. For most measurements the bandwidth of the intermediate-frequency (30 Mc/s) amplifier of the noise-measuring receiver was 500 kc/s; for the lowest pressure investigated, and for low values of the discharge current, however, the bandwidth of match was found to be so narrow that a second

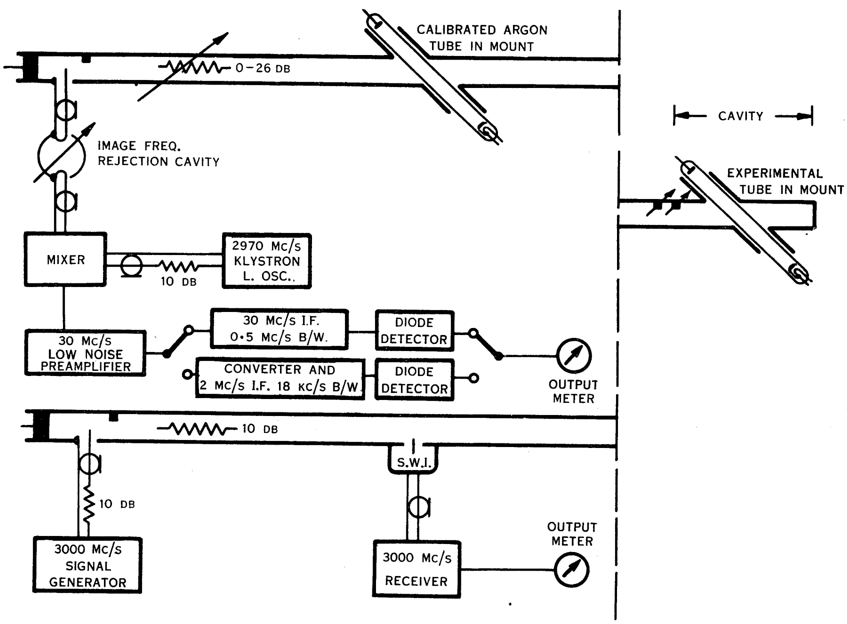

Fig. 2.—Experimental arrangement for measurement of noise temperature of discharge at 3000 Mc/s.

conversion to 2 Mc/s was employed, with a bandwidth of 18 kc/s between points 3 db down from the central frequency. Frequency stability was realized by using regulated voltage supplies throughout the electronic equipment. The noise figure of the receiver, including image rejection cavity, was of the order of 12 db.

The value of T_n was determined by measuring firstly M_n, the ratio of receiver output powers with cavity and room-temperature attenuator separately contributing noise signal to the input, followed by a measurement of M_s, the corresponding ratio for the substandard noise source. It can be shown that

$$T_n=\frac{M_n-(1-F^{-1})}{M_s-(1-F^{-1})}\cdot T_s, \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

where F is the noise figure of the receiver, obtained from the formula

$$F=\frac{1}{(M_s-1)}\left\{\frac{T_s}{T_r}-1\right\}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{6})$$

and T_s is the noise temperature of the substandard noise source. Several determinations were made of M_n and M_s for each set of discharge conditions investigated.

With discharge off, the cavity was then transferred to the impedance measurement equipment where the relevant standing-wave ratios σ_r, σ_0, were determined. Since the presence of the discharge generally gave rise to a small change in resonant frequency of the cavity with discharge on and off, σ_r was determined by measuring V.S.W.R. as a function of frequency (cf. Fig. 3) and

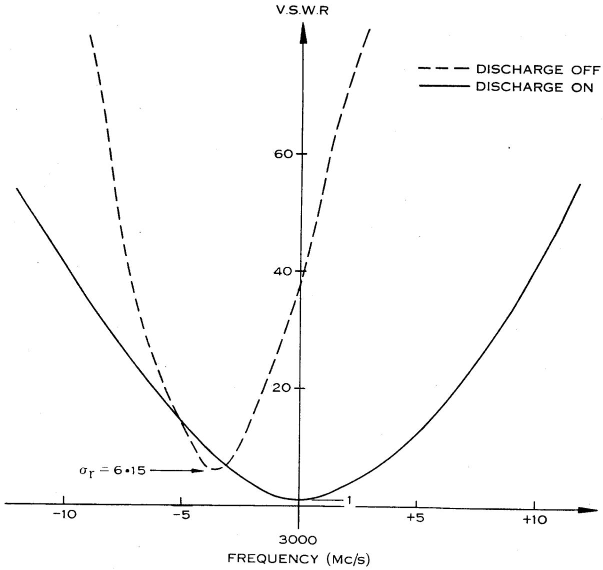

Fig. 3.—Voltage standing-wave ratio of cavity as a function of frequency near 3000 Mc/s. Experimental conditions: $p_0=2\cdot 80$ mm Hg, discharge current 10 mA.

taking σ_r to be the V.S.W.R. at resonance, with discharge off. At 3000 Mc/s σ_0 was found to be greater than 200, and accordingly σ_0^{-1} was neglected in equation (**3**), giving

$$T'=\{\sigma_r/(\sigma_r-1)\}T_n. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{7})$$

By determination of the position of the standing-wave minimum as a function of frequency at the same time, it was verified that the Q of the unloaded cavity (i.e. with discharge off) was less than the external Q (see Slater 1950), so that the equivalent circuits of Figure 1 and the derivation of equation (**3**) are valid.

(*b*) *Calibration of* 3000 *Mc/s Substandard Noise Source*

The determination of the noise temperature of the substandard argon discharge tube was performed by comparing its noise output with the noise output from a wedge-shaped waveguide termination heated uniformly to a temperature of approximately 450 °K. A second termination was used to obtain a room temperature reference.

The large difference between the temperature of the thermal standard (450 °K) and the effective temperature (10,000 °K) of the discharge, led to considerable experimental difficulties ; the most troublesome of these were :

1. The receiver was particularly sensitive to mismatch of the room temperature termination and the 450 °K thermal standard. It may be shown (F. F. Gardner, personal communication) that, when the equivalent input temperature of the receiver is greater than the temperature of the input termination, a change of 1 per cent. in the resistive component of the termination can give rise to a relatively greater apparent change (estimated to be 4 per cent. in this case) in the equivalent temperature of the termination.
2. The difference in noise power from the two sources required that either the variation of the gain of the receiver with power output be known, or that the two sources be brought to the same intensity by means of an attenuator.

