ON THE RADIO EMISSION OF HYDROGEN NEBULAE

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Summary

The interpretation of radio-frequency observations of H II regions is discussed with particular regard for the possible effects of random variations in the electron density and electron temperature through the nebulae. It is shown that such variations serve to alter the optical depth and that the conventional definition of the " emission measure " requires modification if it is to be considered an observable quantity. The radio emission of Strömgren spheres is discussed, and a means of determining their electron temperatures is described. An empirical method for the determination of Strömgren's constant defining the ionized volume as a function of the spectral type and luminosity of the exciting star is described.

I. Introduction

A number of galactic H II regions have been observed at radio frequencies. The observations published to date cover frequencies ranging from 19·7 to 9375 Mc/s. Such observations can provide information about the temperatures and densities of the nebulae, and about the far ultraviolet radiation of the stars exciting them. The objective of the present paper is to examine in some detail the problem of deriving physical data on the nebulae from radio observations.

The H II regions constitute a special class of radio source characterized by their spectra, which are " flat " except at the lower frequencies where they become optically thick. That is, their flux densities are nearly constant over a very wide range of frequencies. This is what is expected if the nebulae are radiating by the thermal process of free-free transitions in an ionized gas. They are readily distinguished from the " non-thermal " radio sources, whose spectra show a strong frequency dependence.

The radio emission of H II regions is well understood theoretically (e.g. Piddington 1951). Discussions of the nebulae as radio sources have generally assumed for simplicity that the objects are uniform throughout, although it has been known that a non-uniform distribution of the nebular gas would tend to increase the radio emission because the emissivity of an ionized gas depends on the square of the electron density. In the present paper we shall give particular attention to the consequences of such a non-uniform distribution.

Section II is devoted to a discussion of the directly observable properties of H II regions. Section III considers the effect of density and temperature variations on the optical depth, and the magnitude of the effect is estimated for some particular nebular models. A modification of the definition of the term

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"emission measure" is suggested. In Section IV, the radio emission of Strömgren spheres is discussed, and a method of deriving their electron temperatures is described. Section V is concerned with the excitation of the nebulae.

II. Directly Observable Properties

Mills, Little, and Sheridan (1956) have used the term "apparent flux density" to describe the observed radiation of an H II region of uniform electron temperature T_e, lying between the observer and a background of uniform brightness temperature T_b. This quantity is

$$F_{app}=(2kf^2/c^2)(T_e-T_b)\int(1-e^{-\tau})d\Omega, \quad \dots\dots\dots\dots \quad (1)$$

where the integration is extended over the entire solid angle Ω subtended by the nebula. In the above equation, k is Boltzmann's constant, c is the velocity of light, f is the frequency in hertz, and τ is the optical depth at the frequency f. The "apparent flux density" as defined here is the difference between the actual flux density coming from the nebula plus the transmitted portion of the background radiation, and the flux density which would be incident from the area covered by the nebula if the latter were absent. It should not be confused with the actual flux density coming from the area covered by the nebula, which is

$$(2kf^2/c^2)\{T_e\int(1-e^{-\tau})d\Omega+T_b\int e^{-\tau}d\Omega\},$$

provided no emission arises between the nebula and the observer. This assumption is reasonable for frequencies above about 100 Mc/s for nebulae near enough to be observed optically. The quantity "apparent flux density" offers the advantage that it describes the "visibility" of a nebula in a convenient manner. If $T_e>T_b$, the object appears in emission; if $T_e<T_b$, it is seen in absorption. It will not be detectable at all if $T_e=T_b$. It is important to note that, while the actual flux density is always positive, the "apparent flux density" may be either positive or negative, depending on whether the object is seen in emission or absorption.

The apparent flux density incident from an H II region may be measured at various frequencies. Existing observations have been made with aerials having beamwidths of the same order as the angular sizes of the nebulae observed. In the future it will be possible, presumably, to resolve a number of the larger nebulae with very large aerials. This will permit direct determinations of the distributions of radio brightness across these objects. At present, however, the radio data usually consist only of measurements of F_{app} and T_b at various frequencies. Information about the angular sizes and apparent shapes of most of the nearer H II regions may be obtained from photographs, although in some cases these are severely affected by interstellar absorption. In the present paper we shall consider only the quantities derivable from these data.

