WAVENUMBER REPRODUCIBILITY OF THE RADIATION $2p_{10}$–$5d_5$ OF KRYPTON 86

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Summary

The vacuum wavenumber of the radiation $2p_{10}$–$5d_5$ (6056 Å) of krypton 86 emitted by a hot cathode Engelhard-type discharge lamp has been established for the temperature range 58–70 °K and current density range 0–1·0 A/cm². Wavenumbers relative to the value for the unperturbed state of the radiation have been measured with a reliability of 1 part in 10^9 using a photoelectric recording and electromechanical scanning Fabry–Perot interferometer. However, this reproducibility is not possible for different lamps without a more exact specification for the form and operation of the lamp and for the interferometric system used. With the present international specification for the lamp, the reproducibility of this new primary standard of length is better than 1 part in 10^8.

I. Introduction

The vacuum wavelength of the $2p_{10}$–$5d_5$ radiation of krypton 86 emitted by unperturbed atoms at rest relative to the observer is now the primary standard of length (International Committee of Weights and Measures (CIPM) 1960). The International Metre is 1 650 763·73 vacuum wavelengths of this unperturbed radiation and all wavelength standards in spectroscopy will in future be measured in terms of the primary standard.

At present the standard is realized in practice with a hot cathode discharge lamp developed by Engelhard (1952) of the Physikalisch–Technische–Bundesanstalt (PTB). The International Committee of Weights and Measures has recommended specifications for this lamp and its operation (CIPM 1960). The d.c. discharge is viewed from the anode end of a capillary whose internal diameter is 2–4 mm and whose wall thickness is about 1 mm. The lamp is operated at the triple point of nitrogen (63 °K) corresponding to a krypton vapour pressure of about 0·030 mmHg. The tolerances placed on temperature and current density are ±1 degK and ±0·1 A/cm² respectively.

The characteristics of this radiation have been examined by several national laboratories in recent years with particular reference to how the wavenumber emitted by the hot cathode lamp differs from the wavenumber for the unperturbed state of the atoms (CIPM 1958). The various formulae (Baird and Smith 1959; Engelhard 1959; Engelhard and Terrien 1960; Bruce and Hill 1961; Rowley 1961) that have been proposed have established the wavenumber shifts at the prescribed temperature (63 °K) and current density (0·3 A/cm²) to better than 1 part in 10^8, but at other temperatures and current densities the wavenumber variation has been less reliably established.

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The earlier work in this Laboratory (Bruce and Hill 1961) was done with a photoelectric recording Fabry–Perot interferometer using electromechanical scanning. The limiting factor in determining the small wavelength shifts was the precision with which a setting could be made on a fringe maximum. With a view to determining the reproducibility of the primary standard, the limiting precision of the scanning interferometer was investigated both theoretically and experimentally (Hill and Bruce 1962). This work indicated the most suitable values of the various parameters in the interferometer and detection systems in order to obtain optimum precision of setting. Modifications and additions were made to the detection system in order to read fringe settings to 0·000 1 fringe or better. This order of setting sensitivity was found to be possible at a path difference of 150 mm which is a precision of 10^9, where precision is defined as the ratio of the order of interference, n, to the smallest measurable change in order δn.

With these modifications and refinements, the wavenumber shifts in the hot cathode Kr 86 lamp, whose form and operation were in accordance with the CIPM recommendation, were carefully measured. This paper reports the results of these measurements.

II. Experimental Procedure and General Remarks

The equipment used has been fully described in the earlier work (Bruce and Hill 1961). To obtain direct readings of fringe settings to 0·000 1 fringe, using an oscillograph or meter as a null indicator, it was necessary to construct a fourth decade of resistors for the electrical controller. This was not completed in time for these measurements, so a sensitive meter in combination with a phase detection system (Bruce and Hill 1962) was used to read off-settings from a fringe maximum to better than 0·000 1 fringe over a range of a few millifringes. Interferometric tests showed that the meter readings were linear over this range.

