A MODEL OF NUCLEAR SHAPE OSCILLATIONS FOR γ–TRANSITIONS AND ELECTRON EXCITATION

By L. J. Tassie*

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Summary

A model of nuclear shape oscillations is set up for an arbitrary nuclear charge distribution. For a uniform charge distribution the model reduces to the liquid drop model. The model is used to consider γ-transitions and electron excitation of nuclei. Explicit expressions are obtained for four charge distributions : (*a*) uniform, (*b*) Gaussian, (*c*) exponential, (*d*) uniform with Gaussian " edge ". The theory predicts a relative angular distribution of electrons scattered by the $4 \cdot 43$ MeV level of ^{12}C in agreement with the experimental results of Fregeau and Hofstadter (1955), but gives a scattered intensity seven times too large.

I. Introduction

The liquid drop model (Fierz 1943 ; Bohr 1952 ; Jekeli 1952) of the nucleus involves the assumption that the nucleus has a well-defined surface and a uniform charge and mass density. Experiments on the elastic scattering of high energy electrons (Hofstadter, Fechter, and McIntyre 1953 ; Hofstadter *et al.* 1954) have demonstrated that these assumptions are not valid even for heavy nuclei. An attempt is made to obtain a modified hydrodynamical model which allows for non-uniform nuclear charge and mass density distributions. In the liquid drop model distortions of the surface of the nucleus are considered ; in this model similar distortions are considered not of the surface but of the shape of the nucleus, the distortions having effect throughout the whole nucleus. Excited states of the nucleus are then due to oscillations of the shape of the nucleus, i.e. oscillations involving departures from spherical symmetry of the mass and charge distributions of the nucleus.

Longitudinal compressional waves may be expected in the nucleus, involving nodes in the radial density distributions. As in the liquid drop model (Bohr 1952), the energy of excitation of these modes of radial oscillation should be much greater than the energy of excitation of the shape oscillations. Thus, in treating low-lying nuclear energy levels, it is assumed that the radial dependence of the density distributions does not alter appreciably.

This shape oscillation model is then used to consider γ-transitions and electron excitation of nuclei. For a uniform charge distribution this model reduces to the usual liquid drop model.

* Physics Department, University of Melbourne ; present address : Research School of Physical Sciences, Australian National University, Canberra.

II. Formulation of the Model

It is assumed that the nucleus is spherically symmetric in the equilibrium position. This confines our treatment to nuclei with spin 0 in the ground state. (To treat other nuclei similar modifications to those used to incorporate the liquid drop model in the collective model (Bohr 1952 ; Bohr and Mottelson 1953) would be necessary.) The charge and mass density distributions are

$$\left.\begin{aligned} &Ze\rho_0^C(r), \\ \text{and}\quad &Am\rho_0^M(r) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

respectively, allowing for a possible difference in these two distributions. For convenience, the " size " of a distribution is defined as the root mean square radius S,

$$S=\int r^2\rho(r)\mathrm{d}V. \quad \ldots\ldots\ldots\ldots \quad (2)$$

From the analysis (Yennie, Ravenhall, and Wilson 1954 ; Brown and Elton 1955) of high energy electron scattering experiments (Hofstadter, Fechter, and McIntyre 1953 ; Hofstadter *et al.* 1954) S is given approximately by

$$S=(3/5)^{\frac{1}{2}}\times 1\cdot 20A^{\frac{1}{3}}\times 10^{-13}\ \text{cm}. \quad \ldots\ldots\ldots\ldots \quad (3)$$

For the excited states of the nucleus, shape oscillations occur. The distortion throughout the nucleus is treated in the same way as the distortion of the nuclear surface in the liquid drop model. We assume that under distortion, an element of mass and charge moves from $\mathbf{r}_0$ to $\mathbf{r}$ *without alteration of the volume it occupies*, i.e.