In order to overcome the first of these difficulties a telescopic waveguide was placed at the input of the receiver. The impedance of the terminations was adjusted until a negligible change in output occurred as the length of the input guide was changed.

Several different arrangements of noise measuring equipment were used to overcome the second of these difficulties.

Initially a straight superheterodyne receiver capable of receiving both the image and signal frequencies was used. The gain versus power output law of the I.F. amplifier and second detector was determined by means of a 30 Mc/s noise diode. The crystal mixer was assumed linear for the small signals involved. Owing to the reception at the image frequency difficulty was experienced in obtaining a suitable match with the various terminations.

A more satisfactory experimental set-up (Fig. 4) used a receiver with a Dicke modulating system (Dicke 1946). The increased stability of the output signal level which was obtained with this arrangement permitted the use of an image rejection cavity in front of the mixer. This eliminated the matching difficulties experienced with the straight receiver. However, since this type of receiver is essentially non-linear, a calibrated waveguide attenuator was placed between the discharge tube mount and the input of the receiver. By this means the equivalent noise temperature was reduced to the same level as that of the thermal standard.

As a check on the above measurement, the noise power output versus plate current law of a coaxial noise diode was measured at 3000 Mc/s, using the previously calibrated " straight " receiver. This was possible since the only departure from linearity of the coaxial diode occurred at very high noise outputs.

The noise diode was then used as a transfer instrument, being calibrated against the hot load at low intensities and compared with the discharge at high intensities.

The results of each of these measurements are given in Table 1 together with the estimated errors.

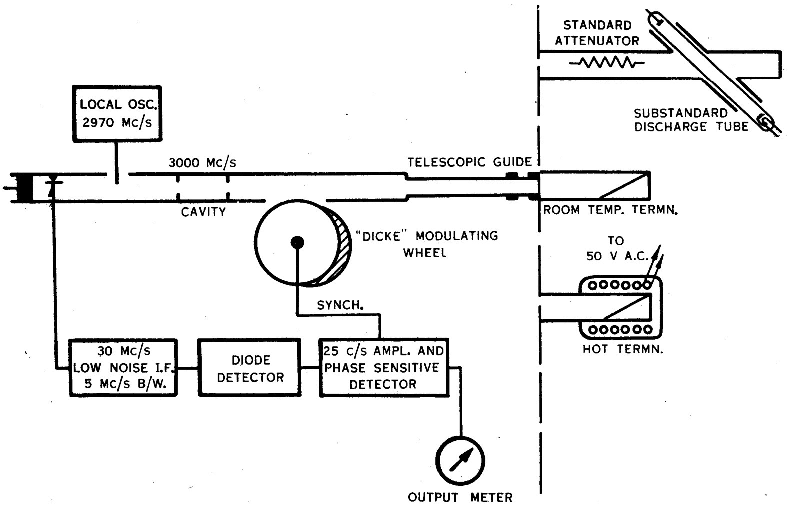

Fig. 4.—Measurement of noise temperature at 3000 Mc/s of substandard discharge tube.

(c) *Measurement of* T′ *at* 200 *Mc/s*

Measurements of the noise radiated by the positive column at 200 Mc/s were made by inserting the positive column into the high-field region of a re-entrant, tunable resonant cavity, as shown in the block diagram of Figure 5. In this case also, the portion of the positive column projecting beyond the resonant cavity was surrounded by cylindrical guide beyond cut-off, to prevent radiation generated in the electrode regions passing into the cavity.

Table 1

DETERMINATION OF SUBSTANDARD NOISE TEMPERATURE AT 3000 Mc/s

Method of Calibration	Equivalent Noise Temperature (°K)	Estimated Error (%)
1. " Straight " receiver	10,300	±10
2. Dicke system and attenuator	10,200	± 5
3. Dicke system and noise diode	10,300	± 5

The effective noise temperature of the cavity was determined by comparison with a diode noise generator whose internal reactance was tuned out at 200 Mc/s (Love 1948) and impedance matched to the characteristic impedance (75 Ω) of the transmission cable. With discharge on, the cavity was attached to the impedance measuring gear and matched to unit standing-wave ratio

(V.S.W.R. $\leqslant 1 \cdot 06$) by means of the coarse and fine capacitive tuning of the cavity and the degree of coupling of the loop. The signal frequency used for matching was accurately aligned beforehand with that of the noise measuring receiver, to which the coaxial lead from the cavity was transferred after matching. The noise measuring receiver was made up of a tuned 200 Mc/s preamplifier, followed by intermediate-frequency amplification at 30 Mc/s and at 2 Mc/s, with an effective bandwidth of 18 kc/s. The noise figure of the receiver was of the order of 10 db.

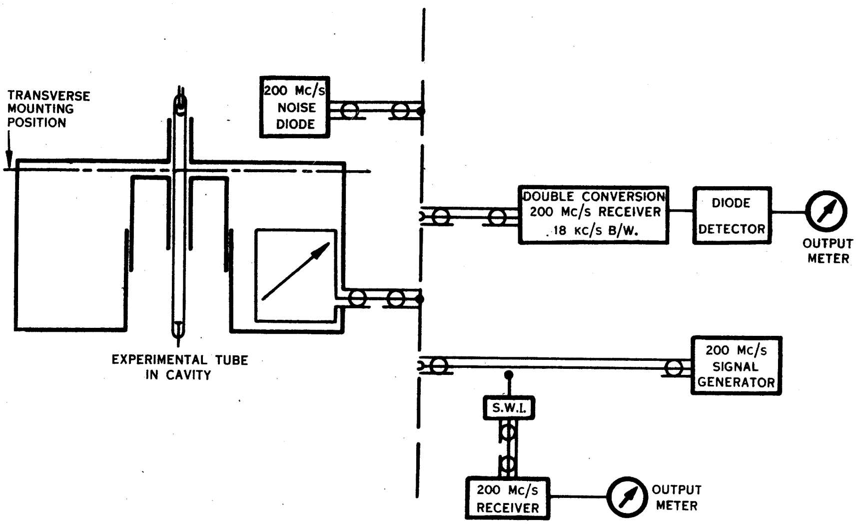

Fig. 5.—Measurement of noise temperature of discharge at 200 Mc/s.

