DETERMINATION OF PHOTONUCLEAR CROSS SECTIONS*

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Introduction

A typical photonuclear experiment is considered in which the γ-ray source is the bremsstrahlung from an electron accelerator. It is assumed that the electrons incident on the radiator have kinetic energy k_0 and total energy $k_0+\mu$ (where μ is the electron rest energy) so that the bremsstrahlung contains photons of all energies $k \leqslant k_0$. The yield $A(k_0)$ for some particular photonuclear reaction is measured as a function of the electron energy and the problem is to use the data to determine the cross section $\sigma(k)$ as a function of the photon energy.

It is evident that $A(k_0)$ and $\sigma(k)$ are related by a Volterra equation of the form

$$A(k_0)=g\int_T^{k_0} \sigma(k)P(k,k_0)\mathrm{d}k, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where g is a constant which normalizes the equation to unit electron current, T is the threshold energy for the reaction, and

$$P(k,k_0)=(1/k)J(k,k_0), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where $J(k,k_0)$ is proportional to the bremsstrahlung intensity. The normalizing constant g is a function of the particular experimental arrangement and its evaluation depends on such factors as the method of beam monitoring, the thickness of the target, counting efficiency, etc. The particular method of normalization used in the present experiments is described on page 317.

Several methods have been proposed for the solution of equation (**1**) (e.g. Baldwin and Klaiber 1948 ; Johns *et al.* 1950 ; Katz and Cameron 1951 ; Wilson 1953*a*) and of these the " photon difference method " of Katz and Cameron has been the most widely used. The present " iterative method " was developed for the analysis of data obtained with the Canberra 33 MeV electron synchrotron.

The Bremsstrahlung Spectrum

It is assumed that the shape of the bremsstrahlung spectrum is as given by the theory of Bethe and Heitler (Heitler 1944) for fast electrons incident on a thin radiator. Although the Born approximation used in the Bethe-Heitler calculations is known to be in error for the heavy elements, the error is principally in the magnitude of the cross section and its shape is virtually unaffected by more exact calculations (e.g. Bethe and Maximon 1954).

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In the form given by Schiff (1951) the bremsstrahlung intensity in the forward direction is

$$J(k,k_0)=[1+(1-z)^2]\ 2\log\alpha-(2-z)^2, \quad \ldots\ldots\ldots \quad (3)$$

where $z=k/(k_0+\mu)$,

$$1/\alpha^2=1/\alpha_1^2+1/\alpha_2^2,$$
$$\alpha_1=2(k_0+\mu)(1-z)/\mu z,$$
$$\alpha_2=C/Z^{\frac{1}{3}}.$$

C=constant=111 and Z=atomic number of the radiator=73, for the present computations. The Schiff distribution, differentiated with respect to the electron energy k_0, is also required for the present analysis. It is given by

$$P'(k,k_0)=\mathrm{d}P(k,k_0)/\mathrm{d}k_0$$
$$=\frac{2}{(k_0+\mu)k}\left\{2z(1-z)\log\alpha+z(z-2)\frac{2-z}{1-z}(1-\alpha^2/\alpha_2^2)[1+(1-z)^2]\right\},$$

and the calculated values are given in Table 1 at 1 MeV intervals.

The assumption of a thin radiator is of course rather artificial and the spectrum will be modified by multiple scattering and radiation loss (see e.g. Wilson 1953*b*) but a precise specification of the radiator conditions is difficult to obtain for a machine with an internal, circulating, electron beam. If an external electron beam is available the radiator conditions can be fixed more precisely and suitable corrections applied to the thin target formulae, as was done for example in the Stanford experiments of Berman and Brown (1954).