$$\rho(\mathbf{r})=\rho_0(r_0). \quad \ldots\ldots\ldots\ldots \quad (4)$$

This assumes that each element of mass and charge is incompressible, i.e. the nucleus is composed of an inhomogeneous incompressible fluid. The " shape " of the distortion can be given by

$$r-r_0=\sum_{l=2,\,m}\alpha_{lm}(r_0)\mathrm{Y}_{lm}(\theta,\,\varphi)r_0^{l-1}, \quad \ldots\ldots\ldots\ldots \quad (5)$$

where r and r_0 are as shown in Figure 1, and Y_{lm} is the spherical harmonic. As in the liquid drop model, it is assumed that the motion in the nucleus is irrotational. Then

$$\mathbf{v}=\nabla\Phi, \quad \ldots\ldots\ldots\ldots \quad (6)$$

where Φ is the velocity potential. We have

$$\partial\Phi/\partial r=(\partial r/\partial t)_\rho. \quad \ldots\ldots\ldots\ldots \quad (6a)$$

The equation of continuity

$$\nabla\,.\,(\rho\mathbf{v})+\partial\rho/\partial t=0,$$

becomes

$$\nabla\rho\,.\,\nabla\Phi+\rho\nabla^2\Phi+\partial\rho/\partial t=0.$$

We assume that the deformations of the nucleus are small, i.e. α_{lm} small, and consider all expressions only to the first non-vanishing term in α_{lm}. Then to the first order in α_{lm},

$$(\partial\rho/\partial r)(\partial\Phi/\partial r)+\rho\nabla^2\Phi+\partial\rho/\partial t=0.$$

Using equation (**6a**),

$$(\partial\rho/\partial r)_t(\partial\Phi/\partial r)_t=(\partial\rho/\partial r)_t(\partial r/\partial t)_\rho=-\partial\rho/\partial t,$$

and thus

$$\nabla^2\Phi=0. \qquad \text{(7)}$$

Then

$$\Phi(\mathbf{r})=\Sigma l^{-1}\beta_{lm}Y_{lm}(\theta,\ \varphi)r^l, \qquad \text{(8)}$$

and from equation (**6a**)

$$\beta_{lm}=\dot{\alpha}_{lm}, \qquad \text{(9)}$$

and α_{lm} is independent of r.

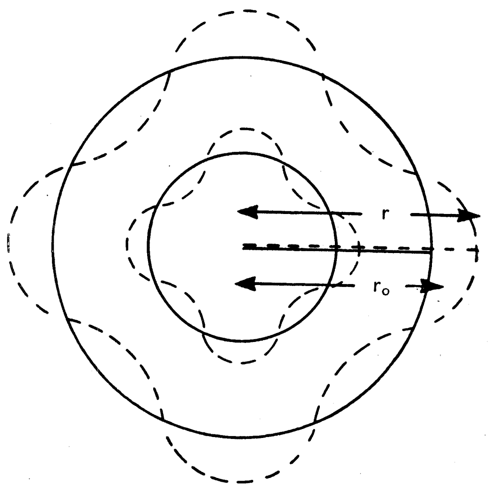

Fig. 1.—The shells of charge and mass in the equilibrium position of the nucleus, shown as continuous lines, move to the positions shown by the dotted lines in the distorted nucleus.

Since in the following, ρ will always appear multiplied by terms of the first order in α_{lm}, we need take $\rho(r)$ only to the zeroth order in α_{lm}, i.e. as $\rho_0(r)$ which we will now write as $\rho(r)$.

III. Electron Excitation and γ-Transitions

Using the first Born approximation, Schiff (1954) has developed expressions for the differential cross sections for the inelastic scattering of high energy electrons with excitation of various nuclear multipole transitions, and has shown that for collective modes of excitation only electric multipole transitions with $m=0$ need be considered. Then

$$d\sigma/d\omega=(d\sigma/d\omega)_P\,|\,\mathscr{F}_L\,|^2, \qquad \text{(10)}$$

where

$$(\mathrm{d}\sigma/\mathrm{d}\omega)_P=\tfrac{1}{4}Z^2(e^2/\hbar c)^2k^{-2}\cos^2\tfrac{1}{2}\theta\ \mathrm{cosec}^4\tfrac{1}{2}\theta \quad \ldots\ldots \quad (\mathbf{11})$$

is the point charge scattering cross section, and

$$\mathscr{F}_L=\{4\pi(2L+1)\}^{\frac{1}{2}}\int \mathrm{j}_L(qr)\mathrm{Y}_{L0}\rho_{\mathrm{trans}}\mathrm{d}V. \quad \ldots\ldots \quad (\mathbf{12})$$