As in the measurements at 3000 Mc/s, the noise temperature T_n of the cavity at 200 Mc/s was determined by observing the receiver output power ratios, M_n and M_s, and inserting these values in equations (**5**) and (**6**). In this case the noise temperature T_s of the standard noise source (diode generator) is given by

$$T_s = eIR/2k + T_r. \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

For the experimental conditions $I=20$ mA, $R=75\ \Omega$, and $T_r=300$ °K ; this gives $T_s=9010$ °K.

With discharge off, standing-wave measurements were then made at the resonant frequency of the cavity and at a frequency well removed from resonance, and the noise temperature T' of the discharge calculated from equation (**3**). Owing to losses in the coaxial cable connecting cavity and receiver input, σ_0 was found to be of the order of 20 db and was not negligible at this frequency.

(*d*) *Determination of Discharge Parameters*

The measurements at 3000 and 200 Mc/s were made on the positive column of discharges in neon. Several tubes were fabricated of Pyrex glass, filled at pressures in the range $0 \cdot 5$–10 mm Hg and sealed off. The positive column in each case was $1 \cdot 0$ cm in diameter and of approximate length 80 cm ; each tube was provided with a thermionic cathode, which was mounted in a buffer volume of neon at the sealing-off pressure. Care was taken to remove foreign gases thoroughly before sealing-off, and no significant changes were detected in the electrical characteristics of the tube during its experimental life. Spectroscopic examination of the radiation from the discharge was also used as a check on the purity of the gas.

A determination of the mean energy of the electrons in the discharge, available over the whole range of discharge conditions, is made possible by the existence of theoretical relationships between the mean energy of the electrons and E/p_0, the electric field strength divided by the pressure of neon at 0° C. If there is sufficient interaction between the electrons in the discharge, they will have a Maxwellian distribution of velocities with electron temperature T_{e1}, and Mierdel (1938) has given the relationship between T_{e1} and E/p_0 in this case. For the case in which there is negligible interaction between the electrons, Druyvesteyn has derived a theoretical distribution of electron velocities (given in equation (**A10**) of Appendix I), and the relationship between T_{e2}* and E/p_0 for this distribution in neon has been given by Druyvesteyn and Penning (1940). These writers also show that there is good agreement between experimental values of T_e and those predicted theoretically, for both sets of conditions.

The value of the electric field strength for insertion in these theoretical relationships was taken to be the longitudinal electric field in the positive column. Because of the great length of the positive column, this was determined with sufficient accuracy by dividing the potential drop across the discharge by its total length ; it was assumed that the cathode fall at the thermionic cathode was negligible. Measurement of the voltage gradient in one case by the use of two axial probes showed this to be a valid approximation. These measurements enabled estimates to be made of the electron temperature T_e for the two separate cases in which either a Maxwellian or a Druyvesteyn distribution was assumed to exist in the plasma.

IV. Experimental Results

Measured values of the effective noise temperature at 3000 Mc/s and at 200 Mc/s are shown in Figures 6–8 as a function of the current in each of four discharge tubes.

For low values of the current in the two tubes of lowest pressure there were, initially, oscillations in the discharge current at approximately 2 kc/s. These oscillations gave rise to anomalous standing-wave patterns at 3000 Mc/s, which were superpositions of the standing-wave patterns corresponding to the varying load presented by the discharge. It was found possible to suppress these oscillations in the case of one tube by inserting a resistor in the tube circuit at the anode, and no difficulty was then experienced in matching the discharge to V.S.W.R. $\leqslant 1\cdot03$. However, the tube of lowest filling pressure ($0\cdot495$ mm Hg at 0 °C) could not be prevented from oscillating below currents of 150 mA.

* In the non-Maxwellian case, the electron temperature is defined (Chapman and Cowling 1951) by : mean energy of electrons $=3kT_e/2$.

Corrections to the observed noise level of the cavity for cavity losses were given by equation (7). At 3000 Mc/s the correction factor $\sigma_r/(\sigma_r-1)$ ranged from unity for currents greater than approximately 20 mA to values as high as 5 as the discharge current was reduced to a few hundred microamperes. Thus

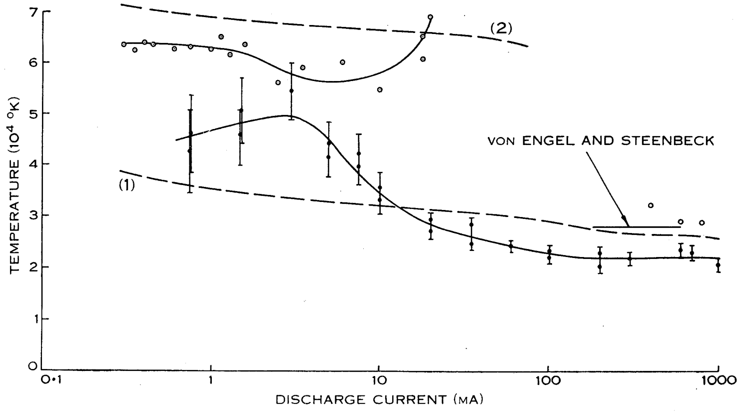

Fig. 6.—Measurements of effective noise temperature of discharge for $p_0=2\cdot80$ mm Hg : · 3000 Mc/s ; ⊙ 200 Mc/s ; ○ probe measurements of electron temperature.

the estimated experimental errors are greater at lower values of the current, as shown in Figures 6–8 ; these errors do not include possible errors in the determination of the temperature of the substandard, which amounted to ±5 per cent.