Method of Solution

If equation (1) is differentiated with respect to the electron energy k_0 one obtains, after some rearrangements,

$$\sigma(k_0)=\sigma_0(k_0)+\frac{1}{S(k_0T)}\int_T^{k_0}\{\sigma(k_0)-\sigma(k)\}P'(k,k_0)\mathrm{d}k, \quad \ldots. \quad (4)$$

where

$$\sigma_0(k_0)=A'(k_0)/gS(k_0,T), \quad \ldots\ldots\ldots \quad (5)$$

$$S(k_0,T)=P(k_0,k_0)+\int_T^{k_0}P'(k,k_0)\mathrm{d}k, \quad \ldots\ldots\ldots \quad (6)$$

and the dash denotes $\mathrm{d}/\mathrm{d}k_0$. The values of $S(k_0,T)$ are given in Table 2. In principle one needs only one set of $S(k_0,T)$ calculated for a single value of T, below the thresholds for all of the reactions which may be studied. However, the convergence of the iterations is greatly improved if T is close to the actual reaction threshold so the table has been prepared for a range of T values. It may be noted that the zero-order approximation, equation (5), would be exact if the bremsstrahlung spectrum were of constant intensity, i.e. if

$$P'(k,k_0)=(1/k_0)\delta(k,k_0).$$

Equation (4) can be solved by iteration in the form

$$\sigma(k_0)=\sum_{n=0}^{\infty}\sigma_n(k_0), \quad \ldots\ldots\ldots \quad (7)$$

TABLE 1

THE SCHIFF BREMSSTRAHLUNG DISTRIBUTION, DIFFERENTIATED WITH RESPECT TO PEAK ENERGY

$100 \times P'(k,k_0)$

k (MeV)	k_0 (MeV) 35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
35	11·144														
34	3·742	11·468													
33	2·104	3·848	11·874												
32	1·366	2·172	3·966	12·194											
31	0·944	1·418	2·256	4·086	12·586										
30	0·682	0·978	1·462	2·330	4·216	12·998									
29	0·512	0·706	1·018	1·528	2·408	4·354	13·452								
28	0·400	0·532	0·738	1·052	1·578	2·488	4·510	13·920							
27	0·332	0·422	0·558	0·766	1·096	1·634	2·586	4·678	14·438						
26	0·290	0·352	0·448	0·582	0·806	1·142	1·704	2·692	4·862	15·008					
25	0·266	0·308	0·378	0·476	0·710	0·836	1·192	1·784	2·816	5·060	15·598				
24	0·258	0·282	0·330	0·402	0·498	0·642	0·876	1·250	1·870	2·948	5·276	16·262			
23	0·254	0·278	0·306	0·356	0·428	0·522	0·676	0·920	1·314	1·962	3·064	5·510	16·958		
22	0·260	0·280	0·304	0·336	0·392	0·452	0·552	0·714	0·972	1·362	2·038	3·216	5·766	17·742	
21	0·270	0·284	0·304	0·332	0·366	0·416	0·488	0·592	0·754	1·032	1·456	2·152	3·380	6·046	18·588
20	0·282	0·294	0·310	0·332	0·360	0·398	0·444	0·526	0·640	0·808	1·094	1·550	2·280	3·560	6·360
19	0·296	0·308	0·322	0·340	0·362	0·392	0·408	0·484	0·562	0·684	0·866	1·164	1·648	2·400	3·748
18	0·314	0·326	0·338	0·354	0·374	0·394	0·402	0·464	0·526	0·612	0·736	0·928	1·250	1·756	2·556
17	0·332	0·344	0·358	0·372	0·388	0·406	0·432	0·470	0·514	0·572	0·668	0·802	1·006	1·242	1·876
16	0·352	0·362	0·380	0·394	0·412	0·430	0·454	0·482	0·518	0·566	0·632	0·728	0·878	1·100	1·448
15	0·372	0·384	0·402	0·420	0·436	0·456	0·480	0·502	0·534	0·568	0·626	0·696	0·802	0·968	1·196
14	0·394	0·408	0·426	0·450	0·460	0·486	0·508	0·532	0·562	0·596	0·638	0·694	0·776	0·896	1·056
13	0·416	0·430	0·450	0·470	0·488	0·514	0·538	0·564	0·598	0·626	0·670	0·716	0·782	0·880	0·994
12	0·438	0·456	0·476	0·496	0·518	0·542	0·572	0·602	0·632	0·666	0·704	0·762	0·826	0·890	0·984
11	0·460	0·480	0·502	0·526	0·550	0·576	0·608	0·638	0·674	0·708	0·750	0·802	0·876	0·952	1·006
10	0·484	0·504	0·528	0·554	0·582	0·610	0·644	0·676	0·714	0·754	0·800	0·856	0·934	0·990	1·070
9	0·504	0·528	0·554	0·584	0·610	0·646	0·682	0·720	0·758	0·804	0·852	0·906	0·974	1·060	1·126
8	0·530	0·554	0·584	0·616	0·646	0·682	0·724	0·760	0·804	0·854	0·908	0·968	1·048	1·116	1·208
7	0·554	0·582	0·610	0·646	0·680	0·718	0·758	0·808	0·852	0·906	0·966	1·032	1·106	1·186	1·280
6	0·576	0·608	0·640	0·676	0·714	0·754	0·798	0·848	0·900	0·960	1·024	1·098	1·178	1·268	1·370
5	0·606	0·636	0·668	0·706	0·748	0·790	0·840	0·892	0·950	1·014	1·086	1·162	1·252	1·352	1·458

