THE MEASUREMENT OF THE THERMAL CONDUCTIVITIES OF ROCKS BY OBSERVATIONS IN BOREHOLES

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Summary

The various possible methods of measuring thermal conductivity by observations in a borehole are discussed in the light of practical considerations. Three designs of measuring head are described and methods of reducing the observations are given. The possibility of obtaining results by heating a considerable length of a borehole is discussed.

I. Introduction

For the determination of the geothermal flux and similar measurements a knowledge of the thermal conductivities of rocks is necessary. In most cases these have been determined in the laboratory by the use of a steady state apparatus of the " comparison " or " divided bar " type. Such methods can easily give an accuracy of 1 per cent. for an individual specimen, but it is doubtful how far this represents the conductivity of the rock in bulk. For example measurements, made by a divided bar apparatus (cf. Benfield (1939), three and sometimes four disks were used) on 20 samples from a mass of petrologically very uniform coarse-grained granite, gave values varying apparently at random from 0·0069‡ to 0·0101 with a mean of 0·0082 ; this was confirmed by measurement of an additional 50 samples in a comparison apparatus (Beck 1955). These variations are probably caused in part by the fact that the diameters of the individual mineral grains (or aggregates of grains) are greater than the thicknesses of at least the thinner disks used, but they probably also represent to some extent random fluctuations in conductivity. It is difficult to know how to use these results in the calculation of heat flux, for it is clear that the association of a measured temperature with a single measured conductivity at (say) 50 ft intervals may not lead to representative results. Probably the best that can be done is to use the simple mean value, and this suggests that a method for measuring conductivities in bulk which was accurate to within 5 per cent. or even 10 per cent. would be more useful than the high precision laboratory methods. There are other important reasons for preferring measurements of rock conductivity *in situ* to laboratory determinations : firstly, the latter cannot take into account the effect of veins or open joints in the rock which may well be important, and, secondly, they are selective both in the sense that disks

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‡ Units used throughout this paper are c.g.s., calorie, and °C.

can only be cut from fairly sound rock and that core may not be recovered at all from highly altered or sheared zones. The last is probably the most important consideration of all, for example as remarked in Section IV, a conductivity of 0·0042 was measured *in situ* in a highly altered and vesicular basalt, the massive portions of which (constituting the bulk of the core recovered) gave a value of 0·0052 in the laboratory.

It is clear that the measurement of thermal conductivity of rock *in situ* from a borehole would have many advantages : it would enable the conductivity to be measured at the same time as the rock temperature ; it would give a result representing the average conductivity of the rock over a region determined by the length of the apparatus and the depth of penetration of heat into the rock ; and, finally, it would avoid the tedious preparation of disks for use in the divided bar apparatus.

The obvious method of making such experiments is by lowering into the borehole a long probe containing a temperature measuring element and a heater which acts as a line source for supply of heat to the surrounding rock. This system has frequently been used in laboratory measurements (e.g. van der Held, Hardebol, and Kalshoven 1953 ; Maxwell, personal communication) and has also been used by Blackwell (1954) for measurements in short holes in tunnel walls.

The object of this paper is to discuss the possible methods of measuring rock conductivity from a single hole. Practical requirements of any method are stated in Section II. The apparatus used in field experiments is described in Section III. The results of some probe measurements are analysed in Section IV. The possibility of measuring conductivity by suddenly heating the contents of the hole is discussed in Section V, and, finally, the heating of the whole length of a hole is discussed in Section VI.

II. Experimental Conditions

Almost all the discussion of the present paper refers to field conditions : there would be little point in using apparatus of the present dimensions in the laboratory, and there is no point in developing methods which may be applicable to laboratory conditions but which cannot be used in the field. The requirements on a method are listed below.

(*a*) *Diameter of Holes*

The smallest drill commonly used for exploratory diamond drilling has a diameter of $1\frac{1}{2}$ in., which is also approximately the size of the usual jackhammer hole drilled in a tunnel wall. The apparatus described below was built for use in holes of this diameter since these are most common in Tasmania where the field work was carried out. In many holes the diameter varies, larger drill diameters being used in the upper parts of the hole, so that it is essential that it be possible to modify the apparatus in the field to take account of this ; also, towards the top of a hole the diameter is usually a little greater than that of the drill and the apparatus should be sufficiently flexible to adapt itself to small changes of this sort. In addition, holes are almost always irregular, frequently there are small shearing movements across them after drilling, and occasionally loose pieces of material protrude into the hole. It is thus essential that, to provide clearance, the diameter of the measuring head should be some millimetres less than the rated diameter of the hole.

