SHORT COMMUNICATIONS

AN ANALOGY BETWEEN THE EFFECTS OF FLUCTUATING ELECTRIC FIELDS AND STEADY MAGNETIC FIELDS IN ISOTROPIC CONDUCTORS WHEN A UNIVERSAL RELAXATION TIME CANNOT BE DEFINED*

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It is well known that when a " universal " time of relaxation (τ) exists, the influence of a harmonically varying electric field ($\mathbf{F} \propto e^{i\omega t}$) on the transport properties of a solid may be taken into account by replacing τ by $\tau/(1+i\omega\tau)$. Dingle (1956*a*) demonstrated that, for an isotropic solid, the effect of a steady magnetic field may similarly be obtained by replacing τ by $\tau/(1+j\Omega\tau)$ with an applied d.c. electric field, and by $\tau/[1+(i\omega+j\Omega)\tau]$ with an a.c. field. (Here $j^2=-1$, $ij \neq -1$, and $\Omega=(-\varepsilon)H/mc$ is the circular frequency of precession of an electron.) The object of the present note is to show that this analogy between a high frequency electric field and a steady magnetic field still exists, even when a " universal " relaxation time cannot be defined.

For ease of exposition, we consider a one-band model and assume that the applied fields are an electric field $\mathbf{F}=(F_x,F_y,0) \propto e^{i\omega t}$ and a constant magnetic field $\mathbf{H}=(0,0,H)$. The acceleration of a charge carrier (charge $-\varepsilon$, effective mass m, energy $E=\frac{1}{2}(\hbar/m)^2 \mid \mathbf{k} \mid^2$, wave vector $\mathbf{k}$, and velocity $\mathbf{v}=\hbar\mathbf{k}/m$) is then

$$\dot{\mathbf{v}} = -\frac{\varepsilon}{m}\left(\mathbf{F}+\frac{1}{c}\mathbf{v}\wedge\mathbf{H}\right).$$

As usual, we start with the Boltzmann equation

$$\frac{\partial f}{\partial t} - \frac{\varepsilon}{m}\left(\mathbf{F}+\frac{1}{c}\mathbf{v}\wedge\mathbf{H}\right)\cdot\nabla_{\mathbf{v}} f = \left[\frac{\partial f}{\partial t}\right]_{\text{coll.}}, \quad \ldots\ldots\ldots\ldots\ldots (1)$$

and assume a first-order solution of the form (Wilson 1953, p. 210)

$$f = f_0 - \mathbf{k}\cdot\mathbf{c}(E)\frac{\partial f_0}{\partial E}. \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Here, f is the electron distribution function, $[\partial f/\partial t]_{\text{coll.}}$ is the rate of change of f due to the various scattering mechanisms, f_0 is the equilibrium distribution function, and $\mathbf{c}(E)$ is a function of the electron energy and the applied fields.

Now, for the more important of those scattering mechanisms for which a universal relaxation time does not exist—acoustic phonon scattering in metals below the Debye temperature (Wilson 1953, p. 263) and optical phonon scattering

* Manuscript received December 2, 1959.

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in polar semiconductors (Howarth and Sondheimer 1953)—the collision term may be written

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll.}} = \mathbf{k}\cdot \mathrm{L}(\mathbf{c})\frac{\partial f_0}{\partial E}, \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where $\mathbf{c}=(c_1,c_2,0)$, $\mathrm{L}(\mathbf{c})=(\mathrm{L}[c_1],\mathrm{L}[c_2],0)$, and L is a linear operator, i.e. if α and β are any functions independent of the electron energy, then

$$\mathrm{L}(\alpha\mathbf{c}+\beta\mathbf{d})=\alpha\mathrm{L}(\mathbf{c})+\beta\mathrm{L}(\mathbf{d}). \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Substituting (2) and (3) in (1) we find, correct to linear terms in the electric field, that the components of $\mathbf{c}$ satisfy the equations

$$\mathrm{L}(c_1)+\frac{\partial c_1}{\partial t}-\Omega c_2=-\frac{\varepsilon\hbar}{m}F_x, \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$\mathrm{L}(c_2)+\frac{\partial c_2}{\partial t}+\Omega c_1=-\frac{\varepsilon\hbar}{m}F_y. \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

Now, by introducing a second imaginary j ($\mathrm{j}^2=-1$, $\mathrm{ij}\neq -1$) equations (5) and (6) may be combined into the single equation

$$\mathrm{L}(\mathbf{C})+\frac{\partial \mathbf{C}}{\partial t}+\mathrm{j}\Omega\mathbf{C}=-\frac{\varepsilon\hbar}{m}\mathbf{F}, \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

where $\mathbf{C}=c_1+\mathrm{j}c_2$, and $\mathbf{F}=F_x+\mathrm{j}F_y$.

It is now convenient to introduce a formal relaxation time $\tau(\omega,\Omega)$ defined by

$$\mathbf{C}=-\frac{\varepsilon\hbar}{m}\mathbf{F}\tau(\omega,\Omega). \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

Since $\mathbf{F}\propto e^{\mathrm{i}\omega t}$, equation (8) is a solution of (7) provided $\tau(\omega,\Omega)$ satisfies

$$\mathrm{L}[\tau(\omega,\Omega)]+(\mathrm{i}\omega+\mathrm{j}\Omega)\tau(\omega,\Omega)=1. \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

With $\omega=\Omega=0$, equation (9) reduces to

$$\mathrm{L}[\tau(0,0)]=1, \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

where $\tau(0,0)$ corresponds to one of the formal relaxation times introduced by Dingle (1956*b*).

Solution of the Boltzmann equation (9) is extremely difficult. The object of the present note is to direct attention to one interesting simplifying feature—that the solutions of (9) for the three field combinations ($\omega=0$, $\Omega\neq 0$), ($\omega\neq 0$, $\Omega=0$), and ($\omega\neq 0$, $\Omega\neq 0$) are essentially the same, so that solution of the Boltzmann equation for any one of the above cases in fact implies a full solution of the problem for all the cases.

Finally, it is of interest to compare the "universal" and the "formal" relaxation time theories. As mentioned above, the influence of a uniform magnetic field and a high frequency electric field on the transport properties may be obtained by replacing τ^{-1} by $\tau^{-1}+\mathrm{i}\omega+\mathrm{j}\Omega$. Analogously, for the formal relaxation time there is the replacement of the collision operator L by $\mathrm{L}+\mathrm{i}\omega+\mathrm{j}\Omega$.

The author is indebted to the Commonwealth Scientific and Industrial Research Organization for the award of a Studentship.

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