k (MeV)	k_0 (MeV) 20	19	18	17	16	15	14	13	12	11	10	9	8	7	6
20	19·522														
19	6·708	20·546													
18	3·976	7·090	21·700												
17	2·724	4·238	7·456	22·972											
16	2·012	2·922	4·526	8·016	24·426										
15	1·594	2·174	3·162	4·858	8·576	26·048									
14	1·322	1·736	2·384	3·428	5·252	9·228	27·912								
13	1·192	1·466	1·920	2·600	3·764	5·724	9·988	30·074							
12	1·130	1·332	1·638	2·132	2·886	4·136	6·260	10·880	32·600						
11	1·126	1·276	1·504	1·910	2·390	3·236	4·524	6·904	11·976	35·590					
10	1·160	1·292	1·440	1·736	2·120	2·708	3·666	5·182	7·802	13·292	39·182				
9	1·224	1·344	1·514	1·714	2·004	2·464	3·140	4·226	5·968	8·882	14·948	43·602			
8	1·292	1·422	1·580	1·756	2·016	2·352	2·878	3·682	4·936	6·972	10·300	17·140	49·148		
7	1·394	1·522	1·666	1·878	2·078	2·394	2·818	3·426	4·386	5·932	8·292	12·168	20·020	56·382	
6	1·486	1·620	1·844	1·960	2·160	2·494	2·830	3·850	4·156	5·356	7·200	10·200	14·884	24·140	66·104
5	1·590	1·734	1·934	2·104	2·340	2·632	3·004	3·464	4·188	5·108	6·624	9·296	13·020	18·960	31·080

TABLE 2

THE NORMALIZING FUNCTION $100S(k_0,T)=100\left\{P(k_0,k_0)+\int_T^{k_0} P'(k,\,k_0)\mathrm{d}k\right\}$