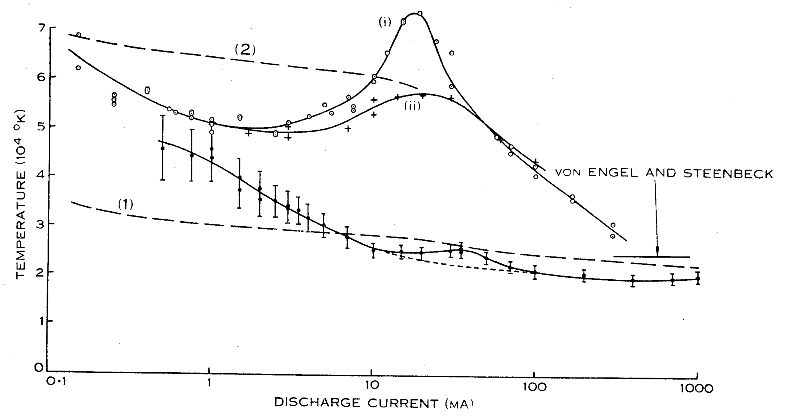

Fig. 7.—Measurements of effective noise temperature of discharge for $p_0=4\cdot75$ mm Hg : · 3000 Mc/s ; ⊙ 200 Mc/s, with axis of discharge tube parallel to E-vector in cavity ; + 200 Mc/s, with discharge tube in transverse mounting position.

The observed noise levels at 200 Mc/s were corrected for cavity and series losses by use of equation (3). The correction factor ranged approximately from $1\cdot20$ at higher values of the current investigated, up to $2\cdot5$ for low currents. The estimated errors in the measurements at 200 Mc/s amounted to ±10 per cent.

In both discharge tubes investigated at 200 Mc/s it was observed that the value of T' at first increased rapidly as the discharge current rose above a value of approximately 10 mA. In the case of the tube with filling pressure 4·75 mm Hg (Fig. 7), the measurements were extended to higher values of the discharge current ; it was found that the value of T' passed through a maximum at about 20 mA, and decreased monotonically for increase in current above this value. Also shown in Figure 7 are some results obtained with the discharge tube mounted transversely in the resonant cavity, in the position shown in Figure 5.

Further experiments were carried out with the tube of Figure 7 in order to investigate the considerable enhancement of the values of T' at 200 Mc/s over those at 3000 Mc/s, for the range of discharge current 1–100 mA. When a signal at 200 Mc/s was fed into the cavity and a pick-up loop placed in the vicinity of the positive column outside the cavity, the guides beyond cut-off

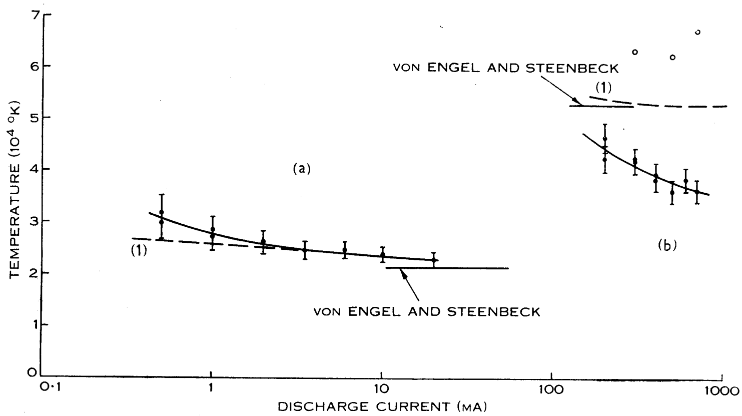

Fig. 8.—Measurements of effective noise temperature at 3000 Mc/s of discharge ; (*a*) $p_0=9\cdot42$ mm Hg, (*b*) $p_0=0\cdot46$ mm Hg. ○ Probe measurements of electron temperature.

surrounding the discharge as it passed into the cavity were found to conduct 200 Mc/s radiation as the current rose above 5 mA. This conduction presumably took place with the discharge acting as the centre conductor of a coaxial line (Rosen 1949). Measurements were made of the attenuation of the signal along the positive column when used in conjunction with an earthed, plane conductor as a strip transmission line ; it amounted to 2 db/cm for a current of 10 mA in the discharge tube of Figure 7, and decreased with increase in current. Thus the attenuation provided by each copper tube surrounding the positive column as it entered the cavity was estimated to decrease from approximately 100 db to less than 15 db as the discharge current increased through the critical value for coaxial transmission.

With the discharge current at 20 mA, the noise level at 200 Mc/s could be increased by amounts up to 10 per cent. by bringing up a magnet to the cathode region of the discharge, or by decreasing the filament supply voltage. By moving the tube longitudinally with respect to the cavity, so that the distance of the cathode from the cavity was reduced by approximately 60 cm, the noise level from the cavity could be increased monotonically up to a maximum of $1 \cdot 14$ times the level observed with maximum separation ; this experiment was carried out with the discharge current at 20 mA. Measurements were made of the 200 Mc/s noise generated (by fluctuations in the discharge current) in a 75 Ω resistor placed in series with the discharge tube ; in the range of discharge current 20–100 mA the effective noise temperature of the resistor increased from 3000 to 6000 °K.

From measured values of the longitudinal electric field in the discharge, values of the electron temperature were calculated as discussed in Section III (*d*). The calculations were carried out for two assumed distribution functions, namely, Maxwell's and Druyvesteyn's. Calculations given in Appendix I show that the effective noise temperature of a plasma with a Druyvesteyn distribution, of mean energy $\bar{\eta}=3kT_{e2}/2$, is $0 \cdot 874\ T_{e2}$ when the radio-frequency radiation arises as bremsstrahlung in collisions between electrons and positive ions ; the values of T_{e1} for a Maxwellian distribution and of $0 \cdot 874\ T_{e2}$ for a Druyvesteyn distribution are given in Figures 6–8 as a function of the discharge current.

Probe measurements of electron temperature were attempted, the results being shown in Figures 6 and 8. The measurements were not felt to be particularly reliable, because of the small diameter of the positive column and the resultant disturbance to the properties of the discharge in its immediate vicinity by the probe. For comparison, theoretical values of the electron temperature, calculated from an expression given by von Engel and Steenbeck (1934), are also shown in Figures 6–8.