$\mathrm{j}_L(z)\equiv(\pi/2z)^{\frac{1}{2}}\mathrm{J}_{L+\frac{1}{2}}(z)$ is the spherical Bessel function. $|\mathscr{F}_L|^2$ is called the scattering form factor. $\hbar k$ is the momentum of the incident electron, $q=2k\sin\frac{1}{2}\theta$, and $Ze\rho_{\mathrm{trans}}$ is the transition charge density of the nucleus. In equation (**10**) it is assumed that the energy of the transition is small compared with the energy of the incident electron, and that the angle of scattering θ is not small. Both these requirements are satisfied in experiments (Fregeau and Hofstadter 1955), as the inelastic scattering at low energies or small angles is masked by the bremsstrahlung tail of the elastic scattering.

From the equation of continuity,

$$\begin{aligned}\rho_{\mathrm{trans}}&=(\mathrm{i}\omega)^{-1}\nabla\cdot\mathbf{j}\\&=(\mathrm{i}\omega)^{-1}\nabla\cdot(\rho^C(r)\mathbf{v})\\&=(\mathrm{i}\omega)^{-1}\nabla\rho^C(r)\cdot\nabla\Phi\\&=(\mathrm{i}\omega)^{-1}(\mathrm{d}\rho^C/\mathrm{d}r)(\partial\Phi/\partial r), \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{13})\end{aligned}$$

using $\nabla^2\Phi=0$. $\hbar\omega$ is the energy of the transition. From equation (**8**) it follows that

$$\rho_{\mathrm{trans}}=(\mathrm{i}\omega)^{-1}\Sigma\beta_{lm}\mathrm{Y}_{lm}r^{l-1}\mathrm{d}\rho^C/\mathrm{d}r. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{14})$$

From the orthogonality of the Y_{lm}, only the term $l=L$, $m=0$ will contribute to (**12**), giving

$$\begin{aligned}\mathscr{F}_L&=\{4\pi(2L+1)\}^{\frac{1}{2}}(\mathrm{i}\omega)^{-1}\beta_{L0}\int_0^\infty \mathrm{j}_L(qr)r^{L+1}(\mathrm{d}\rho^C/\mathrm{d}r)\mathrm{d}r\\&=\{4\pi(2L+1)\}^{\frac{1}{2}}(\mathrm{i}\beta_{L0}/\omega)q\int_0^\infty \mathrm{j}_{L-1}(qr)r^{L+1}\rho^C(r)\mathrm{d}r. \quad \ldots\ldots \quad (\mathbf{15})\end{aligned}$$

The transition probability for the emission of a photon of electric multipole order L, and energy $\hbar\omega$ is given by (Blatt and Weisskopf 1952)

$$T_E(L,M)=\frac{8\pi(L+1)}{L\{(2L+1)!!\}^2}\ \frac{(\omega/c)^{2L+1}}{\hbar}\ |Q_{LM}|^2, \quad \ldots\ldots \quad (\mathbf{16})$$

where

$$Q_{LM}=Ze\int r^L\mathrm{Y}_{LM}{}^*\rho_{\mathrm{trans}}\mathrm{d}V \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{17})$$

is the transition 2^L-pole moment. For the shape oscillation model, using equation (**14**), this becomes

$$\left.\begin{aligned}Q_{LM}&=(\mathrm{i}\beta_{LM}/\omega)Ze(2L+1)\int_0^\infty \rho^C(r)r^{2L}\mathrm{d}r\\&=(\mathrm{i}\beta_{LM}/\omega)Ze(2L+1)\overline{r_C^{2(L-1)}}/4\pi.\end{aligned}\right\} \quad \ldots\ldots \quad (\mathbf{18})$$

Comparing equations (**12**) and (**17**), electron excitation and γ-emission are related by

$$\lim_{q\to 0} \mathscr{F}_L=\{4\pi(2L+1)\}^{\frac{1}{2}}\frac{q^L}{(2L+1)!!}Q_{L0}/Ze. \quad \ldots\ldots\ldots \quad (\mathbf{19})$$

It is convenient to put

$$\mathscr{F}_L=S^{-L}(Q_{L0}/Ze)G_L(qS). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{20})$$