T (MeV)	k_0 (MeV) 35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
35	0·93														
34	7·29	0·96													
33	9·79	7·52	0·98												
32	11·8	10·1	7·74	1·02											
31	13·1	12·2	10·4	7·96	1·05										
30	14·0	13·5	12·6	10·7	8·21	1·08									
29	14·7	14·5	14·0	12·9	11·0	8·48	1·11								
28	15·2	15·2	14·9	14·4	13·4	11·4	8·75	1·15							
27	15·6	15·7	15·7	15·4	14·9	13·8	11·7	9·15	1·20						
26	15·9	16·1	16·2	16·2	15·9	15·3	14·3	12·3	9·48	1·24					
25	16·2	16·4	16·6	16·7	16·8	16·5	15·9	14·9	12·8	9·84	1·29				
24	16·5	16·7	17·0	17·2	17·3	17·2	17·0	16·6	15·5	13·3	10·3	1·35			
23	16·7	17·0	17·3	17·6	17·8	17·9	17·9	17·8	17·3	16·1	13·8	10·7	1·41		
22	17·0	17·3	17·6	17·9	18·2	18·4	18·5	18·6	18·5	18·0	16·7	14·3	11·1	1·47	
21	17·2	17·6	18·0	18·3	18·6	18·9	19·1	19·4	19·5	19·3	18·5	17·2	14·9	11·7	1·54
20	17·5	17·9	18·3	18·6	19·0	19·2	19·5	20·0	20·2	20·3	20·1	19·5	18·2	15·6	12·2
19	17·8	18·2	18·6	19·0	19·3	19·7	20·1	20·5	20·9	21·1	21·2	21·1	20·4	19·3	16·6
18	18·1	18·5	18·9	19·2	19·7	20·0	20·5	21·0	21·4	21·8	22·1	22·4	22·3	21·4	20·2
17	18·4	18·8	19·2	19·7	20·1	20·5	20·9	21·5	21·9	22·4	22·8	23·1	23·3	23·5	22·7
16	18·7	19·2	19·6	20·0	20·5	20·8	21·4	22·0	22·5	23·0	23·4	24·1	24·6	24·5	24·5
15	19·1	19·5	20·0	20·4	20·9	21·3	21·9	22·4	23·0	23·5	24·1	24·9	25·2	26·0	26·0
14	19·5	19·9	20·4	20·8	21·3	21·7	22·3	23·0	23·5	24·1	24·7	25·5	26·3	26·7	27·1
13	19·9	20·3	20·8	21·3	21·8	22·3	22·9	23·5	24·1	24·7	25·4	26·3	26·9	27·9	28·2
12	20·3	20·8	21·3	21·8	22·3	22·7	23·4	24·1	24·7	25·3	26·0	27·0	27·9	28·5	29·1
11	20·7	21·2	21·8	22·3	22·8	23·3	23·9	24·7	25·3	26·0	26·7	27·9	28·5	29·7	30·2
10	21·2	21·7	22·3	22·8	23·4	23·8	24·6	25·3	26·0	26·7	27·5	28·5	29·9	30·4	31·2
9	21·7	22·2	22·8	23·4	23·9	24·5	25·2	26·0	26·7	27·5	28·3	29·0	30·3	31·7	32·3
8	22·2	22·7	23·3	24·0	24·6	25·1	25·9	26·7	27·5	28·3	29·2	30·3	31·5	32·4	33·4
7	22·7	23·3	23·9	24·6	25·2	25·9	26·7	27·5	28·3	29·2	30·1	31·4	32·3	33·9	34·7
6	23·3	23·9	24·6	25·2	25·9	26·6	27·4	28·3	29·2	30·1	31·1	32·4	33·7	34·8	35·9
5	23·9	24·5	25·2	25·9	26·6	27·3	28·3	29·2	30·1	31·1	32·1	33·3	34·6	36·4	37·3

T (MeV)	k_0 (MeV) 20	19	18	17	16	15	14	13	12	11	10	9	8	7	6
20	1·62														
19	12·8	1·70													
18	17·4	13·6	1·80												
17	21·3	18·4	14·3	1·91											
16	23·8	22·6	19·2	15·0	2·02										
15	25·9	25·3	23·9	20·5	16·1	2·16									
14	27·3	27·5	26·7	25·2	21·9	17·4	2·31								
13	28·8	29·2	30·0	28·6	27·2	23·5	18·4	2·49							
12	29·8	30·7	30·9	31·1	30·6	29·3	25·5	20·0	2·70						
11	31·1	31·9	32·8	33·3	33·7	33·0	31·6	25·9	21·8	2·94					
10	32·1	33·3	34·0	35·1	35·8	36·5	36·0	34·5	30·1	24·7	3·23				
9	33·4	34·5	35·8	36·8	38·2	38·9	39·6	38·9	38·2	32·5	26·2	3·60			
8	34·5	36·0	37·1	38·8	39·9	41·7	42·7	43·8	43·5	43·3	36·8	29·4	4·05		
7	35·9	37·3	38·9	40·4	42·2	43·7	45·6	47·0	48·9	48·1	47·6	43·6	33·1	4·53	
6	37·2	38·9	40·4	42·3	44·1	46·5	48·4	51·0	52·9	56·1	55·2	54·4	49·9	37·9	5·42
5	38·6	39·1	42·5	44·2	46·5	48·7	51·3	52·7	57·4	59·3	62·0	64·4	63·7	58·3	44·8

where

$$\sigma_n(k_0)=\frac{1}{S(k_0,T)}\int_T^{k_0}\{\sigma_{n-1}(k_0)-\sigma_{n-1}(k)\}P'(k,k_0)\mathrm{d}k. \quad \ldots . \quad (8)$$

The suggested procedure for solving photonuclear yield curves is then as follows:

(1) The observed yield curve is differentiated numerically with respect to the electron energy.