V. Discussion

As an introduction to the discussion of the results presented in Section IV, we consider the expected level of thermal radio-frequency radiation from the plasma of a discharge in neon. For a sufficiently high value of the electron density, the electrons have a Maxwellian distribution of velocities (with electron temperature T_{e1}) due to the interaction between electrons, while for values of the electron density so low that electron interaction is negligible, we assume the distribution to have the form derived by Druyvesteyn, with mean energy equal to $3kT_{e2}/2$. In Appendix I it is shown that the values of the equivalent noise temperature T' of the plasma in the two cases are respectively T_{e1} and $0 \cdot 874\ T_{e2}$, when the radiation is emitted as bremsstrahlung in the collisions between electrons and positive ions. If measurements are made of the electric field strength in the positive column and values of the mean energy of the electrons calculated on the basis of one or the other distribution as the true one, we arrive at two curves describing the relationship between the value of T' and the current in the discharge, to which the electron density is related. These curves are shown schematically in Figure 9 ; curves (1) and (2) give the values of T_{e1} and $0 \cdot 874\ T_{e2}$ respectively. If the electrons have a Maxwellian distribution of velocities for values of the discharge current greater than I_1, and electron-electron interaction is negligible for values less than I_2, the value of T' is equal to T_{e1} for values of current greater than I_1, and $0 \cdot 874\ T_{e2}$ for values less than I_2. Between these values of the discharge current the electron velocity distribution undergoes a transition from one form to the other, and the equivalent noise temperature at all radio frequencies is accordingly expected to have the form shown in Figure 9 over the full range of discharge current.

The results presented in Figures 6–8 are in qualitative agreement with the theory outlined above, and we shall take the point of view that the observed increase (Figs. 6, 7) in T' at 3000 Mc/s to values greater than T_{e1} indicates a departure from a Maxwellian distribution of the electron velocities as the discharge current, and with it the electron density, decreased from the highest values. There are several features of quantitative departure from the theoretical picture, particularly in the measurements at 200 Mc/s, and we shall discuss them in turn.

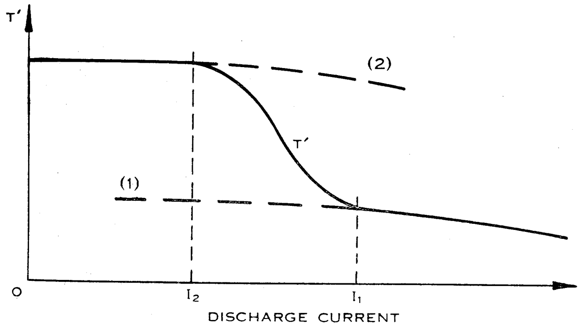

Fig. 9.—Theoretical value of the noise temperature of the positive column, T', as a function of current in the discharge.

At values of the discharge current less than 3 mA in Figure 6 and 1·5 mA in Figure 7, the correction factor for cavity losses at 3000 Mc/s was in excess of 2. Consequently the values of T' at 3000 Mc/s shown in Figures 6 and 7 for these respective ranges of discharge current may be lower than the true values, in accordance with equation (4). This is the probable reason for the observed difference in the values of T' at 3000 and 200 Mc/s for the lowest values of the discharge current.

It is probable that the electron distribution function, in the absence of electron interaction, is not exactly that derived by Druyvesteyn ; it is also possible that the effective noise temperature of a Druyvesteyn plasma may differ from $0 \cdot 874\ T_{e2}$ for other processes of emission and absorption of radiation than those considered in Appendix I. In the range of discharge conditions in which the electron density is so low that electron interaction is negligible the agreement between observed values of T' and the calculated value $0 \cdot 874\ T_{e2}$ may thus be poor.

A feature of the results shown in Figures 6–8 is the relationship between the noise temperature at 3000 Mc/s and the electron temperature in the region of higher discharge currents, where the electrons may be assumed to have a Maxwellian distribution of velocities. The values of T' and T_{e1} at different pressures are shown in Table 2, together with estimated values of the mean frequency with which an electron collides with atoms of the gas under the given conditions.

It is seen that the value of T' decreased progressively below T_{e1} as the discharge pressure was reduced, and that the collision frequency of the electrons simultaneously fell to values less than the frequency of observation (3×10^9 sec^{-1}). Johnson and DeRemer (1951) and Knol (1951) have also reported values of $T' < T_{e1}$, but it is not clear from their papers whether they have taken into account room temperature contributions to the microwave radiation from cavity losses.

If the radiation arises from discrete microscopic processes in the plasma, such as the bremsstrahlung emitted by electrons in their encounters with positive ions, there is no dependence of the effective noise temperature on the time rate at which these processes occur in a medium of infinite optical depth, other than that they should occur at random. The effective noise temperature for the radiation arising from microscopic processes involving the electrons in such a

Table 2

Electron temperature, noise temperature at 3000 Mc/s, and estimated collision frequency for Maxwellian distribution of electron energies

p_0 (mm Hg)	Current (mA)	T' (°K)	T_e (°K)	Z (sec^{-1})
9·42	10	23,800	23,800	$\sim 1 \cdot 0 \times 10^{10}$
4·75	100	21,300	24,400	$5 \cdot 1 \times 10^9$
2·80	150	22,200	28,000	$3 \cdot 5 \times 10^9$
0·46	~700	36,400	53,000	$7 \cdot 4 \times 10^8$

medium will be the electron temperature T_{e1}, provided only that the motion of the other partners in the process (e.g. ions, atoms) may be neglected relative to that of the electrons. This is a valid assumption for almost all the electrons of a Maxwellian distribution in the positive column. The observed decrease in the value of T' at 3000 Mc/s below T_{e1} leads to the conclusion that there are mechanisms for the generation of radio-frequency noise in the positive column other than microscopic processes involving the electrons, and that these additional mechanisms have an effective noise temperature at 3000 Mc/s which may fall below T_{e1}. An example of such mechanisms might be the fluctuations in the current through the discharge which arise from the collisions of the charge carriers (electrons) with the atoms of the gas.

For the case in which the electrons have a Maxwellian distribution the results given here are not sufficiently comprehensive to compare the values of T' at 3000 and 200 Mc/s and thus to establish the significance of the decrease in collision frequency below 3×10^9 sec^{-1} as the value of T' at 3000 Mc/s decreased below T_{e1} (Table 2).

We now return to a consideration of the limiting values of discharge current below which the electrons in each tube first showed departures from a Maxwellian distribution of velocities. These values were derived from the measurements of T' at 3000 Mc/s, shown in Figures 6–8 ; they were taken to be the values at which T' first showed a significant upward tendency in relation to the value of T_{e1} as the discharge current was reduced from the highest values. The limiting values of discharge current are given in Table 3, together with the corresponding estimated values of the axial electron density. The latter were calculated using values for the mobility of electrons in neon determined by Nielson (1936) ; it was assumed that the mean density of electrons in the positive column was equal to $0 \cdot 7$ times the axial density. Also shown in Table 3 are the relevant values of the parameter E/p_0, which is a measure of the energy gained from the electric field by an electron between collisions. The results show that a Maxwellian distribution is established for smaller values of the electron density as the energy gained by an electron during one free path is decreased.