Then the relative angular distribution of the inelastically scattered electrons depends on the function $G_L(qS)$, and the actual magnitude of the cross section is determined by the Q_{L0}. This gives us a method of determining the $|Q_{L0}|^2$ from electron scattering measurements by comparing the experimental form factor, i.e. the ratio of the actual scattering to point charge scattering, with G_L^2.

$$G_L(qS)=4\pi\{4\pi/(2L+1)\}^{\frac{1}{2}}(\overline{r_C^{2(L-1)}})^{-1}S^Lq \times\int_0^\infty \mathrm{j}_{L-1}(qr)r^{L+1}\rho^C(r)\mathrm{d}r. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{21})$$

For all charge distributions,

$$\lim_{q\to 0} G_L=\{4\pi(2L+1)\}^{\frac{1}{2}}(qS)^L/(2L+1)!! \quad \ldots\ldots\ldots\ldots \quad (\mathbf{22})$$

In Section VI we obtain a theoretical estimate of Q_{L0}.

IV. Calculation of G_L

The inelastic scattering has been considered for four different charge distributions.

(*a*) Uniform distribution

$$\left.\begin{array}{ll} \rho(r)=(3/4\pi)R_0^{-3}, & r\leqslant R_0, \\ \rho(r)=0 \quad\quad\quad\;\; , & r\geqslant R_0, \\ R_0=(5/3)^{\frac{1}{2}}S. & \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{23})$$

For the uniform distribution, the results are the same as those obtained from the usual liquid drop model.

$$G_L(qS)=\{4\pi(2L+1)\}^{\frac{1}{2}}(3/5)^{L/2}\mathrm{j}_L\{(5/3)^{\frac{1}{2}}qS\}. \quad \ldots\ldots \quad (\mathbf{24})$$

(*b*) Gaussian distribution

$$\left.\begin{array}{l} \rho(r)=(\pi^{\frac{1}{2}}g)^{-3}\exp(-r^2/g^2), \\ g=(2/3)^{\frac{1}{2}}S. \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (\mathbf{25})$$

The integral in equation (**21**) can be obtained using equation 2, p. 393 of Watson (1944), the confluent hypergeometric function involved reducing to the exponential function, finally giving

$$G_L(qS)=\{4\pi(2L+1)\}^{\frac{1}{2}}\frac{(qS)^L}{(2L+1)!!}\exp\{-(qS)^2/6\}. \quad \ldots\ldots \quad (\mathbf{26})$$

(c) Exponential distribution

$$\left.\begin{aligned}\rho(r)&=(8\pi d^3)^{-1}\exp(-r/d),\\ d&=12^{-\frac{1}{2}}S.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (27)$$

For this case, the integral in equation (**21**) is obtained using equation 3, p. 393 of Watson (1944), the required hypergeometric function reducing to $(1-z)^{\frac{1}{2}}$, finally giving

$$G_L(qS)=\{4\pi(2L+1)\}^{\frac{1}{2}}\frac{(qS)^L}{(2L+1)!!}\{1+(qS)^2/12\}^{-(L+1)}. \quad \ldots. \quad (28)$$

(d) Uniform distribution with a Gaussian " edge "

$$\left.\begin{aligned}\rho(r)&=\rho_0 && r\leqslant a,\\ \rho(r)&=\rho_0\exp\{-(r-a)^2/b^2\} && r\geqslant a,\\ \rho_0&=(3/4\pi)a^{-3}X(b/a)\\ S^2&=\frac{3}{5}a^2X(b/a)\left\{1+\frac{5}{2}\pi^{\frac{1}{2}}b/a+10(b/a)^2+\frac{15}{2}\pi^{\frac{1}{2}}(b/a)^3\right.\\ &\qquad\left.+10(b/a)^4+\frac{15}{8}\pi^{\frac{1}{2}}(b/a)^5\right\},\end{aligned}\right\} \quad \ldots \quad (29)$$

where

$$X(b/a)=\left\{1+\frac{3}{2}\pi^{\frac{1}{2}}b/a+3(b/a)^2+\frac{3}{4}\pi^{\frac{1}{2}}(b/a)^3\right\}^{-1}.$$

$$\mathscr{F}_L=\{4\pi(2L+1)\}^{\frac{1}{2}}(i\beta_{L0}/\omega)\rho_0a^{L+1}\{j_L(qa)+qaf_L(b/a,qa)\}, \quad \ldots \quad (30)$$

where

$$f_L(x,y)=\int_0^\infty\exp\{-(t-1)^2/x^2\}t^{L+1}j_{L-1}(yt)dt. \quad \ldots\ldots\ldots\ldots \quad (31)$$