(b) *Contents of Holes*

Most vertical boreholes are filled with water, but occasionally a hole is encountered which is dry for portion of its length. The presence of water implies that an extremely good seal is necessary at the ends of the heating element to prevent loss of heat by convection. Experiments in glass tubes indicate that, at the rate of heat supply used, convection will be set up through a gap of $0 \cdot 5$ mm or even less. Holes which are not filled with water have other disadvantages, notably the high thermal resistance of the air between the heater and the rock, and the possibility of evaporation from the surface of the rock (which is usually damp) and of movement of moisture within the rock.

(c) *Time of Experiment*

Half an hour has been regarded as a reasonable time to be aimed at for a single measurement. If longer times are used the task of logging a deep hole becomes intolerably tedious. Also, it was felt that the danger of heat loss by convection with the seal of Figure 1 (*a*) would be greater for long runs.

The choice of this relatively short time of experiment rules out the possibility of using the approximations to the theoretical values of the temperature which are valid for large values of the time. Longer times of experiment in water-filled holes have the advantage that the depth of penetration of heat is greater, so that a more representative value of the conductivity should be obtained. For damp holes, longer times of experiment have the same disadvantage as steady state methods, namely that moisture movement and evaporation at the surface may have important and unknown effects.

III. Experimental Measuring Heads

As remarked above, the core of the problem is the provision of an adequate seal at the ends of the heating element in order to prevent convection. Three different measuring heads have been constructed which have proved satisfactory and are shown schematically in Figure 1.

The head of Figure 1 (*a*) consists of a brass rod $1\frac{1}{4}$ in. diameter and 3 ft long wound with the heating wire on its external surface. Temperature is measured by a thermistor T inside the rod, the internal arrangements being similar to that of the temperature measuring head shown in Newstead and Beck (1953). Measurement is by an A.C. bridge connected to the thermistor by a coaxial cable, the sheath of this and the supporting wire forming the heater leads L. The seals R at the ends are composed of a large number of thin rubber sheets cut radially to form a " bottle brush " ; these are attached to a poorly conducting " Perspex " cylinder and can easily be changed if holes of different diameters have to be logged. They adapt themselves very well to small changes of shape and diameter and have been observed to inhibit convection completely when the head has been operated in a glass tube. In one or two cases theoretically impossible temperature curves have been obtained which have been attributed to convection occurring through an open joint that was impossible to seal.

The head of Figure 1 (*b*) is similar to that of Figure 1 (*a*) except in length (5 ft) and in that the seals *D* are pneumatic. They consist of 8 in. lengths of inflatable rubber tube (" Ductube ") connected to an air-line hose *A* in which the heater and temperature measuring leads are carried. The " Ductubes " can be unscrewed and changed to different sizes when necessary. This arrangement will not merely give a seal which prevents convection but will completely stop flow through the hole under a considerable head. It has been used for measuring water flow in and across boreholes and will be described in detail elsewhere. The disadvantage of this head is that the air-line hose and gas cylinder for inflating it are cumbersome and that the depths to which it can be used are limited ; 300 ft has been used without difficulty.

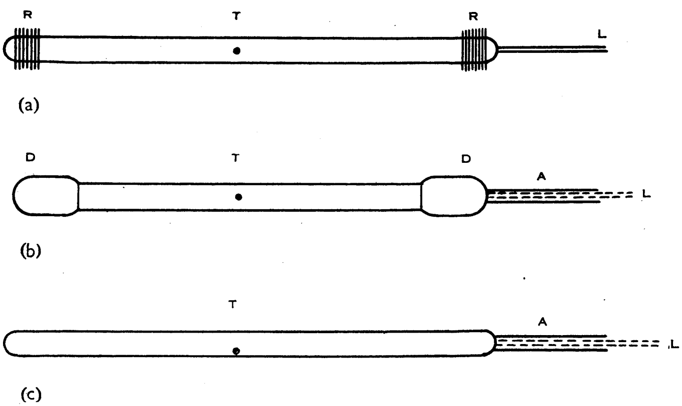

Fig. 1.—Schematic diagrams of measuring heads.