Equation (1) implies that we may learn the electron temperature of a nebula simply by finding the frequency at which F_{app} is zero, and measuring T_b at this frequency. Then $T_e=T_b$. Mills, Little, and Sheridan (loc. cit.), following this line of thought, have shown that the commonly accepted value of $T_e\approx 10{,}000$ °K is consistent with the observational evidence. There are two considerations which prevent this approach being a sensitive method of determining electron temperatures. Firstly, current calibration methods do not permit the absolute accuracy of brightness temperature measurements to be better than about ± 20 per cent. Secondly, the random noise fluctuations in the output of contemporary receivers prevent an accurate determination of the frequency at which F_{app} vanishes. Therefore at present this method can give us only the order of magnitude of the electron temperatures. Nevertheless, future improvements in instrumental calibration techniques and receiver characteristics may make an approach of this kind useful.

III. The Radio Emission of a Non-uniform Hydrogen Nebula

(a) *The Optical Depth*

We shall consider the optical depth in a cloud of completely ionized hydrogen gas. We assume that the emission and absorption of radiation in the gas are completely described by the mechanism of free-free transitions. According to Piddington (1951), at low densities the absorption coefficient per centimetre in a gas consisting of equal numbers of protons and electrons is

$$\left.\begin{aligned}\varkappa &= \zeta n_e^2/f^2T_e^{3/2},\\ \zeta &= 9\cdot 70\times 10^{-3}\ln(3kT_e/2hf).\end{aligned}\right\} \qquad (2)$$

In these equations, n_e is the number of free electrons per unit volume and h is Planck's constant. The optical depth along a particular line of sight in a cloud of ionized hydrogen is

$$\tau = \int_0^S \varkappa \mathrm{d}s, \qquad (3)$$

where S is the length traversed by the line in the ionized region. If the distribution of matter and temperature is perfectly uniform, we have simply

$$\tau_u = \zeta n_e^2 S/f^2 T_e^{3/2}. \qquad (4)$$

The subscript u denotes the uniform case.

In general, there is no physical justification for an *a priori* assumption that an actual nebula is entirely uniform. The electron density and electron temperature at a point s on a particular line of sight may be written

$$\begin{aligned}n_e(s) &= \overline{n_e} + \delta n_e(s),\\ T_e(s) &= \overline{T_e} + \delta T_e(s),\end{aligned}$$

where $\overline{n_e}$ and $\overline{T_e}$ are the average values for the nebula, and $\delta n_e(s)$ and $\delta T_e(s)$ are the local deviations from the average. Defining

$$\begin{aligned}m &\equiv \delta n_e(s)/\overline{n_e},\\ t &\equiv \delta T_e(s)/\overline{T_e},\end{aligned}$$

we have

$$\left.\begin{aligned}n_e(s) &= n_e(1+m),\\ T_e(s) &= T_e(1+t).\end{aligned}\right\} \qquad (5)$$

Neglecting the slow variation of ζ as a function of T_e, we may rewrite the equations (2) as

$$\left.\begin{aligned}\varkappa&=\frac{\zeta(\overline{n}_e)^2}{f^2\overline{T}_e^{3/2}}\cdot\frac{(1+m)^2}{(1+t)^{3/2}},\\ \zeta&=9\cdot 70\times 10^{-3}\ln(3k\overline{T}_e/2hf).\end{aligned}\right\} \qquad (6)$$

The optical depth is then

$$\tau=\frac{\tau_u}{S}\int_0^S\frac{(1+m)^2}{(1+t)^{3/2}}\mathrm{d}s. \qquad (7)$$

The optical depth therefore differs from that in the uniform case by a factor

$$Q=\frac{1}{S}\int_0^S\frac{(1+m)^2}{(1+t)^{3/2}}\mathrm{d}s. \qquad (8)$$

We shall call this the "amplification factor". If $Q>1$, the optical depth is greater than that corresponding to a uniform density and temperature; if $Q<1$, it is less. The value of Q is independent of the frequency, and is fixed by the distribution of matter and temperature within the nebula. Equation (8) is quite general; it may be applied for either systematic or random variations.