For electric quadrupole transitions the $G_2^2(qS)$ are shown in Figure 2. The values for distribution (d) were obtained for $b/a=0\cdot4$ by graphical integration of $f_2(x,y)$. For $b/a=0\cdot4$, this distribution is a reasonable approximation to the smoothed uniform charge distribution, for which the calculated elastic scattering of high energy electrons by gold (Brown and Elton 1955) agrees with experiment (Hofstadter, Fechter, and McIntyre 1953 ; Hofstadter *et al.* 1954).

V. Comparison with Experiment for ^{12}C

Because of its statistical nature, this shape oscillation model should be valid only for medium and heavy nuclei. Also, the inelastic electron scattering has been considered using the Born approximation, which breaks down for scattering by heavy elements. However, Yennie, Wilson, and Ravenhall (1953) have shown that the Born approximation is not very far out for the elastic scattering by copper, except in the neighbourhood of diffraction minima. Thus, our results would be expected to apply to nuclei with atomic numbers about 60. Because of the large uncertainty in theoretical nuclear matrix elements, there is little justification for using more accurate methods than the Born approximation at present.

However, the only experimental results available to date are for beryllium (McIntyre, Hahn, and Hofstadter 1954) and carbon (Fregeau and Hofstadter

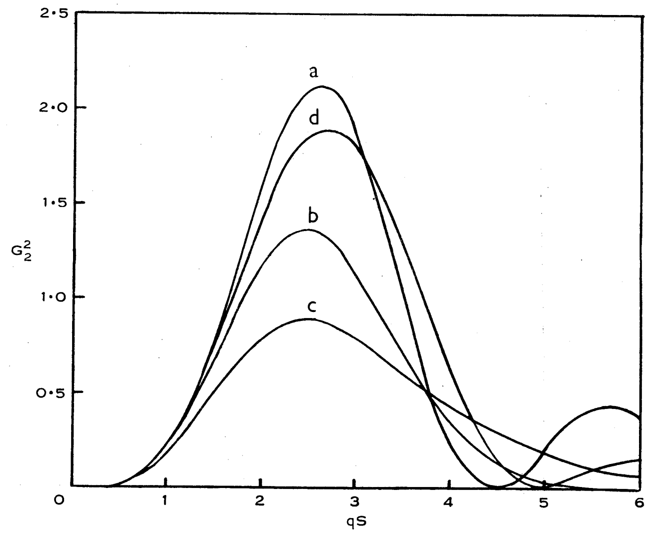

Fig. 2.—$\{G_2(qS)\}^2$ (see equation (**21**)) for electric quadrupole transitions for the following charge distributions : (*a*) uniform, (*b*) Gaussian, (*c*) exponential, (*d*) uniform with Gaussian " edge ", $b/a=0\cdot4$.

1955). Comparison has been made with the experimental results of Fregeau and Hofstadter (1955) for the excitation of the 4·43 MeV level of carbon,

TABLE 1

EXCITATION OF THE 4·43 MeV LEVEL OF ^{12}C

Column 4 gives the results for $|Q_{20}|^2/e^2$ obtained by comparing the theoretical scattering form factors with the form factor obtained experimentally by Fregeau and Hofstadter (1955). Column 5 gives the theoretical value of $|Q_{20}|^2/e^2$ obtained from equation (**41**) in Section VI

Charge Distribution	S ($\times 10^{-13}$ cm)	Agreement of Theoretical with Experimental Form Factor	$\|Q_{20}\|^2/e^2$ From Comparison with Experimental Scattering Results ($\times 10^{-52}$ cm^4)	$\|Q_{20}\|^2/e^2$ From Theory of Section VI ($\times 10^{-52}$ cm^4)
(*a*) Uniform ..	2·20*	Good	7·7	55
	2·23	Good	7·9	56
(*b*) Gaussian ..	2·47*	Unsatisfactory	11·8	69
	2·04	Good	7·8	47
(*c*) Exponential ..	2·58*	No agreement		
	1·9	Good	10	41
(*d*) Uniform with	2·4	Fair	10	65
Gaussian " edge " $b/a=0\cdot4$	2·20	Good	8·4	55