TABLE 3

ELECTRON DENSITY BELOW WHICH ELECTRON DISTRIBUTION FUNCTIONS SHOW DEPARTURES FROM THE MAXWELLIAN

p_0 (mm Hg)	Limiting Current (mA)	Calculated Axial Electron Density (cm^{-3})	E/p_0 ($\mathrm{V\ cm^{-1}\ (mm\ Hg)^{-1}}$)
$9 \cdot 42$	4	4×10^{10}	$\sim 0 \cdot 7$
$4 \cdot 75$	10	7×10^{10}	$1 \cdot 37$
$2 \cdot 80$	30	$1 \cdot 4 \times 10^{11}$	$1 \cdot 93$
$0 \cdot 46$	$\geqslant 700$	$\geqslant 8 \times 10^{11}$	$\sim 9 \cdot 0$

Calculations on the interaction of electrons in a plasma (Cahn 1949) show that it should be sufficient to maintain a Maxwellian distribution for electron densities greater than 10^{11}–10^{12} cm^{-3}, for particular values $E=1$ V/cm and electron mean free path $\lambda=10^{-2}$ cm. This corresponds in the case of neon to $E/p_0 \simeq 0 \cdot 2$ V cm^{-1} (mm Hg)$^{-1}$. The lowest value of E/p_0 investigated here was $0 \cdot 7$, for which the critical density was estimated to be 4×10^{10} cm^{-3}. In view of the approximate nature of this value, it cannot be said that there is wide disagreement between these results and Cahn's theory. The observation that the critical density increased as the energy gained by an electron during one free path became larger (Table 3) is in accord with the idea that electrostatic interaction between electrons is primarily responsible for the establishment of a Maxwellian distribution.

We discuss finally the measurements of the effective noise temperature at 200 Mc/s, the results of which are shown in Figures 6 and 7. For discharge currents below 1 mA the value of T' is believed to be accounted for by the absence of electron interaction at low electron densities, so that the distribution of electron velocities is non-Maxwellian. For values of the discharge current above 1 mA we shall confine our discussion to the more extensive results of Figure 7.

The fact that a decrease in the attenuation for externally generated signals entering the cavity occurred at a discharge current of approximately 5 mA suggests that the increase in T' at 200 Mc/s, observed in the range 1–100 mA, may have been due in part to the penetration into the cavity of radiation generated at the electrodes. Labrum and Bigg (1952) have shown that the cathode of a discharge can act as a generator of radio-frequency noise at a high level. The observation of an increase in T' as the cathode moved closer to the cavity in this case is evidence that 200 Mc/s radiation from the cathode region probably played some part in the enhancement of the values of T'.

From measurements of the noise in the resistor placed in series with the discharge tube it was shown that there were fluctuations in the current in the discharge, with a component at 200 Mc/s which increased monotonically in the range 20–100 mA. This modulation may have originated in the electrode regions outside the resonant cavity. If it is assumed that the fluctuations in the discharge current at 200 Mc/s interact with the electric field in the cavity for the longitudinal position of the tube, the fluctuations will make a contribution to the noise level in the cavity. Experiments were carried out with the same discharge tube inserted in the cavity in such a position that this interaction was reduced, i.e. with the direction of current flow in the discharge normal to the electric field vector in the high-field region of the cavity : the noise level was in fact found to be reduced (Fig. 7, curve (ii)) by an amount comparable with the current noise developed in the series resistance. It is concluded that fluctuations in the discharge current at 200 Mc/s also contributed to the enhancement of T' in the range 1–100 mA.

There remains the possibility that the increase in values of T' at 200 Mc/s above those at 3000 Mc/s, in the range of discharge current 1–100 mA in Figure 7, was in part due to an intrinsic contribution to the noise level from the positive column ; i.e. the plasma within the cavity generated noise in excess of the thermal level. This possibility is supported by a re-examination of the results at 3000 Mc/s, which show some evidence of a small maximum in the value of T' at a discharge current of 35 mA. This increase in the value of T' at 3000 Mc/s must almost certainly originate within the positive column enclosed by the resonant cavity ; using the theory of Rosen (1949) it was calculated that the cylindrical tubes surrounding the positive column on either side of the cavity acted as waveguides beyond cut-off at 3000 Mc/s, for discharge currents less than 70 mA. Under this condition noise generated externally did not enter the cavity. A similar limiting value of the discharge current (70 mA) was deduced from standing wave measurements at 3000 Mc/s with the discharge as load. It is assumed that the additional source of noise in the positive column which is evident at 3000 Mc/s had a spectrum extending to 200 Mc/s.

It may be concluded that there are at least three mechanisms which contribute to the enhanced values of T' at 200 Mc/s shown in Figure 7 :

(i) radiation generated in the vicinity of the electrodes, which may enter the cavity by coaxial transmission ;

(ii) fluctuations in the discharge current at 200 Mc/s which may contribute to the radiation field within the cavity ; and

(iii) generation by the plasma within the cavity of radiation which is in excess of the thermal level.

The experimental results do not permit us to state quantitatively the relative magnitudes of the contributions from these three mechanisms.

VI. Acknowledgments

It is a pleasure to thank Mr. F. J. Lehany, Dr. J. L. Pawsey, and Mr. L. G. Dobbie for discussions ; Mr. F. C. James for his assistance in the preparation of the discharge tubes ; and Dr. R. N. Bracewell for a critical reading of the manuscript.

VII. References

Cahn, J. H. (1949).—*Phys. Rev.* **75** : 293.

Chapman, S., and Cowling, T. G. (1951).—" The Mathematical Theory of Non-uniform Gases." 2nd Ed. (Cambridge Univ. Press.)

Dicke, R. H. (1946).—*Rev. Sci. Instrum.* **17** : 268.

Druyvesteyn, M. J., and Penning, F. M. (1940).—*Rev. Mod. Phys.* **12** : 87.