It is evident that, in the absence of systematic density and temperature variations, the optical depth is the same as if the nebula were at a uniform temperature $\overline{T}_e$ with a uniform density equal to $\overline{n}_e\sqrt{Q}$. An unfortunate consequence of these considerations is that the average density of an H II region cannot be found from radio-frequency observations without an independent knowledge of the distribution of matter and temperature. We may, however, derive an "equivalent density" defined by

$$n_{eq}=\overline{n}_e\sqrt{Q}. \qquad (9)$$

This is "equivalent" in the sense that it is the density the nebula would need to have in order to produce the observed radio emission if it were uniform. Strictly speaking, it is a parameter which depends in an undetermined manner on the average electron density and the variations in density and temperature. In the special case that the electron temperature is uniform the equivalent density is equal to the root mean square density.

(b) *The Apparent Flux Density*

The apparent brightness temperature at a point on the projected surface of a nebula is

$$\begin{aligned}T_B&=\mathrm{e}^{-\tau}\int_0^\tau T_e\mathrm{e}^{\tau'}\mathrm{d}\tau'+T_b\mathrm{e}^{-\tau}\\ &=\overline{T}_e(1-\mathrm{e}^{-\tau})+\overline{T}_e\mathrm{e}^{-\tau}\int_0^\tau t\mathrm{e}^{\tau'}\mathrm{d}\tau'+T_b\mathrm{e}^{-\tau}.\end{aligned}$$

If the temperature fluctuations are random along the line of sight, and if their linear scale is small compared to the length of the line of sight in the ionized region, $\bar{t}=0$, and consequently the integral vanishes. Therefore, the apparent brightness temperature is

$$T_B=\overline{T_e}(1-e^{-\tau})+T_b e^{-\tau}.$$

This differs from T_b by

$$\Delta T=(\overline{T_e}-T_b)(1-e^{-\tau}).$$

The apparent flux density is then

$$F_{app}=(2kf^2/c^2)(\overline{T_e}-T_b)\int(1-e^{-\tau})d\Omega. \quad \ldots\ldots\ldots \quad \mathbf{(10)}$$

This expression is equivalent to equation (**1**) if the electron temperature is uniform.

(c) On the Evaluation of the Amplification Factor

There is no general solution to the equation for the amplification factor (equation (**8**)). A considerable simplification results, however, if the variations are purely random and if there is a functional relationship between electron temperature and density. In an actual nebula, it is likely that such a functional relationship exists, at least approximately. In the present subsection we shall consider two extreme possible cases: an isothermal nebula (uniform electron temperature) and an adiabatic nebula (uniform entropy). If the density variations within a nebula arise from turbulent motions of the gas, we may expect the actual relationship to lie somewhere between these two extremes.

In an isothermal nebula, $t=0$ everywhere and equation (**8**) gives

$$Q=\overline{(1+m)^2},$$

where the average is taken over the part of the line of sight lying in the nebula. Since we are assuming that the density variations are random, the above expression may be written

$$Q=1+\overline{m^2}.$$

In an adiabatic nebula, we have

$$1+t=(1+m)^{\gamma-1},$$

where γ is the ratio of specific heats. The electrons and protons comprising the gas have no communicable internal degrees of freedom, so $\gamma=5/3$. We obtain the result that

$$Q=\overline{1+m}=1.$$

Thus the optical depth is unaffected by the presence of random density variations if they are adiabatically related to the temperature variations.