(2) A zero order approximation to the cross section is obtained using equation (5) and Table 2 for $S(k_0,T)$, selecting the nearest value of T below the reaction threshold.

(3) Improved solutions are obtained by iteration using equations (7) and (8) and the values of $P'(k,k_0)$ given in Table 1. The integrals in equation (8) are of course replaced by one of the usual simple numerical approximations.

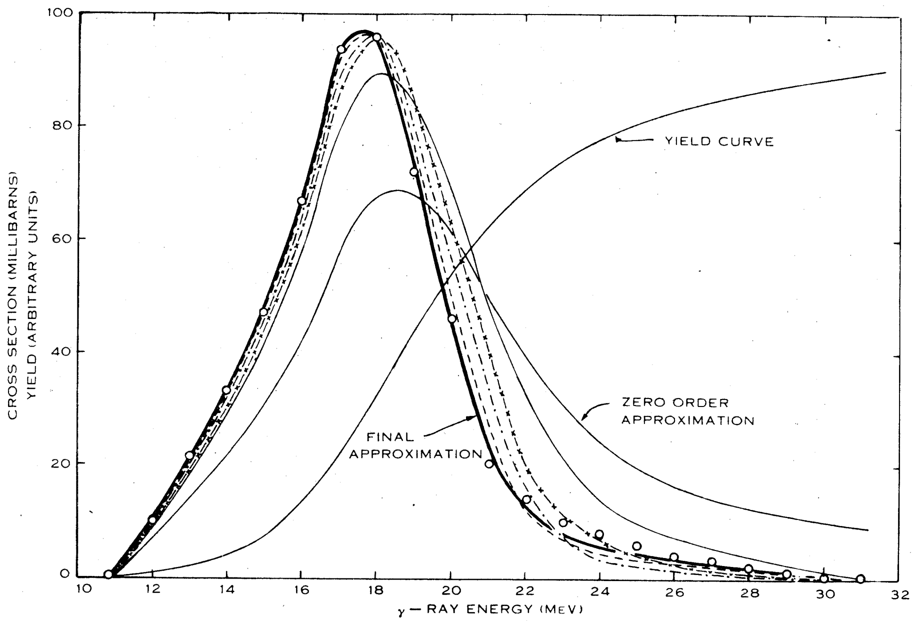


Fig. 1.—Illustration of the iterative method: (*a*) the ^{63}Cu (γ,n) cross section as determined by Berman and Brown (1954) (circles ○); (*b*) an artificial yield curve constructed from this cross section; (*c*) the analysis of the yield curve showing the successive iterations converging to the original cross section

The Reactions ^{12}C (γ,n), ^{16}O (γ,n), *and* ^{54}Fe (γ,n)

The Canberra electron synchrotron was used to irradiate samples of carbon, oxygen (boric acid), and iron at different bremsstrahlung energies and the induced positron activities ^{11}C (20·4 min), ^{15}O (2·1 min), and ^{53}Fe (8·9 min) were measured with thin-window Geiger counters. The γ-ray flux was monitored by means of the well-known ^{63}Cu (γ,n) reaction, copper foils being exposed at the same time as the samples. Berman and Brown (1954) have used the Stanford linear accelerator to measure the ^{63}Cu (γ,n) reaction absolutely and their result, which is independent of a calculated R-meter response, has been taken as standard for the present work. The methods of Baker and Katz (1953) were used to compare the positron activities of the samples with that of the standard.

The measured yield curves were analysed by the iterative method described above. In order to check the procedure an artificial yield curve was constructed from the ^{63}Cu (γ,n) cross section curve of Berman and Brown (1954) using values of $P(k,k_0)$ calculated from equations (**2**) and (**3**). The constructed