Easley, M. A., and Mumford, W. W. (1951).—*J. Appl. Phys.* **22** : 846.

von Engel, A., and Steenbeck, M. (1934).—" Elektrische Gasentladungen." Vol. 2. (Springer: Berlin.)

Fowler, R. H. (1936).—" Statistical Mechanics." 2nd Ed. (Cambridge Univ. Press.)

Johnson, H., and DeRemer, K. R. (1951).—*Proc. Inst. Radio Engrs., N.Y.* **39** : 908.

Knol, K. S. (1951).—*Philips Res. Rep.* **6** : 288.

Labrum, N. R., and Bigg, E. K. (1952).—*Proc. Phys. Soc. Lond.* B **65** : 356.

Love, A. W. (1948).—*J. Instn. Engrs. Aust.* **20** : 33.

Mierdel, G. (1938).—*Wiss. Veröff. Siemens* **17** : 71.

Mumford, W. W. (1949).—*Bell Syst. Tech. J.* **28** : 608.

Nielsen, R. A. (1936).—*Phys. Rev.* **50** : 950.

Rosen, P (1949).—*J. Appl. Phys.* **20** : 868.

Slater, J. C. (1950).—" Microwave Electronics." (D. van Nostrand & Co. Inc. : New York.)

Smerd, S. F., and Westfold, K. C. (1949).—*Phil. Mag.* **40** : 831.

Westfold, K. C. (1950).—*Phil. Mag.* **41** : 509.

Westfold, K. C. (1953).—*Phil. Mag.* **44** : 711.

Woolley, R. v. d. R. (1947).—*Aust. J. Sci.* **10** (2) : Suppl.

Appendix I

By L. W. Davies

Radio-frequency Radiation from Non-equilibrium Discharges in Neon

A discussion is given here of the intensity level of the radio-frequency radiation from a plasma in which the electrons are assumed to have a distribution of energies derived by Druyvesteyn (Chapman and Cowling 1951, Sect. 18.71) ; the distribution applies to a plasma in which there is negligible interaction between electrons. The problem is approached by deriving the effective temperature for radio-frequency radiation of a plasma of infinite optical depth, when the radiation (bremsstrahlung) is emitted and absorbed during free-free encounters of electrons with positive ions.

The equation of transfer of radio-frequency radiation along a ray trajectory passing through an ionized medium may be written (Woolley 1947)

$$\mu_f^2 \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{I_f}{\mu_f^2}\right)=\eta_f-K_f I_f, \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A1})$$

where I_f is the intensity of radiation in the frequency range $(f, f+\mathrm{d}f)$ in the direction of the ray, η_f is the volume emissivity, K_f the absorption coefficient, μ_f the refractive index of the medium, and s the distance measured along a ray trajectory.

Upon the introduction of the optical depth τ_f, defined by

$$\tau_f(s)=\int_0^s K_f \mathrm{d}s, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A2})$$

the solution of (**A1**) is

$$\frac{I_f}{\mu_f^2}=\left(\frac{I_f}{\mu_f^2}\right)_0 \mathrm{e}^{-\tau_f}+\mathrm{e}^{-\tau_f}\int_0^{\tau_f} F_f \mathrm{e}^{\tau_f}\mathrm{d}\tau_f, \quad \ldots\ldots\ldots \quad (\mathbf{A3})$$

where

$$F_f=\frac{\eta_f}{K_f\mu_f^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A4})$$

is the ergiebigkeit of the medium for radiation in the frequency range $(f, f+\mathrm{d}f)$ (Smerd and Westfold 1949), and the subscript 0 refers to the zero level for τ_f.

If the ionized medium is uniform in its properties, so that the ergiebigkeit F_f is independent of optical depth, it follows from equation (**A3**) that

$$\frac{I_f}{\mu_f^2}=\left(\frac{I_f}{\mu_f^2}\right)_0 \mathrm{e}^{-\tau_f}+F_f(1-\mathrm{e}^{-\tau_f}), \quad \ldots\ldots\ldots\ldots \quad (\mathbf{A5})$$

and for a medium of infinite optical depth,

$$\frac{I_f}{\mu_f^2}=F_f. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A6})$$

Hence the intensity of radiation is directly related to the ergiebigkeit for such a medium. In the particular case of a medium in thermodynamic equilibrium at temperature T we have from the Kirchhoff relationship

$$F_f=B_f(T)=\frac{2k}{c^2}f^2T, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A7})$$

where $B_f(T)$ is the intensity of black radiation at temperature T. Thus in this case

$$I_f=\mu_f^2\frac{2k}{c^2}f^2T. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A8})$$

For non-equilibrium conditions we define an effective temperature T' for the emergent radiation by means of the relationship

$$I_f=\mu_f^2\frac{2k}{c^2}f^2T'. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{A9})$$

Smerd and Westfold (1949) and Westfold (1950) have calculated the ergiebigkeit for an ionized medium in equilibrium from the microscopic processes occurring in the plasma ; they assumed that the contributions to the r.f. radiation field arose from the bremsstrahlung emitted by electrons during their collisions with positive ions. Making this same assumption, we derive the ergiebigkeit, in this case, for an ionized medium in which the electrons have a distribution of energies $\mu(\eta)$ given by Druyvesteyn, namely,

$$\mu(\eta)=B\eta^{\frac{1}{2}}e^{-\alpha\eta^2/\bar{\eta}^2}, \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(A10)}$$

where $\alpha=[\Gamma(5/4)/\Gamma(3/4)]^2$, $\bar{\eta}$ is the mean energy, and from the relationship

$$\int_0^\infty \mu(\eta)d\eta=N_1,$$

where N_1 is the electron density, it follows that

$$B=2N_1\alpha^{3/4}/\{\bar{\eta}^{3/2}\Gamma(3/4)\}.$$

When the motion of the positive ions in the plasma is neglected, the number of collisions per unit time per unit volume between ions and electrons such that the electron energy η lies in the range $(\eta, \eta+d\eta)$ and the collision parameter b lies in $(b, b+db)$ is given by (Fowler 1936, Sect. 19.32)

$$N_2 \,.\, 2\pi b \,.\, db \,.\, \left(\frac{2\eta}{m}\right)^{\frac{1}{2}}\mu(\eta)d\eta,$$

where N_2 is the number-density of positive ions, and m is the mass of the electron.