(d) The Discrete Cloud Model

The value of the amplification factor depends not only on the relation between m and t but also on the form of the function $m(s)$. A particularly simple model is one in which a fraction β of the mass of a nebula resides in condensations occupying a fraction α of the nebular volume, the densities being uniform and equal in the condensations, and also uniform outside them. This situation corresponds to having

$$m=\beta/\alpha-1$$

inside the condensations and

$$m=(\alpha-\beta)/(1-\alpha)$$

outside. If the electron temperature is uniform, and if the condensations are distributed at random, we find

$$Q=\beta^2/\alpha+(1-\beta)^2/(1-\alpha). \quad \ldots\ldots\ldots\ldots \quad \textbf{(11)}$$

A consequence of this result is that

$$Q(\alpha,\ \beta)=Q(1-\alpha,\ 1-\beta).$$

This means that if, instead of condensations, we have uniform regions of sub-average density scattered through a nebula, we obtain a positive amplification factor equal to that in the inverse case. Figure 1 shows Q as a function of α and β

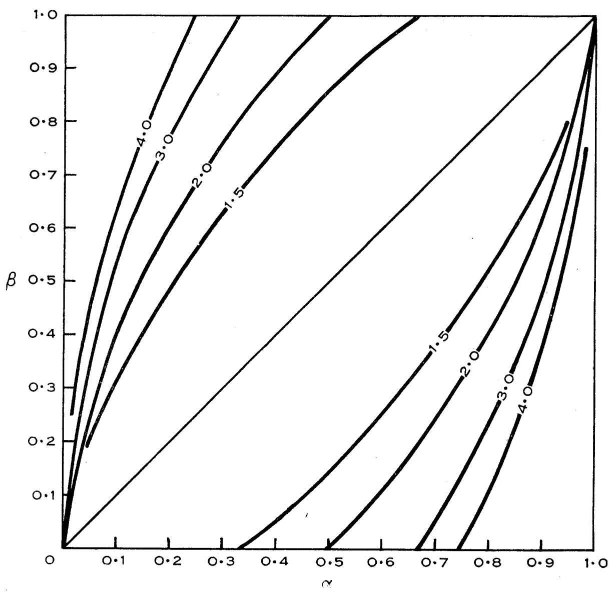

Fig. 1.—$Q(\alpha,\ \beta)$ for an isothermal nebula.

for constant electron temperature. The part of the diagram above the diagonal corresponding to $Q=1$ applies to the case of true condensations ($\alpha<\beta$); the part below refers to " negative condensations " ($\alpha>\beta$). If $\alpha=\beta$, the nebula is uniform and perforce $Q=1$.

(*e*) *The Emission Measure*

Strömgren (1948) introduced the term *emission measure* to describe the monochromatic intensity of nebular radiation in the Balmer lines. The emission measure is defined as the product of the square of the density (which Strömgren assumed to be uniform) and the length of the emission path in parsecs ;

$$\varepsilon=(3\cdot 08\times 10^{18})^{-1}n_e^2S. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

The numerical factor is required because we are using c.g.s. units.

The emission measure has also been used by some authors in discussing the radio emission of H II regions. According to equation (4), we have a simple relationship between emission measure and optical depth for perfectly uniform nebulae :

$$\tau_u=3\cdot 08\times 10^{18}\zeta\varepsilon/f^2T_e^{3/2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

For a non-uniform nebula, however,

$$\tau=\zeta(\bar{n}_e)^2SQ/f^2\overline{T}_e^{\,3/2}=\zeta n_{eq}^2S/f^2\overline{T}_e^{\,3/2}. \quad \ldots\ldots\ldots \quad (14)$$

If we define

$$\varepsilon'=(3\cdot 08\times 10^{18})^{-1}n_{eq}^2S, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

we obtain an expression analogous to (13):

$$\tau=3\cdot 08\times 10^{18}\zeta\varepsilon'/f^2\overline{T}_e^{\,3/2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

The emission measure as defined by Strömgren is not appropriate for discussing a non-uniform nebula, since it cannot be determined from observations without reference to a physical model of the nebula. However, the analogous quantity ε' defined by equation (15) can be related directly to the optical depth, provided the average electron temperature is known. In the next section we shall outline a means of obtaining the average electron temperature and the equivalent density of a spherical nebula from radio data. Therefore ε' is in principle an observable quantity, whereas ε is not. We suggest that the term " emission measure " should be defined by (15) instead of (12). The two definitions are equivalent for a uniform nebula because then $Q=1$.