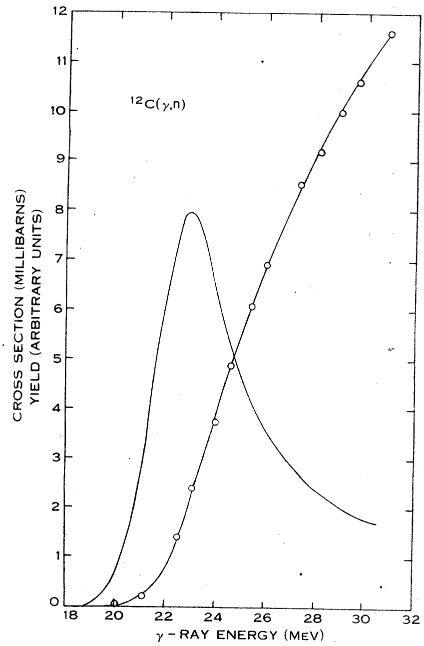


Fig. 2.—The measured yield curve and the derived cross section curve for the reaction ^{12}C (γ,n).

yield curve* was analysed to recompute the cross section. The results of the calculation are presented in Figure 1, which shows the assumed cross section, the constructed yield curve, and the successive iterations, which are seen to converge to the assumed cross section curve.

The present results are shown in Figures 2, 3, and 4, and some important parameters are summarized in Table 3.

* This constructed yield curve has a slightly different shape from the actual measured yield curve of Berman and Brown (1954), since they used a thick target bremsstrahlung formula in the analysis of their results. The constructed yield curve is used to monitor the present data, a procedure which gives some compensation for any difference between the actual bremsstrahlung shape and the assumed thin target formula.

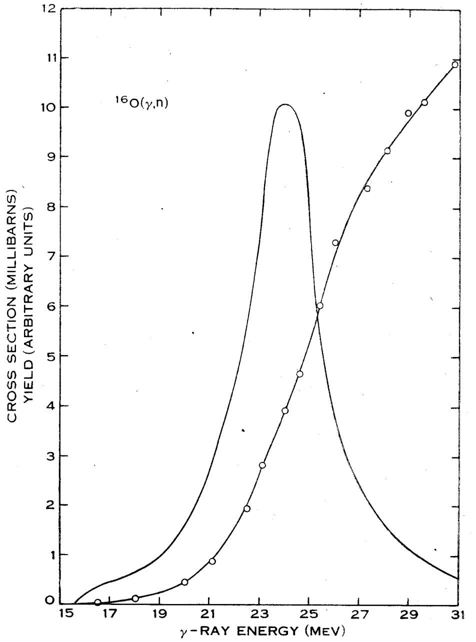


Fig. 3.—The measured yield curve and the derived cross section curve for the reaction ^{16}O (γ,n).