The head of Figure 1 (*c*) is an obvious development of that in Figure 1 (*b*). It consists of a rubber tube on which the heater wires are laid longitudinally, covered with thin sheet rubber, and the whole vulcanized. There are 50 wires around the circumference. The thermistor is attached to the inside of the tube. This head has the great advantage of completely eliminating the air or water space between the heater and the surface of the hole. It has been used in auger holes for measurements in soils and is very suitable for measurements in short drill holes. In such cases it can be inflated by a hand pump. As remarked earlier, in air-filled holes a gap of more than 1 mm is undesirable though gaps as large as 3 or 4 mm can be used. This implies that with any other design of head a number of heads with different diameters must be constructed if holes are at all irregular. In water-filled holes the situation is not so bad since substantial transfer of heat by convection takes place in a large water space.

It should be mentioned that in the heads of Figures 1 (*a*) and 1 (*b*) the thermistor was separated from the wall of the tube by an air- (or oil-) filled space. This was done in order to minimize the possibility of thermistors being broken or changing their constants by mechanical shocks during lowering or raising. This has had the effect of introducing a time-lag into the thermistor readings. This has been approximately corrected for as follows: if M' and c' are the mass and specific heat of the thermistor, R is the thermal resistance between it and the surrounding region, and V and v are the temperatures of this region and the thermistor, then

$$M'c'R\ \mathrm{d}v/\mathrm{d}t+v=V. \qquad (1)$$

The "time constant" $M'c'R$ of the system is measured by immersing the head in water and is of the order of $\frac{1}{2}$ min. All experimental readings are then corrected by (**1**), but only those for times greater than 3 or 4 min, where this correction is small, have been used.

IV. Discussion of Experimental Results

The necessary theory has been given in a companion paper (Jaeger 1956*a*) which will be referred to as paper I. The system may be idealized as follows: the rock outside the hole of radius a has thermal conductivity K, density ρ, specific heat c, and diffusivity $\varkappa=K/\rho c$; the hole contains perfect conductor of thermal capacity S per unit length, to which heat is supplied at the rate Q per unit time per unit length, and there is a thermal contact resistance $1/H$ per unit area at $r=a$.

Then the temperature rise V of the perfect conductor at time t after switching on the current is given by

$$V=(Q/K)G(h,\ \alpha,\ \tau), \qquad (2)$$

where G is a complicated integral for which numerical values are given in paper I, and h, α, τ are the dimensionless parameters:

$$h=K/aH, \quad \alpha=2\pi a^2\rho c/S, \quad \tau=\varkappa t/a^2. \qquad (3)$$

The order of magnitude of these parameters is of great importance. For a hole of radius 2 cm in rock of diffusivity $\varkappa=0\cdot01$, $\tau=0\cdot3$ after 2 min and is $4\cdot5$ after $\frac{1}{2}$ hr: this is just the range in which neither the approximate solutions for small nor large values of the time (paper I, equations (**21**), (**22**), (**23**)) apply. Even if two terms of the asymptotic expansion for large values of the time are used, as is done by Blackwell (1954), larger values of the time are needed, particularly for large values of h.

The values of h to be expected may be seen from the fact that $h=5$ for a film of air 1 mm thick in a hole of diameter 4 cm in rock of conductivity $0\cdot0053$. Thus, in a dry hole values of h greater than this are to be expected. For a water-filled hole values will be correspondingly smaller. In both air- and water-filled holes some of the transport of heat through the region between the heater and the rock will be by conduction and some by convection, and, since this latter will be non-linear, the theory can only be regarded as approximate: the fact that the experimental curves agree so well with the theoretical ones indicates that it is a good approximation.

For a hole completely filled with water, α is about 1; for a hole completely filled with brass, it is about $1\cdot5$; if the head is not solid but tubular, higher values of α occur, for example, for the head of Figure 1 (*c*), α is about 6. In water-filled holes there is some uncertainty as to whether the whole of the thermal capacity of the water should be included in S: this implies that, even if ρc is measured in the laboratory, the value of α is not precisely known.

In principle, all of h, α, and $\varkappa$ have to be found from a single experimental curve. Blackwell proposes to find h from an approximate solution valid for small values of the time (paper I, equation (**21**)) and α and $\varkappa$ from the asymptotic expansion (paper I, equation (**23**)) for large values of the time. In the present work, using only moderate values of the time, a method of curve-fitting has usually been employed.