IV. Strömgren Spheres

Strömgren (1939, 1948) has studied theoretically the ionized region which would surround a hot star imbedded in an extended uniform cloud of hydrogen gas. He showed that the Lyman continuum radiation of the star would cause almost complete ionization of the hydrogen out to a quite sharply defined boundary. The radius of the ionized sphere was shown to depend upon the density of the gas and the Lyman continuum flux emitted by the star. Strömgren's theory provides a convenient basis for discussing hydrogen nebulae, although it treats a highly idealized case. Many galactic H II regions are very nearly spherical in shape and are reasonably concentric with the stars exciting them. Such nebulae evidently approximate to the case considered by Strömgren and are frequently referred to by the convenient designation " Strömgren spheres ". In the present section we consider the radio emission of these objects.

(a) *The Apparent Flux Density*

We assume that the average density of the nebular hydrogen does not change as a function of distance from the exciting star and that the density and temperature variations have a linear scale small compared to the radius of the ionized sphere. We shall let τ_0 denote the optical depth of the ionized sphere at its apparent centre, and θ_0 be the apparent angular radius of the nebula in radians. We also assume that the radius of the ionized region is small compared with its distance, so that we may set

$$\theta_0 = \sin \theta_0.$$

The optical depth at an angular distance θ from the centre of the nebula will be

$$\tau(\theta) = \tau_0(1 - \theta^2/\theta_0^2)^{\frac{1}{2}}.$$

For a spherical nebula, equation (**10**) takes the form

$$F_{app} = 2kf^2/c^2(\overline{T}_e - T_b)\int_0^{\theta_0} (1 - e^{-\tau(\theta)})2\pi\theta d\theta$$

$$= \frac{4\pi kf^2(\overline{T}_e - T_b)}{c^2}\int_0^{\theta_0} \theta(1 - e^{-\tau(\theta)})d\theta. \quad \ldots\ldots\ldots\ldots (\mathbf{17})$$

Completion of the integration gives the result

$$F_{app} = \{4\pi k\theta_0^2 f^2(\overline{T}_e - T_b)/c^2\}Y(\tau_0), \quad \ldots\ldots\ldots\ldots\ldots\ldots (\mathbf{18})$$

where

$$Y(\tau_0) = \tfrac{1}{2} + \tau_0^{-2}[e^{-\tau_0}(\tau_0 + 1) - 1]. \quad \ldots\ldots\ldots\ldots (\mathbf{19})$$

An equivalent expression is

$$Y(\tau_0) = \sum_{n=1}^{\infty} (-1)^{n+1}\frac{\tau_0^n}{n!(n+2)}. \quad \ldots\ldots\ldots\ldots\ldots (\mathbf{20})$$

This series may be shown to converge for all values of $\tau_0 > 0$. If $\tau_0 < 0 \cdot 1$, the approximation

$$Y(\tau_0) = \tfrac{1}{3}\tau_0$$

is sufficiently accurate. We also note that $Y(\infty) = 0 \cdot 500$. Figure 2 shows $Y(\tau_0)$ for $0 \cdot 1 \leqslant \tau_0 \leqslant 10 \cdot 0$.

Equation (**18**) takes the limiting forms

$$F_{app} = \{4\pi k\theta_0^2 f^2(\overline{T}_e - T_b)/3c^2\}\tau_0, \quad \ldots\ldots\ldots\ldots (\mathbf{21})$$

for $\tau_0 \ll 1$ and

$$F_{app} = 2\pi k\theta_0^2 f^2(\overline{T}_e - T_b)/c^2, \quad \ldots\ldots\ldots\ldots\ldots\ldots (\mathbf{22})$$

for $\tau_0 \gg 1$. These correspond to expressions given by Mills, Little, and Sheridan (loc. cit.).

(*b*) *The Electron Temperature*

Equation (**18**) provides a means of estimating the average electron temperature of a Strömgren sphere from observations of the apparent flux density at two well-separated frequencies. The central optical depth as a function of electron temperature may be calculated at each frequency. The ratio of the optical depths at each temperature is then found. The ratio corresponding to the correct electron temperature will be equal to the ratio of the opacities at the two frequencies, so

$$\frac{\tau_{0,1}}{\tau_{0,2}}=\frac{\zeta_1}{\zeta_2}\cdot\frac{f_2^2}{f_1^2}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

where the subscripts 1 and 2 denote the two frequencies.