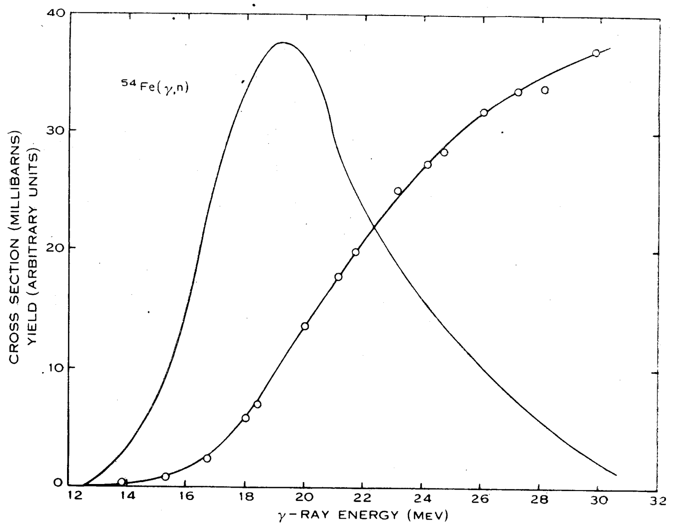
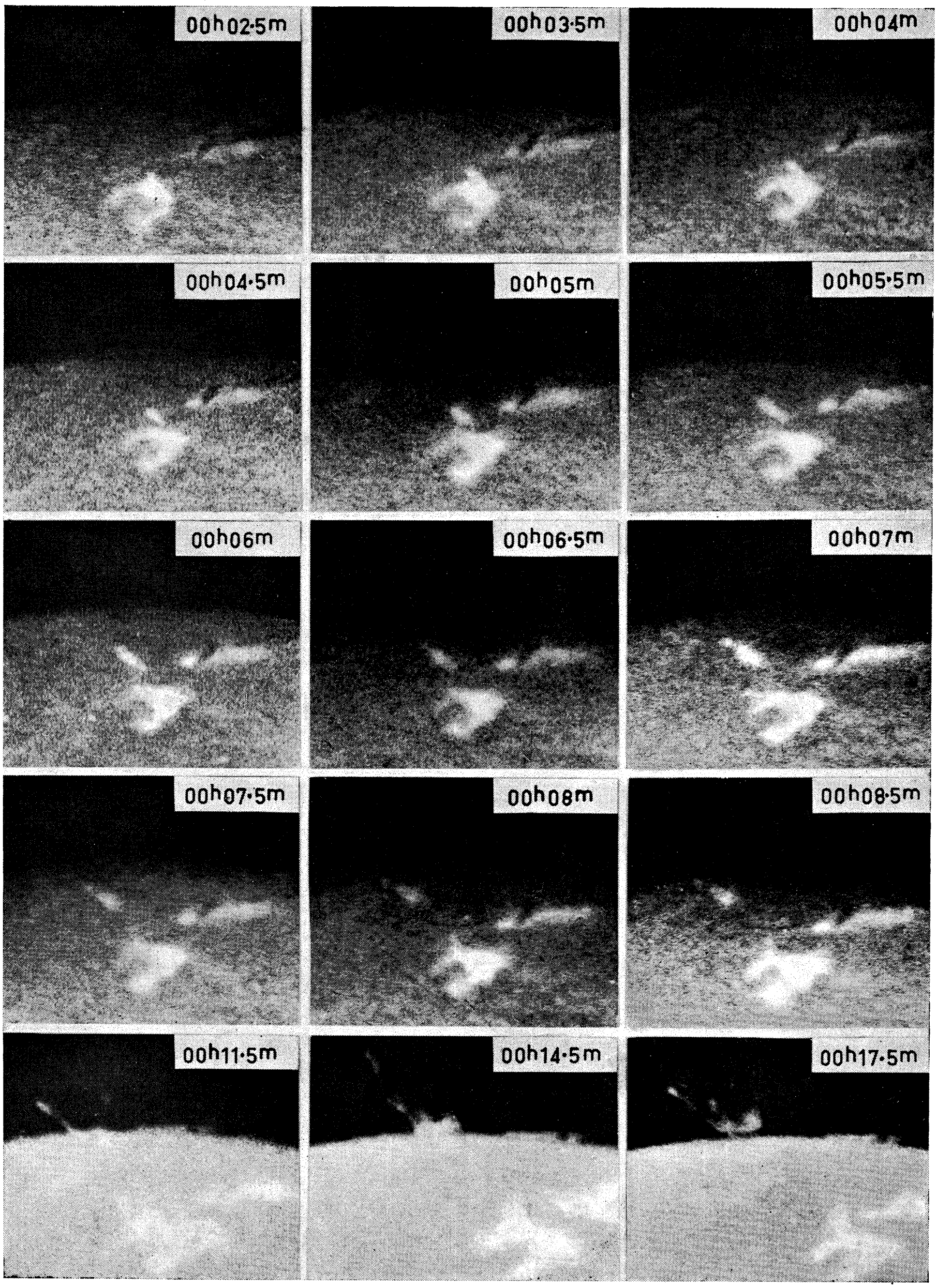


Fig. 4.—The measured yield curve and the derived cross section curve for the reaction ^{54}Fe (γ,n).

A NEW TYPE OF FLARE



On the original negative the diameter of a solar image is 15 mm.

All the cross section curves have finite tails extending beyond 30 MeV; the very pronounced tail in the ^{12}C (γ,n) curve is in accordance with the observations of Barber, George, and Reagan (1955).

TABLE 3

PARAMETERS OF THE MEASURED (γ,n) CROSS SECTIONS

Reaction	Peak Energy (MeV)	Width at Half Maximum (MeV)	Integrated Cross Section to 31 MeV (MeV mb)
^{12}C (γ,n)	23	4·2	42 ± 7
^{16}O (γ,n)	24	3·4	46 ± 7
^{54}Fe (γ,n)	19	6·9	290 ± 50

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