* Indicates value of S giving best fit to the elastic scattering form factor for that particular charge distribution.

although one hardly expects this nuclear model to be valid for such a light element. Fregeau and Hofstadter obtain the following r.m.s. radii from their elastic scattering results,

$$\text{Uniform distribution } S=2\cdot 20\times 10^{-13} \text{ cm}$$
$$\text{Gaussian distribution } S=2\cdot 47\times 10^{-13} \text{ cm.}$$

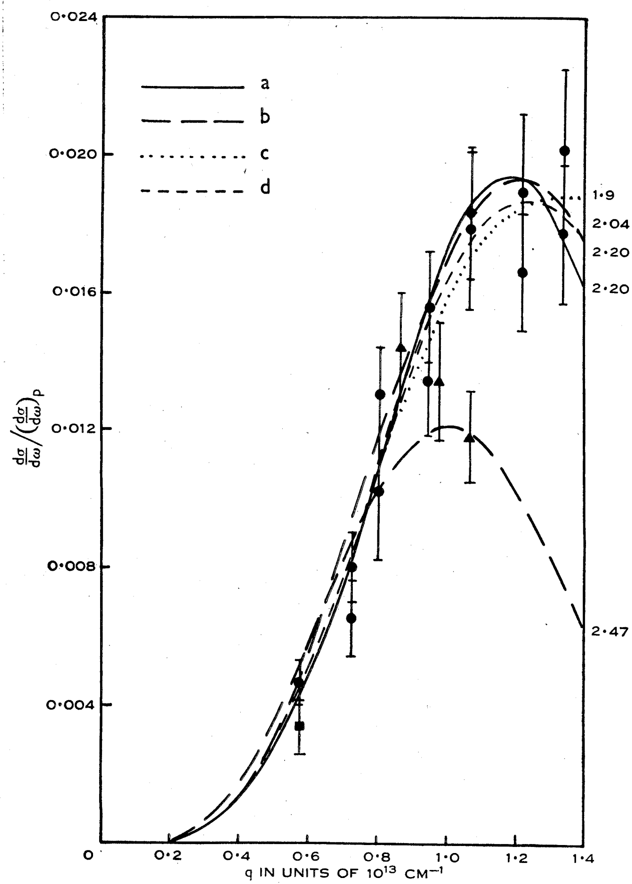

Fig. 3.—Ratio of cross section for the quadrupole excitation of the $4\cdot 43$ MeV level to the point charge scattering cross section of ^{12}C. Theoretical curves are $(S^2Ze)^{-2}\mid Q_{20}\mid^2\{G_2(qS)\}^2$ for the charge distributions: (*a*) uniform, (*b*) Gaussian, (*c*) exponential, (*d*) uniform with Gaussian " edge ", $b/a=0\cdot 4$. The curves are labelled with the value of S in units of 10^{-13} cm, and the $\mid Q_{20}\mid^2$ are given in column 4 of Table 1. The experimental points are those of Fregeau and Hofstadter (1955) for the scattering of electrons with energies 187 MeV (full circles), 150 MeV (triangles), and 80 MeV (square).

For an exponential charge distribution, the best fit to these elastic scattering results is obtained with $S=2\cdot 58\times 10^{-13}$ cm, but the fit is not satisfactory as the theoretical scattering cross section falls off too slowly with angle.

The $4\cdot 43$ MeV transition is electric quadrupole 0^+-2^+. The results obtained for $\mid Q_{20}\mid^2$ by comparing our $G_2^2(qS)$ using the above values of S, with the experimental form factor given by Fregeau and Hofstadter are given in Table 1, and the results for the form factor $(S^2Ze)^{-2} \mid Q_{20} \mid^2 G_2^2$ are shown in Figure 3 in comparison with the experimental form factor. The experimental form factor could not be fitted using an exponential distribution with $S=2\cdot58\times10^{-13}$ cm.