The plate separation of the interferometer was 75 mm and the plates were coated with silver films whose reflectance was 83%. The plates were oscillated at a frequency of 76 c/s and the amplitude of scan was about one-eighth of an order which meant that about the upper one-third of the profile was used in determining fringe settings. The photodetector was a tri-alkali cathode photomultiplier with a cathode sensitivity of 140 μA/lumen. An image of the circular interference pattern was projected onto the entrance slit of a high dispersion spectrograph. The central spot of this image was viewed at unit amplification at the exit end of the spectrograph through a circular aperture whose diameter was about 0·4 mm.

The procedure was to set on a fringe maximum to 0·000 1 fringe using the electrical controller decade dials to adjust the plate separation. The meter was then read to give off-setting from the maximum to 0·000 1 fringe or slightly better than this figure. The combined readings of controller and meter gave the fringe setting to this accuracy. Earlier work had shown that the controller could be relied upon to this order of magnitude.

The controller provided a very convenient means of adjusting two sets of decade dials (A and B) to give almost simultaneous settings in two channels corresponding to two different conditions of operation of the lamp (A and B). The determination of wavenumber shift was obtained from three settings A–B–A, each taken at accurate time intervals of 10 sec. Twelve (12) independent determinations were made by each of two observers. At the beginning and end of each set of 12 determinations, calibration checks were made of the meter displacement in millifringes. Less frequent checks were made of the controller readings corresponding to one fringe displacement, as these were found to be extremely constant. This measurement procedure A–B–A satisfactorily eliminated any errors arising from thermal drift. Every point plotted on Figures 1–6 is the mean of 24 independent determinations.

As in the earlier work, one lamp was used and the pressure-current density shifts measured for specific changes in current density at a range of temperatures. The temperature range covered was 58–70 °K and the current density range 0·15–1·20 A/cm². Some effort was made to use two independent lamps as an alternative procedure. The reproducibility of results to 1 in 10^9 in this case was very dependent on the illumination of the interferometer plates by the capillary of the lamp. There is no doubt that the optical alignment of the lamp and illumination system onto the interferometer plates is critical when two lamps are being compared. The narrow capillary form of the lamp aggravates this difficulty.

The Doppler shift $(\Delta\sigma)_D$ was measured by using two lamps, and also by using one lamp with an appropriate arrangement of mirrors and lenses to enable CAO and ACO viewing at rapid intervals. CAO viewing is such that the direction of light propagation may be said to be from cathode to anode to observer and for ACO viewing to be from anode to cathode to observer. This shift is considered to be due to motion of the radiating atoms in the capillary, and is measured by observing the wavenumber shift when the direction of viewing is reversed. Half this shift is taken to be the Doppler shift, being positive for CAO viewing and negative for ACO viewing. Obviously, optical alignment problems are present whether one or two lamps are used. After considerable work the use of two lamps was preferred.

The wavenumber shifts for different current density changes at a range of temperatures were plotted against current density changes. From these curves absolute values for the pressure-current density shift $(\Delta\sigma)_S$ were obtained.

III. Results

A least-squares analysis was made of all results to determine the degree of proportionality between the wavelength shifts and various functions of the current density j and the vapour pressure p of krypton 86. The vapour pressure p was calculated from the temperature T using the Meihuizen relation

$$\log p_{cm} = -607\cdot69/T + 7\cdot2955 - 0\cdot0026675T.$$

The significance of any deduced relationship between wavelength shifts and the parameters p, T, and j depended on the precision with which the shifts could be measured. This precision was calculated theoretically for the conditions under which the interferometer and detecting systems were used (Hill and

Bruce 1962), and determined experimentally by finding the standard deviation of the 24 independent determinations of the wavelength shift for the various values of p, T, and j used. The values for the theoretical and experimental precision of determination are expressed as indicated in Table 1. The variation

TABLE 1

PRECISION OF DETERMINATION $n/\delta n$

Temperature (°K)	Doppler Shift	Pressure-Current Density Shift	
		Experimental	Theoretical
70	$0\cdot3\times10^9$	$0\cdot2\times10^9$	$0\cdot4\times10^9$
68		0·3	
66	0·5	0·6	0·7
64		0·9	
63	1·0	1·0	1·0
62		1·0	
60		1·0	
58	0·5	1·0	0·7

in the theoretical precision with temperature is due to the different power levels of emission from the lamp at different temperatures. These had been determined in earlier experiments (Bruce and Hill 1961).