It follows that the number ν of collisions per unit time made by one electron with the ions of the medium is

$$\nu=\frac{1}{N_1}\int_0^\infty\int_0^\sigma N_2 \,.\, 2\pi b\left(\frac{2\eta}{m}\right)^{\frac{1}{2}}\mu(\eta)db\,d\eta. \quad \ldots\ldots\ldots \quad \text{(A11)}$$

In this expression the upper limit σ to the variable b is an effective collision distance for the ions. It has been discussed by Smerd and Westfold (1949) and Westfold (1953).

From Smerd and Westfold's treatment we have

$$K_f=4\nu f_0^2/3cf^2\mu_f, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A12)}$$

where

$$f_0^2=N_1e^2/\pi m. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A13)}$$

The spontaneous emission per unit volume per unit time is given by

$$4\pi\eta_f \,.\, df=\int_0^\infty\int_0^\sigma Q_f \,.\, df \,.\, N_2 \,.\, 2\pi b\left(\frac{2\eta}{m}\right)^{\frac{1}{2}}\mu(\eta)db\,d\eta, \quad \ldots\ldots \quad \text{(A14)}$$

where $Q_f \mathrm{d}f$ is the energy radiated in the frequency range $(f, f+\mathrm{d}f)$ as bremsstrahlung during a collision between electron and positive ion. Its value is given by Westfold (1950)

$$Q_f \mathrm{d}f = \frac{16e^2}{3c^3} \cdot \frac{2\eta}{m} \cdot \frac{\mu_f}{\{1+(2\eta b/Ze^2)^2\}}, \quad \text{(A15)}$$

where Ze is the charge on the positive ion.

We are now in a position to evaluate the ergiebigkeit for arbitrary electron energy distribution $\mu(\eta)$, and thence the effective temperature T' from equations (**A6**) and (**A9**). We shall do so first for the case of a Maxwellian distribution of electron velocities. In this case

$$\mu(\eta) = 2\pi N_1 \eta^{\frac{1}{2}} (\pi k T_e)^{-3/2} \exp(-\eta/kT_e), \quad \text{(A16)}$$

whence we have from equations (**A11**) and (**A14**):

$$\nu = 2N_2\sigma^2(2\pi kT_e/m)^{\frac{1}{2}}, \quad \text{(A17)}$$

and

$$\eta_f = \frac{4\sqrt{2}N_1N_2Z^2e^6\mu_f}{3\sqrt{\pi}c^3(mkT_e)^{3/2}} \int_0^\infty \left[\ln\left\{1+\frac{4\eta^2\sigma^2}{Z^2e^4}\right\}\right] \exp(-\eta/kT_e)\mathrm{d}\eta,$$

that is,

$$\eta_f = 8N_1N_2Z^2e^6\mu_f A_1(2)/\{3mc^3(2\pi mkT_e)^{\frac{1}{2}}\}, \quad \text{(A18)}$$

where $A_1(2)$ is the mean value of the slowly varying logarithmic function $\ln\{1+4\eta^2\sigma^2/Z^2e^4\}$, and we have made the approximation of replacing this function by its mean value in the integral. Smerd and Westfold give

$$\sigma = \{A_1(2)\}^{\frac{1}{2}} Ze^2/2kT_e. \quad \text{(A19)}$$

From equations (**A4**), (**A12**), (**A13**), (**A16**), (**A17**), and (**A18**) we find for the ergiebigkeit

$$F_f = \frac{2k}{c^2} f^2 T_e$$
$$= B_f(T_e).$$

Thus the effective noise temperature of a plasma, in which the electrons have a Maxwellian distribution of energies with temperature T_e, is equal to the electron temperature.

We now calculate the ergiebigkeit for the Druyvesteyn distribution, equation (**A10**).

From equations (**A10**) and (**A11**) we have

$$\nu = \pi N_2\sigma^2 B\bar{\eta}^2/\{\alpha N_1(2m)^{\frac{1}{2}}\}. \quad \text{(A20)}$$

Similarly from equations (**A10**), (**A14**), and (**A15**),

$$\eta_f = \frac{4BN_2Z^2e^6\mu_f}{3mc^3(2m)^{\frac{1}{2}}} \int_0^\infty \left[\ln\left\{1+\frac{4\eta^2\sigma^2}{Z^2e^4}\right\}\right] \exp(-\alpha\eta^2/\bar{\eta}^2)\mathrm{d}\eta,$$

that is,

$$\eta_f = (2\pi)^{\frac{1}{2}} BN_2Z^2e^6\mu_f A_1(2)\bar{\eta}/\{3mc^3(m\alpha)^{\frac{1}{2}}\}, \quad \text{(A21)}$$

where we have again made the approximation of replacing the slowly varying logarithmic function by its (constant) mean value. The equivalent relationship to equation (**A19**) is now

$$\sigma=3\{A_1(2)\}^{\frac{1}{2}}Ze^2/4\bar{\eta}. \qquad \text{(A22)}$$

From equations (**A4**), (**A12**),* (**A13**), (**A19**), (**A20**), and (**A21**) we have for the ergiebigkeit

$$F_f=8(\pi\alpha)^{\frac{1}{2}}f^2\bar{\eta}/9c^2. \qquad \text{(A23)}$$

Inserting the value $\alpha=[\Gamma(5/4)/\Gamma(3/4)]^2$ we find

$$F_f=\frac{2f^2}{c^2}\left[\frac{4\sqrt{\pi}}{9}\,\frac{\Gamma(5/4)}{\Gamma(3/4)}\right]\bar{\eta},$$

that is,

$$F_f=\frac{2f^2}{c^2}[0\cdot874kT_e] \qquad \text{(A24)}$$
$$=0\cdot874B_f(T_e),$$

where the electron temperature T_e is related to the mean energy by

$$\bar{\eta}=\frac{3}{2}kT_e.$$

Thus the calculated value of effective noise temperature of the plasma in this case is $0\cdot874T_e$.

* Dr. Westfold has kindly pointed out that the use here of equation (**A12**) involves the assumption that the expression for K_f remains unchanged for a Druyvesteyn distribution.