The experimental results were then fitted by varying both S and $\mid Q_{20} \mid^2$ for all the charge distributions in Section IV including (d) with $b/a=0\cdot4$. The results are also shown in Figure 3 and Table 1. Under these conditions, the scattering form factors derived from all the distributions will fit the experimental form factor. However, only distributions (a) and (d) give a result for S consistent with the elastic scattering experiments, and the exponential distribution can be definitely excluded. The model used here is the same as the liquid drop model for the uniform distribution and close to it for distribution (d). The difference in S required to fit the Gaussian distribution to the elastic scattering and inelastic scattering may be due to the inadequacy of our model for this distribution. The experimental form factor can also be fitted by the form factor derived from a uniform transition charge density with a r.m.s. radius $S=2\cdot58\times10^{-13}$ cm, but it would seem that this transition charge density can be excluded since it has an appreciably larger extension than the static charge distribution. For the range of qS of these experiments, approximately $qS=1$ to $qS=3$, it seems that the scattering is largely model independent.

Experiments at larger values of qS, i.e. heavier nuclei, larger scattering angles, and higher electron energies, would give more information about the transition charge density and would provide a more critical test of the theory. However, we conclude from this analysis that the transition charge density has a larger r.m.s. radius than the static charge density and is greatest near the edge of the nucleus.

VI. The Transition Multipole Moments

To obtain numerical values of the multipole moments from equation (**18**), we need the expectation value of β_{LM} for the transition. Using the quantum mechanical relation

$$\langle n \mid \dot{\alpha}_{lm} \mid n' \rangle = \mathrm{i}\omega \langle n \mid \alpha_{lm} \mid n' \rangle, \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{32})$$

we obtain, in a similar way to the quantum treatment of the liquid drop model for small deformations (Bohr 1952), that for transitions from a no phonon to a one phonon state

$$\begin{aligned} \langle 0 \mid \beta_{lm} \mid 1 \rangle &= \mathrm{i}\omega_l(\hbar\omega_l/2C_l)^{\frac{1}{2}} \\ &= \mathrm{i}(\hbar\omega_l/2B_l)^{\frac{1}{2}}, \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{33})$$

where the total energy of the shape oscillations of the nucleus is

$$\left.\begin{aligned} H &= T+V, \\ T &= \tfrac{1}{2}\sum_{l=2,\,m} B_l \mid \beta_{lm} \mid^2, \\ V &= \tfrac{1}{2}\sum_{l=2,\,m} C_l \mid \alpha_{lm} \mid^2. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (\mathbf{34})$$

For this model it is easier to obtain an estimate of B_l than of C_l.

The total kinetic energy of the nuclear motion is

$$T=\tfrac{1}{2}Am\int \rho^M(r)v^2\mathrm{d}V. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

Using equations (6) and (7), this reduces to

$$\begin{aligned} T&=-\tfrac{1}{2}Am\int \nabla\rho^M(r)\,.\,\Phi\nabla\Phi\mathrm{d}V \\ &=-\tfrac{1}{2}Am\int \Phi(\mathrm{d}\rho^M/\mathrm{d}r)(\partial\Phi/\partial r)\mathrm{d}V \\ &=-\tfrac{1}{2}Am\sum_{l=2,\,m} l^{-1}\,|\,\beta_{lm}\,|^2\int_0^\infty r^{2l+1}(\mathrm{d}\rho^M/\mathrm{d}r)\mathrm{d}r, \quad \ldots\ldots \quad (36) \end{aligned}$$

using (Bohr 1952)

$$\beta_{lm}=(-1)^m\beta_{lm}^*. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (37)$$

This gives

$$\left.\begin{aligned} B_l&=l^{-1}(2l+1)Am\int_0^\infty r^{2l}\rho^M(r)\mathrm{d}r \\ &=l^{-1}(2l+1)(Am/4\pi)\overline{r_M^{2(l-1)}}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (38)$$

From equations (18), (33), and (38), we obtain for transitions from a one phonon state to a no phonon state or vice versa,

$$|\,Q_{LM}\,|^2=Z^2e^2\hbar(8\pi Am\omega_L)^{-1}L(2L+1)(\overline{r_C^{2(L-1)}})^2/\overline{r_M^{2(L-1)}}. \quad \ldots\ldots \quad (39)$$