Tests for linearity between wavelength shift $(\Delta\delta)_S$ and various functions of p, T, and j also indicated that it was not very profitable to try to fit the experimental results to any definite relation over an extended temperature range.

TABLE 2

WAVENUMBER SHIFT $(\Delta\sigma)_S$, PRESSURE p, AND CURRENT DENSITY j RELATIONSHIPS

Fitting of lines
$$(\Delta\sigma)_S=m_1j+c_1, \quad (1)$$
$$(\Delta\sigma)_S=m_2j^{\frac{1}{2}}+c_2, \quad (2)$$
$$(\Delta\sigma)_S=m_3j^{\frac{2}{3}}+c_3, \quad (3)$$
$$(\Delta\sigma)_S=m_4(pj)^{\frac{2}{3}}+c_4. \quad (4)$$

Temperature (°K)	Standard Deviation for $(\Delta\sigma)_S$ in m^{-1} using Equations				Difference (2)−(3)
	(1)	(2)	(3)	(4)	
69·9	$4\cdot0\times10^{-3}$	$2\cdot6\times10^{-3}$	$1\cdot5\times10^{-3}$	$4\cdot9\times10^{-3}$	$+1\cdot1\times10^{-3}$
68·4	3·6	2·0	1·7	4·8	+0·3
65·6	3·1	0·9	1·4	1·5	−0·6
63·8	3·8	1·8	2·4	2·5	−0·6
62·9	2·0	0·3	0·8	1·0	−0·5
61·9	1·9	0·5	1·0	1·0	−0·5
60·0	0·5	0·2	0·3	0·6	−0·1
57·6	0·4	0·1	0·2	0·8	−0·1

$1\cdot6\times10^{-3}\,m^{-1}$ is 1 part in 10^9

Table 2 shows some of the results of the least-squares analysis for the pressure-current density effect. It is clear that it is not possible to distinguish between $j^{\frac{1}{2}}$ and $j^{\frac{2}{3}}$ relationships to 1 part in 10^9, the order of accuracy of the measurements.

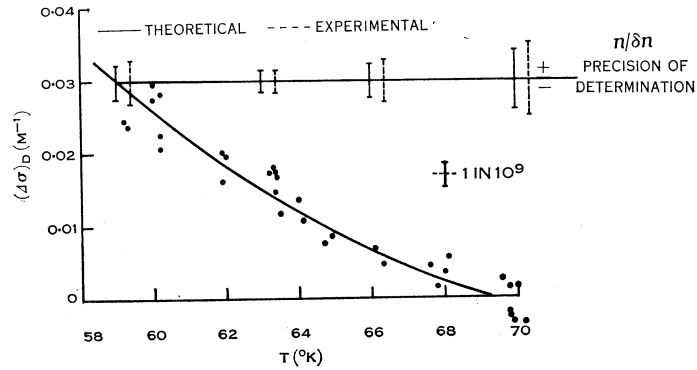

Fig. 1.—Wavenumber shift $(\Delta\sigma)_D$ due to Doppler effect as a function of temperature T. Shift is positive for anode nearest observer (CAO direction) and negative for cathode nearest observer (ACO direction).

The constants m and c in the straight-line relation $y=mx+c$ vary with the temperature T and show fairly close correlation with the light power emitted at the different temperatures.

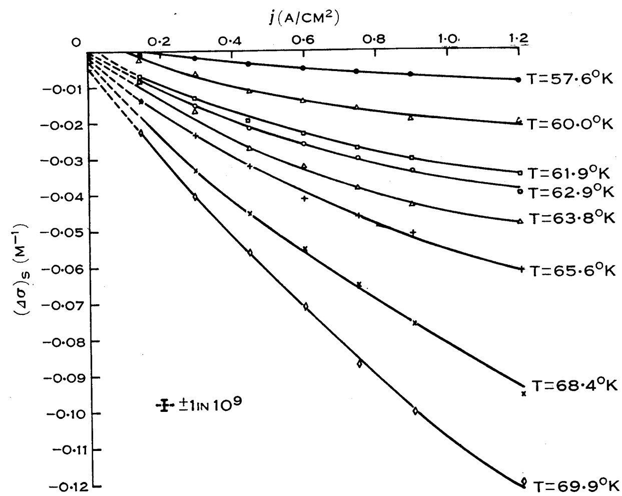

Fig. 2.—Wavenumber shift $(\Delta\sigma)_S$ due to pressure-current density effect as a function of current density j at different temperatures. CAO direction.