It follows from (**2**) that

$$\log V = \log (Q/K) + \log G(h, \alpha, \tau). \qquad \ldots\ldots\ldots\ldots \quad (4)$$

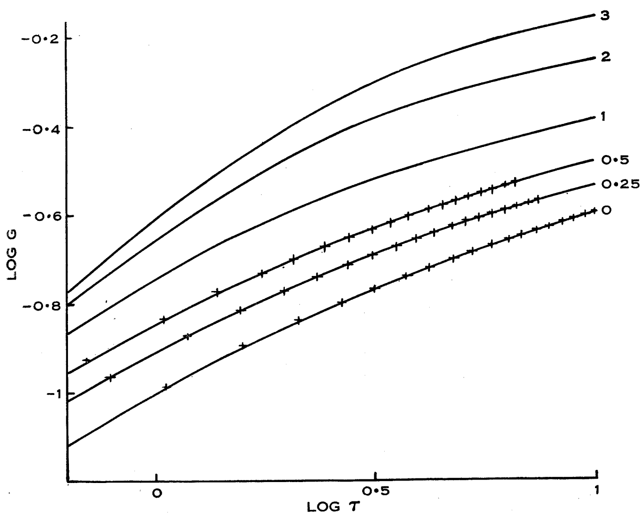

Fig. 2.—Values of log $G(h, 2, \tau)$. The numbers on the curves are the values of h. Crosses denote experimental points fitted to the curves.

A plot of log $G(h, \alpha, \tau)$ against log τ for $\alpha=2$ and various values of h is shown in Figure 2. The experimental values of log V are plotted against log t on transparent graph paper and slid over a family of curves such as that of Figure 2, keeping the axes parallel, until the best fit is obtained. The displacement of the axes then gives log (Q/K) and log $(\varkappa/a^2)$.

When this process is carried out it is found that reasonable fits may be obtained with curves of different parameters but that all these give similar values of K though they may give widely different values of ρc. Since ρc does not vary widely between rocks it is easy to make a best-possible choice of K.

As an example, a number of experimental points from a run of 100 min duration* with the head of Figure 1 (b) in a water-filled hole in vesicular basalt are shown by crosses in Figure 2 fitted to the different curves of the family. The values of K and ρc found from these and other curves are set out in Table 1.

* A number of longer runs such as this was made to check the fit of the experimental curves over a wider range.

Of these, the fit of the points with the curve $\alpha=2$, $h=0\cdot25$ is slightly the better and a rather high value such as $\rho c=0\cdot79$ might well be due to the presence of water filling the vesicles. In any case it is clear that K lies between $0\cdot0040$ and $0\cdot0042$. The present method of reduction is not regarded as being adequate to determine ρc: the value of the latter is simply used to discriminate between the various possible values of K. There are, of course, many other ways in which the experimental curves may be treated. In particular, it greatly simplifies the reduction if the temperatures for small values of the time can be relied on to be completely accurate. If this is the case, H can be determined from the relation

$$2\pi aHV=Q-S\ \mathrm{d}V/\mathrm{d}t, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

which holds until the change in surface temperature of the rock becomes important, and, from H and a trial value of K, h can be found. The curve of Figure 2 which has to be fitted is then known.

TABLE 1

POSSIBLE FITS OF EXPERIMENTAL VALUES

Curve	K	$\varkappa$	ρc
$\alpha=2$, $h=0$ ..	$0\cdot0040$	$0\cdot0073$	$0\cdot55$
$\alpha=2$, $h=0\cdot25$ ..	$0\cdot0042$	$0\cdot0053$	$0\cdot79$
$\alpha=2$, $h=0\cdot5$ ..	$0\cdot0046$	$0\cdot0047$	$0\cdot98$
$\alpha=1\cdot5$, $h=0$ ..	$0\cdot0042$	$0\cdot0094$	$0\cdot44$

Unfortunately no sound holes were available, so that conductivities measured by this method could not be compared with those found in the laboratory using core samples. For example, the basalt core from the hole in which the run above was taken was mostly so highly vesicular that its conductivity could not be measured in the laboratory, while massive samples, which were totally unrepresentative, gave a value of $0\cdot0052$.

In another hole in hard shale conductivities of $0\cdot0079$ and $0\cdot0080$ were measured below the water-table and $0\cdot0076$ above it. Laboratory measurements on surface rock (no core was available) were of this order.

A number of laboratory experiments was made in a large cement cylinder using the head of Figure 1 (*a*). Three separate experiments with the hole filled with water gave $0\cdot0054$, $0\cdot0050$, $0\cdot0050$, while, with the hole filled with air but the cement saturated with water, the measured values were $0\cdot0050$ and $0\cdot0047$. Conductivities for cement from the cylinder measured in the divided bar apparatus were $0\cdot0042$ for the cement dry, and $0\cdot0050$ when saturated with water.