There has been some doubt whether the nuclear charge distribution is the same as the mass distribution. Purely nuclear measurements (Blatt and Weisskopf 1952, p. 15) have indicated a larger nuclear size than the electromagnetic measurements (Hofstadter, Fechter, and McIntyre 1953 ; Hofstadter *et al.* 1954). However, an analysis by Williams (1955) of the experiments of Coor *et al.* (1955) on the absorption cross sections of nuclei for 1·4 BeV neutrons yields a r.m.s. radius in agreement with that determined from electromagnetic measurements. Assuming then, that

$$\rho^C(r)=\rho^M(r)=\rho(r),$$

we obtain

$$|\,Q_{Lm}\,|^2=Z^2e^2\hbar(8\pi\omega_L Am)^{-1}L(2L+1)\overline{r^{2(L-1)}}. \quad \ldots\ldots\ldots\ldots \quad (40)$$

If $\rho^C(r)$ and $\rho^M(r)$ are of the same form but with different S, then the correction factor to the left-hand side of equation (40) would be $(S_C/S_M)^{2(L-1)}$.

For the distributions in Section IV, $\overline{r^{2(L-1)}}$ is given by

(*a*) Uniform $\quad 3(2L+1)^{-1}(5/3)^{L-1}S^{2(L-1)}$,
(*b*) Gaussian $\quad 3^{1-L}(2L-1)!!S^{2(L-1)}$,
(*c*) Exponential $\quad \tfrac{1}{2}\times 12^{1-L}(2L)!S^{2(L-1)}$.

For different distributions with the same r.m.s. radius, the difference in the $|\,Q_{LM}\,|^2$ are small for the first few L and increase with L.

For electric quadrupole transitions,

$$|Q_{2M}|^2=Z^2e^2\hbar(\pi\omega_L Am)^{-1}(5/4)S^2. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{41})$$

The values obtained from equation (**41**) are compared in Table 1 with the values derived from the inelastic electron scattering measurements for the various values of S. It is seen that the theoretical value is approximately seven times too large.

From the experimental results (Fregeau and Hofstadter 1955), we can deduce that $|Q_{20}|^2/e^2$ is approximately 8×10^{-52} cm^4. Inserting this value in equation (**16**), gives 6×10^{-14} sec as the lifetime of the $4\cdot43$ MeV level of ^{12}C for γ-decay to the ground state. This is consistent with the result of the γ-decay experiments of Mills and Mackin (1954) that the lifetime of this state is less than 3×10^{-13} sec. However, Devons, Manning, and Towle (1956) have measured the lifetime of this level for γ-emission using a recoil method and obtain a value of $(2\cdot6\pm0\cdot9)\times10^{-14}$ sec.

VII. Discussion

In conclusion, this theory of nuclear shape oscillations seems capable of explaining the angular distribution of the scattered electrons, but gives a scattered intensity too large by a factor of 7.

The assumption in equation (4) may cause some error because of compression effects, even when there are no compressional waves present. Woeste (1952) has treated compression effects for a density distribution with a sharp edge and only small deviations from uniformity, and shows that the density is greater at the surface than at the centre of the nucleus. However, electron scattering experiments (Hofstadter, Fechter, and McIntyre 1953; Hofstadter *et al.* 1954) show that the nuclear charge density distribution does not have a sharp surface, but that the density decreases smoothly to zero over a finite distance. This non-uniformity of the density distribution is more easily explained by quantum mechanics than by a classical effect such as compressibility. The nucleon wave functions must be smooth, and therefore must decrease smoothly to zero, giving rise to non-uniform nuclear charge and mass density distributions.

It is doubtful whether the similarity of the nucleus to an inhomogeneous fluid is such as to justify a detailed treatment including compression effects. A more accurate theory of the electron excitation of collective oscillations could be obtained by using the theory of an oscillating shell structure given by Araújo (1956). Such a theory would be very complicated for heavy nuclei, and would be restricted to the density distributions which can be obtained from simple nuclear potentials such as the spherical box and parabolic well.

At any rate, the theory developed here shows that the diffuseness of the nuclear surface can be neglected when treating the electron excitation of collective transitions provided that qS is not too large, as the curves for G_2^2 in Figure 2 for charge distributions (a), (b), and (d) do not differ very greatly up to about $qS=2$.

VIII. Acknowledgments

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