The results for the wavenumber shifts are given in Figures 1–6. Figure 1 shows the Doppler shift $(\Delta\sigma)_D$ and its variation with temperature when the lamp is viewed end on. Every point plotted is the mean of at least 24 independent determinations and the experimental precisions indicated are the standard deviations in that number of determinations. Many more determinations were made of the Doppler shift than the pressure-current density shift since the scatter of results was greater. This is attributed to the fact that the observation of

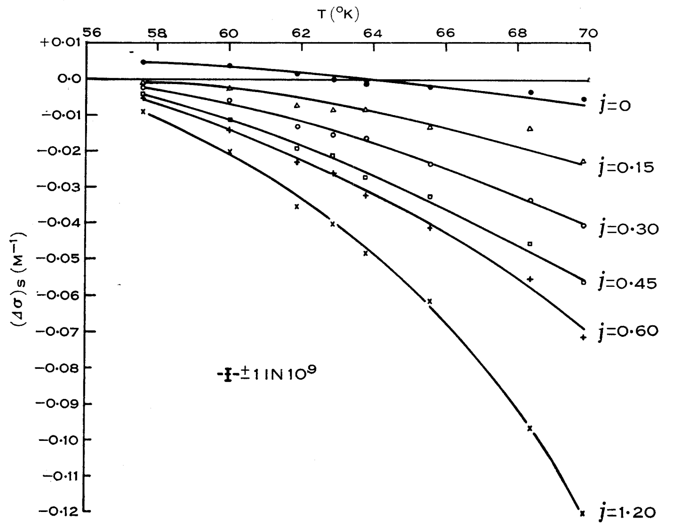

Fig. 3.—Wavenumber shift $(\Delta\sigma)_S$ as a function of temperature T at different current densities. CAO direction.

Doppler shifts involved two independent optical alignments, and the critical nature of the adjustment in such a case has already been discussed. However, it is clear that the curve in Figure 1 gives the Doppler shift to a few parts in 10^9. Variation of current density had a negligible effect on the shift.

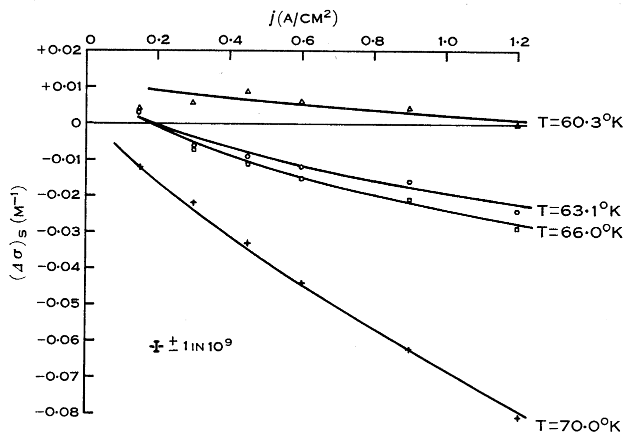

Fig. 4.—Wavenumber shift $(\Delta\sigma)_S$ as a function of current density j at different temperatures. ACO direction.

Figure 2 shows the wavenumber shift $(\Delta\sigma)_S$ due to pressure and current density as a function of current density for different temperatures in the range 58–70 °K.

Figure 3 shows $(\Delta\sigma)_S$ as a function of temperature at different current densities, and is directly derivable from Figure 2. The lamp was viewed CAO for both these results. Similar results for ACO viewing are given in Figure 4. For comparison Figure 5 shows the values of $(\Delta\sigma)_S$ at different current densities