We may illustrate the calculation by considering the Rosette Nebula (NGC 2237). This object resembles a classical Strömgren sphere except for the fact that its central part appears to be much lower in density than the average. According to Mills, Slee, and Hill (1958), the apparent flux density at 85·5 Mc/s is $2\cdot7\times10^{-24}$ W m^{-2} Hz^{-1}. Piddington and Trent (1956) found the apparent

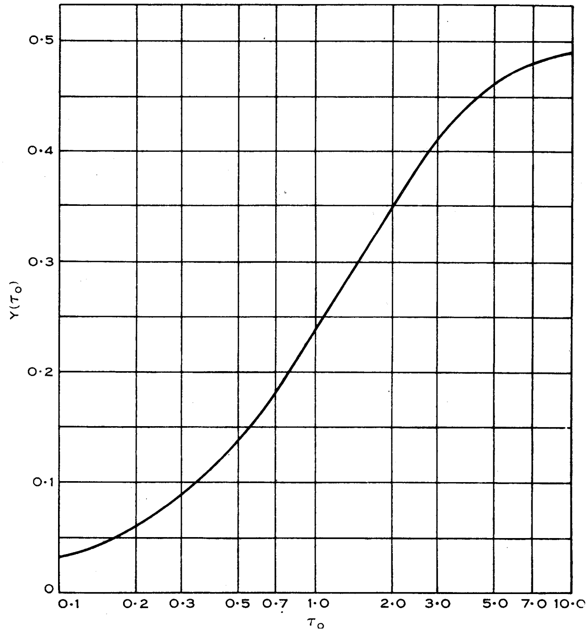

Fig. 2.—The function $Y(\tau_0)$.

flux density at 600 Mc/s to be $4\cdot0\times10^{-24}$ W m^{-2} Hz^{-1}. At 85·5 Mc/s, T_b=1800 °K (Mills, unpublished data), while T_b is negligible at 600 Mc/s. The angular diameter of the nebula is 80 min of arc. Using these data, we find the values given in Table 1. The values of the ratio of $\tau_{0,1}$ to $\tau_{0,2}$ calculated from equation (23) are 56·6 at 8000 °K and 56·4 at 10,000 °K. We can now plot the two determinations of optical depth ratio as a function of electron temperature. The intersection of the two curves gives the required value of the average electron temperature. We do this in Figure 3, obtaining the result $\overline{T}_e$=8600 °K. Unfortunately, this result is sensitive to the errors in the measured flux densities; it is probably within about 30 per cent. of the correct value. The method has other limitations. Firstly, it is not reliable if the central optical depth is too great at one or both frequencies, since the computed values of τ_0 are then too sensitive to errors in $Y(\tau_0)$. Secondly, if the nebula is too thin optically at both frequencies, so that $Y(\tau_0)$ is linear at each, one cannot obtain a reliable result.

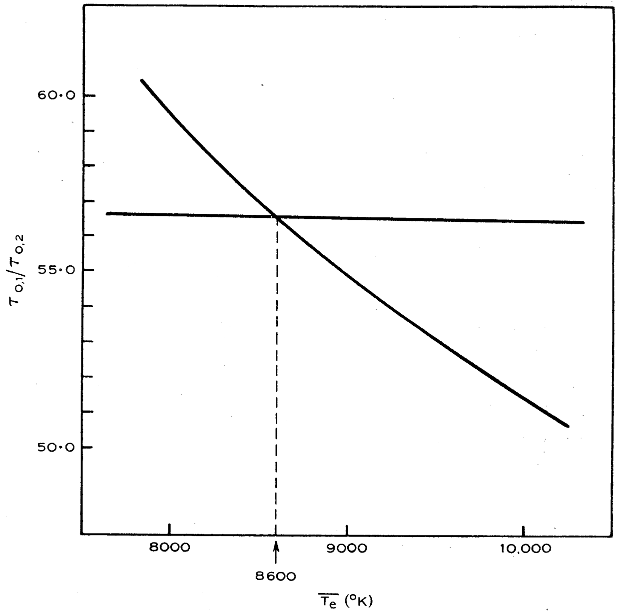

Fig. 3.—Determination of the electron temperature of the Rosette Nebula (NGC 2237).