Finally, it may be mentioned that, in the first series of field tests made, the heater was supplied from the alternator mentioned in Section VI. It was found that it was impossible to measure temperatures accurately because of cross-talk between the A.C. supplies to the heater and bridge. A regime was then adopted of running the heater on for 4 min and off for 1 min, the temperature being measured at the end of the "off" minute. There is no difficulty in principle in using such results since curves similar to those of Figure 2 can be calculated for "on-off" heating. Unfortunately, the time-lag in the thermistor made these experiments too difficult to interpret, but they are mentioned as confirming the observations of a later series (with the heater run from batteries) that the temperature curves are reproducible in successive runs in the same position to a few hundredths of a degree.

V. The Interior of the Hole Suddenly Heated

This method has been discussed by Bullard (1954) in connexion with temperature measurements by the deep-sea probe. He pointed out the possibility of measuring thermal conductivity by heating a probe rapidly by a high current.

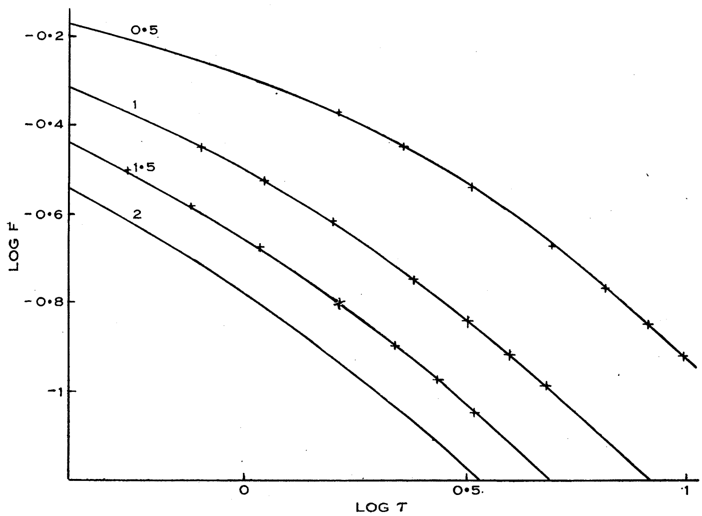

Fig. 3.—Values of $\log F(0, \alpha, \tau)$. The numbers on the curves are values of α. Crosses denote experimental points fitted to the curves.

An alternative, which is attractive because of its simplicity, is suggested by a common method of measuring temperatures in tunnel walls, namely, filling an inclined drill hole with water and measuring the temperature of the water some hours later when it has attained equilibrium with the rock. Clearly, if the water is kept well stirred and the variation of its temperature with time is measured, K can be found. Subject to the obvious practical difficulties, the method has been found to work perfectly in laboratory experiments. In this case the contact resistance may be taken to be zero, and if V_0 is the initial value of the excess of the temperature of the water above the rock and V its value at time t,

$$V/V_0 = F(0, \alpha, \tau), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where $F(0, \alpha, \tau)$ is given in paper I, Table 1, and α and τ are defined in (3). A plot of $\log F(0, \alpha, \tau)$ against $\log \tau$ is shown in Figure 3.

To reduce the experimental results, the logarithms of the observed values of the temperature excess are plotted against $\log t$ on transparent graph paper and slid across the family of Figure 3, keeping the axes parallel, until a best fit is obtained. The displacements of the axes then give $\log (\varkappa/a^2)$ and $\log V_0$; since some heat is usually lost in adding the water it is better to regard V_0 as unknown—if it is known accurately, translation in one direction only is necessary.

In Figure 3, experimental points for the cement cylinder are shown fitted to the curves for $\alpha=0\cdot5$, $1\cdot0$, and $1\cdot5$. It may be anticipated that α is nearer to 1 than the other values and this curve gives a slightly better fit, but all curves give the same value $K=0\cdot0052$. The result by the method of Section IV was $0\cdot0051$ and in the divided bar apparatus $0\cdot0050$.

VI. The Heating of a Whole Borehole

As remarked earlier, experiments with a heating probe for times of the order of an hour, give an indication of the conductivity only in a thin region surrounding the hole (at the end of an hour in a hole of radius 2 cm the temperature at a distance of 8 cm from the surface is less than one-fifth of the surface temperature) and this is not adequate to indicate the effect of large-scale features such as joints. If experiments are carried on for much longer times very long probes must be used to ensure radial flow of heat and also the time involved in measurements at several points would be prohibitive.