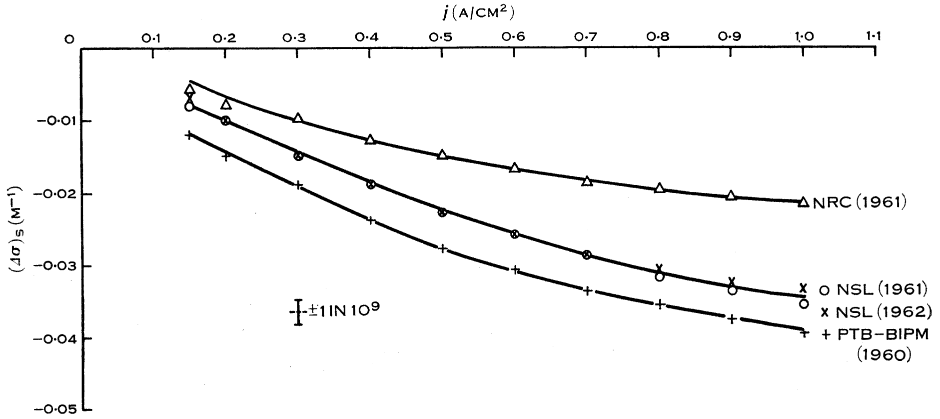

Fig. 5.—Wavenumber shift $(\Delta\sigma)_S$ as a function of current density j for $T=63$ °K. CAO direction.

for the temperature of 63 °K obtained by three different laboratories with lamps conforming to the specification recommended by the CIPM. Figure 6 gives a similar comparison of $(\Delta\sigma)_S - T$ values for $j=0\cdot3$ A/cm^2. These last two figures demonstrate that while individual laboratories obtain results reproducible to 1 part in 10^9 the agreement in absolute values for $(\Delta\sigma)_S$ is of the order of 2–3 parts

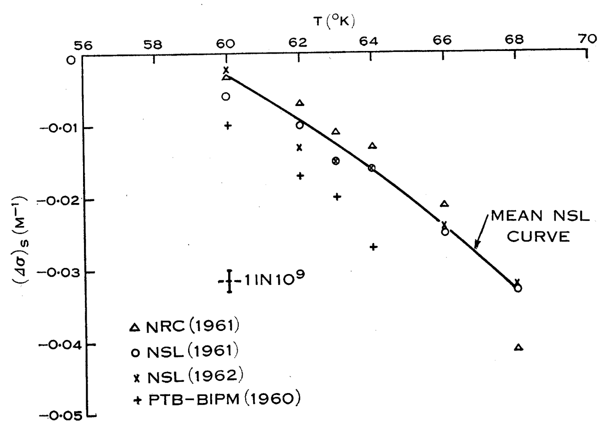

Fig. 6.—Wavenumber shift $(\Delta\sigma)_S$ as a function of temperature T for $j=0\cdot3$ A/cm^2. CAO direction.

in 10^9. This fact is not surprising if one considers that a variation in temperature of 1 degK or in current density of $0\cdot1$ A/cm^2 will introduce wavelength shift variations of 2–3 parts in 10^9. Agreement of a higher order than this could only be expected if the same lamp was used by all the laboratories and if the methods of determining the temperature and current density were specified in much greater detail than in the present recommendation. The interferometric method used is also important since the effects of any asymmetry in the line will depend on the detection methods used. For example, the amplitude of scan will determine how much of the line profile is used in pointing on a fringe and, in general, Michelson systems will differ from Fabry–Perot systems in the amount of profile used in the detection of a fringe maximum.

Results for $(\Delta\sigma)_S$ and $(\Delta\sigma)_D$ agree closely with those from PTB–BIPM (1960), NPL (1961), and NRC (1961), the mean of all determinations differing from the present NSL results by not more than about 1 part in 10^9 of the wavenumber.

Table 3

COMPARISON OF RESULTS

Total wavenumber shift $[(\Delta\sigma)_S+(\Delta\sigma)_D]$ in m^{-1}, at $j=0\cdot3$ A/cm² CAO viewing

Temp. (°K)	Precise Results						Earlier Results		
	PTB-BIPM 1960	NSL 1961	NPL 1961	NRC 1961	NSL 1962	PTB 1958	NRC 1959	PTB 1959	Mean
68		−0·030			−0·031				
64		−0·005		−0·001	−0·004				
63	−0·001	−0·001	−0·007	+0·002	0·000	+0·006	+0·001	−0·003	0·000
62		+0·004		+0·013	+0·008				
60		+0·021		+0·020	+0·023				

IV. Discussion

The results indicate that the wavenumbers in a given lamp can be determined with an accuracy of about 1 part in 10^9 using a photoelectric scanning Fabry–Perot interferometer under vacuum conditions. The interferometer and detection system could, with suitable choice of the best values of the various parameters, and the use of high-reflecting dielectric-coated plates, achieve a precision of determination of the wavenumber shifts of 5×10^9. However, the evidence indicates that other sources of error such as small variations in flatness of the interferometer plates and variations in the operating conditions of the lamp that could still come within the recommended specification, would introduce errors that would make such a precision not meaningful in practice.