(c) The Equivalent Density

We may use equation (**18**) to derive the equivalent density of a spherical nebula from measurements of its radio-frequency flux, provided its central optical depth is not too great. If F_{app}, $\overline{T}_e$, T_b, and θ_0 are known we may solve for $Y(\tau_0)$, and find the corresponding value of τ_0.

Now

$$\tau_0 = 6\cdot 16 \times 10^{18} \theta_0 R \zeta n_{eq}^2 / f^2 \overline{T}_e^{\,3/2},$$

whence

$$n_{eq} = 5\cdot 72 \times 10^{-10} f (\tau_0 \overline{T}_e^{\,3/2} / 2\zeta R \theta_0)^{\frac{1}{2}}. \quad \text{..........} \quad (\mathbf{24})$$

TABLE 1

CENTRAL OPTICAL DEPTH OF THE ROSETTE NEBULA AS A FUNCTION OF FREQUENCY AND ELECTRON TEMPERATURE

Frequency	85·5 Mc/s		600 Mc/s		
$\overline{T}_e$ (°K)	$Y(\tau_{0,1})$	$(\tau_{0,1})$	$Y(\tau_{0,2})$	$\tau_{0,2}$	$\tau_{0,1}/\tau_{0,2}$
8,000	0·229	0·96	0·00535	0·0161	59·5
9,000	0·197	0·78	0·00475	0·0142	55·0
10,000	0·173	0·66	0·00428	0·0128	51·5

R is the distance to the nebula in parsecs, which must be found from optical studies of the exciting star. This ordinarily requires a knowledge of the absolute magnitude and intrinsic colour of the star. These data are not well determined for O, B, and Wolf-Rayet stars. Since only stars of these types are sufficiently hot and luminous to excite an observable Strömgren sphere, it follows that the distance to an H II region is not easy to find accurately. The seriousness of the matter is reduced somewhat for our purposes by the fact that the equivalent density depends only on the square root of R. We note that the derived equivalent density is almost independent of $\overline{T}_e$, since the product $f^2\overline{T}_e^{3/2}\tau_0$ is nearly constant for a given nebula over a very wide range of temperatures.

Johnson (1957) has found that the distance to the cluster NGC 2244, which contains the stars exciting the Rosette Nebula, is 1660 parsecs. Using this distance and the radio data given above, we obtain an equivalent density of 17 cm^{-3}, in good agreement with the value 14 cm^{-3} found by Minkowski (1955) from optical data.

V. The Excitation of the Nebulae

Strömgren (1939) has shown that the quantity

$$U=n_{eq}^{2/3}s_0 \qquad (25)$$

is a constant depending on the spectral type and absolute luminosity of the exciting star, s_0 being the radius of the ionized zone in parsecs. Strömgren assumed a uniform density in his derivation; according to the considerations we have presented in Section III this may be replaced by the equivalent density n_{eq}.

Strömgren computed the excitation constant U for stars of various spectral types from his theory. The calculated values depend on a number of assumptions, however, and it is desirable to have a direct observational determination of these quantities. Substituting (24) in (25) and replacing $R\theta_0$ by s_0, we get

$$U=6\cdot 87\times 10^{-7}s_0^{2/3}f^{2/3}\overline{T}_e^{1/2}(\tau_0/2\zeta)^{1/3}. \qquad (26)$$

This result is independent of the possible presence of density variations in the nebular gas, since τ_0 varies as s_0^{-2} for any value of Q. It is almost independent of the electron temperature, because of the quasi-constancy of the product $f^2\overline{T}_e^{3/2}\tau_0$. Therefore we may find U for a star exciting a spherical nebula if we know s_0 from optical studies and τ_0 from radio observations.

We may apply equation (26) to the Rosette Nebula, using the data of Section IV. We find $U=126$. The nebula is actually excited by four O-stars—one each of types O5 and O6, and two of type O8. The effective U for a group of stars is the cube root of the sum of the cubes of the U's of the individual stars. The effective U found from the data given by Strömgren is 167, which is in fair agreement with the empirical value.

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