One simple method of avoiding these difficulties is to heat a considerable length of the hole for a long time and to observe the temperatures at various depths, preferably with a string of thermistors as used by Balsley (personal communication). If this is done, convection will take place throughout the length of the hole and the effect of this can only be determined by experiment. It might be hoped that a stable temperature gradient would be established fairly rapidly so that the previous theory could be applied subsequently, but at the worst, temperature measurements at any depth give a complete temperature-time curve at that depth, from which the temperature distribution within the rock at the instant of switching off the current can be calculated (Jaeger 1956*b*), and this, together with the observed temperatures in the hole after switching off, gives an integral equation, which can be solved numerically for K.

In view of this, and also of the possibility of obtaining information about the time which must elapse after drilling for a borehole to attain equilibrium, it was decided to try the experiment of heating a length of a borehole. To provide power, a 3 kW alternator was mounted in a " Landrover ". Unfortunately, on examination, the only suitable hole had a small water flow from it, but as it was felt that this might be superficial, the experiment was continued. A length of 285 ft of cable was lowered into the hole, the power to the heater being 2400 W or $0\cdot066$ cal cm^{-1} sec^{-1}. The hole was heated for 700 min, the current being then switched off and the recovery of temperature observed. Temperature measuring heads were located at depths of 75 and 175 ft, their records being shown in Figure 4. The hole was in dolerite whose average conductivity as measured in the laboratory was $0\cdot0048$ and its diameter was $2\frac{5}{16}$ in. from the collar to 128 ft, and $1\frac{7}{8}$ in. below that.

The dotted curves are calculated as in Section IV on the assumption that there is no movement of water. As remarked earlier, there will certainly be convection in the early stages of heating but this is not sufficient to account for the differences between the observed and calculated curves since this difference is also shown by the cooling curves, in which the effect of convection should be absent. Curves tending to a horizontal asymptote were shown in paper I, Section VI, to be characteristic of heat exchange by movement of water in the hole, and a small flow of water up the hole undoubtedly accounts for the present results (this flow must be much smaller than that issuing from the collar of the hole, which must be a surface effect).

These results, particularly the nature of the cooling curves, indicate that the method is well worth trying in a hole in which there is no water movement.

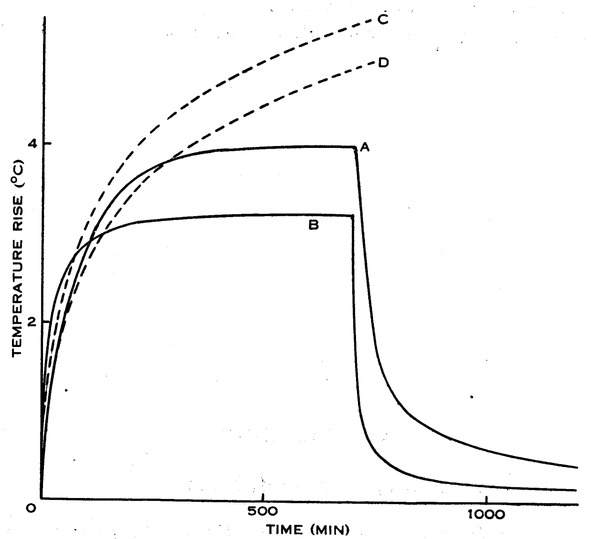

Fig. 4.—Temperatures at 75 ft (curve *A*) and 175 ft (curve *B*) in a borehole heated to 285 ft. Dotted curves *C*, *D* are those expected in the complete absence of water movement.

VII. Movement of Water Along and Across Boreholes

One by-product of the present experiments was the discovery that water movement in boreholes occurs very frequently. It has occasionally been suspected and invoked to explain anomalous temperature gradients, but apparatus of the present type is an extremely sensitive detector of it: the use of the head of Figure 1 (*b*) to measure water flows in or across boreholes will be discussed elsewhere since these measurements are of considerable engineering importance.

In the present series of experiments in the Tasmanian dolerites and basalts, water flow was detected in every hole investigated. In one hole in particular, which was apparently quite normal, being full to the collar and having no water flowing from it, water entered at 400 ft, flowed up the hole and out at 50 ft.

VIII. Acknowledgments

We are indebted to the Hydro-electric Commission of Tasmania and the Snowy Mountains Hydro-electric Authority for permission to use their boreholes and for putting their facilities at our disposal.

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