Baird and Smith (personal communication) referred to the uncertainty in fringe setting and to a very small but detectable asymmetry in the line of the order of 1 part in 2×10^8. Bayer-Helms (1959) has also reported some asymmetric line profiles at various temperatures and current densities. Many profiles have been recorded at this Laboratory in the past few years, under different operating conditions. Figure 7 shows a typical result for the profile for $T=63$ °K and $j=0\cdot3$ A/cm². When corrections were made for instrumental asymmetry the line asymmetry was never found to be more than $0\cdot01$ m^{-1} for the whole profile. Reliance upon the profile record to better than this figure (1 in 10^8) is not really justified but the asymmetry indicates agreement in order of magnitude with Baird and Smith's values. However, in the scanning Fabry–Perot interferometer only the upper third of the profile was used and asymmetry over this part is negligible. At a temperature of 75 °K with $j = 0 \cdot 3$ A/cm^2, the asymmetry of the line over the upper three-quarters of the profile is about $0 \cdot 1$ m^{-1}, while at 60 °K it is negligible. At a temperature of 63 °K the current density needs to be greater than $0 \cdot 5$ A/cm^2 before the asymmetry exceeds $0 \cdot 01$ m^{-1}.

The difficulties that are associated with optical alignment when comparing two lamps are such that a precision of 1 part in 10^9 cannot be expected. This trouble is aggravated by the small size of the capillary source of light, and will

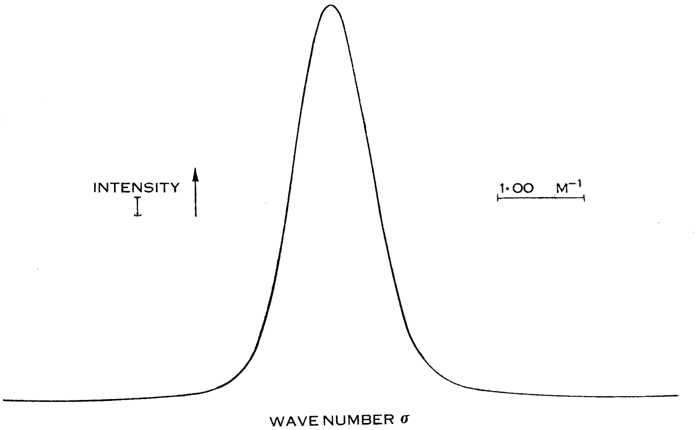

Fig. 7.—Spectral line profile of $2p_{10}$–$5d_5$ radiation of krypton 86. Spectral range $10 \cdot 0$ m^{-1} ($0 \cdot 037$ Å).

also arise when Hg 198 and Cd 114 lamps are being compared with the Kr 86 6056 Å line. Improvement of the quality of Fabry–Perot plates to a degree that is greater than achieved at present (flat to $\lambda/50$–$\lambda/100$) would probably reduce the errors arising from optical illumination of the plates.

Thus, while it is true to state that the wavenumber emitted by any given lamp can be established to 1 part in 10^9 for very specific conditions of operation, it is not possible to obtain this reproducibility for different lamps whose geometry and operating conditions are still within the limits specified by the CIPM. To do this, a more stringent specification is needed for the form and operation of the lamp and the type of interferometer and detection system used. The reproducibility of this new primary standard is certainly better than 1 part in 10^8, and modern interferometric techniques can examine this reproducibility with a precision at least 10 times greater than this value.

V. Acknowledgments

Mr. J. L. Goldberg designed and constructed the phase detection unit. Mr. D. L. Cotterell assisted with the operation of the krypton 86 lamp and the associated measurements of temperature.

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