DISTRIBUTION OF AURORAS IN THE SOUTHERN HEMISPHERE

II. NIGHTLY PROBABILITY OF OVERHEAD AURORA

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Summary

The probability P of overhead occurrence of aurora during a night is estimated for 22 southern hemisphere stations during the I.G.Y. The distribution of probability of occurrence is represented in terms of three measures of colatitude: θ_1 is defined in terms of the eccentric dipole representation of the geomagnetic field; θ_2 is defined in terms of projections from circles in the equatorial plane, along the geomagnetic lines of force, onto the Earth; θ_3 is defined in terms of the integral invariant $\int_S^N (1-F/F_m)^{\frac{1}{2}}dl$ of charged particle motion in the geomagnetic field.

The data are well represented in terms of θ_1, θ_2, or θ_3. θ_1 is useful on the grounds of simplicity. θ_2 is relevant to theories such as that of Alfvén (1940) but comparison of data with theory is not feasible. θ_3 is relevant to scattering from geomagnetically trapped radiation.

Simple models of scattering from trapped radiation in a quiet, longitude independent atmosphere are not consistent with the data. More complex models are not amenable to comparison. Difficulties in accounting for polar cap auroras in this way are noted.

I. Introduction

In an earlier investigation (Bond and Jacka 1960; referred to as Part I of this series) the authors estimated the position of the southern isoaurore of maximum frequency. This estimate was based on data on frequency of occurrence, on the hours and quarter hours, as a function of elevation and on orientation of auroras observed at a small number of stations.

The present investigation is based on observations of overhead appearance, at any time during the night, of auroras at 22 stations and aims to describe the distribution of probability of occurrence of auroras over the whole southern hemisphere. A possible slight inconsistency with the earlier results is noted.

The distribution of probability of occurrence of auroras is represented in terms of three different measures of colatitude and their possible physical significance discussed.

II. The Data

The data used refer to the period of the International Geophysical Year (July 1957–December 1958). They were extracted from the records of visual observations and all-sky photographs of auroras from all southern hemisphere stations from which they were available. At most stations all-sky camera

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operation was limited only by instrument failure. At some stations visual observations were virtually continuous; at others they were limited to periods of non-operation of the all-sky camera.

From these data, for each station, throughout the observing period (defined as between evening and morning nautical twilight) the following basic data were listed for each hour, 0–1, . . ., 23–24h U.T.: The sky was (i) "cloudy", or (ii) "clear", and if "clear" (iii) "overhead" aurora was present at some time during the hour, or (iv) absent throughout the hour. By "cloudy" is meant 6/8 or more of the sky was cloud covered at some time during the hour; by "clear" is meant cloud cover was 5/8 or less throughout the hour. "Overhead" aurora is defined as aurora with lower borders within 60° zenith distance (i.e. within $1\frac{1}{2}°$ great circle distance of the station, assuming lower borders to lie at 105 km height).

The probabilities of occurrence of "overhead" aurora during a night were then estimated by three different methods. The estimates are listed in Table 1.

(*a*) *Estimate* p′

This estimate was based simply on wholly "clear" nights. Denoting by N'_0 the number of nights during which all hours were clear and by N'_a the number of these nights during which overhead aurora was present, $p' = N'_a/N'_0$. Because of extreme cloudiness the amount of data available for this estimate was very small at some stations.

(*b*) *Estimate* p″

Denoting by N''_0 the number of nights during which at least one "clear" hour occurred and by N''_a the number of these nights during which overhead aurora was present during a clear hour, $p'' = N''_a/N''_0$. This estimate makes more complete use of the information contained in the basic data but it may be biased, and differently at different stations, because of possible systematic variations in cloud cover during the night.

(*c*) *Estimate* P

The observing periods were divided into 3-hr intervals (or parts thereof), 0–3, . . ., 21–24h U.T. In each interval during which a clear hour occurred the presence or absence of overhead aurora was noted from the basic data. These clear hour data were then divided into classes according to the value of the planetary geomagnetic disturbance index Kp and 3-hr time interval. The relative frequency of occurrence of auroras $f(Kp, t)$ was then determined for each Kp-time class. It was now assumed that the frequencies $f(Kp, t)$ were representative of the whole population of 3-hr intervals.

For each night during which at least one clear hour occurred the presence or absence of overhead aurora during any one of these hours was noted from the basic data. These nights were then grouped according to the dominant (maximum $f(Kp, t)$) Kp-time class represented by a clear hour during the night and the relative frequency of occurrence of auroras $F(Kp, t)$ determined for each group of nights. The remaining cloudy nights and the cloudy periods of those (few) nights during which clear hours occurred without overhead aurora were now similarly grouped according to the dominant Kp-time class represented and the frequencies $F(Kp, t)$ ascribed to each group of nights.

Denoting by $N(Kp, t)$ the total number (clear and cloudy) of nights in a group characterized by the same dominant Kp-time class, the estimate

$$P = \Sigma N(Kp, t) \,.\, F(Kp, t) / \Sigma N(Kp, t)$$

of the probability of occurrence of overhead aurora during a night was then evaluated.

The estimate P makes full use of the information contained in the list of basic data ; it is an unbiased estimate and the values obtained from the several stations may be meaningfully compared.

TABLE 1

NIGHTLY PROBABILITY OF OVERHEAD AURORA (%)

Station	Symbol	p'	p''	P
Awarua	Aw	6·9	6·7	9·5
Byrd..	By	79·3	81·9	94·1
Camden	Ca	0·5	0·2	0·2
Campbell I. ..	CI	40·0	23·4	25·7
Cape Hallett ..	Ht	80·8	70·1	80·1
Davis	Da	81·1	78·3	87·5
Dumont d'Urville ..	DU	70·4	68·8	67·5
Ellsworth	El	52·2	44·0	54·9
Halley B.	HB	38·2	21·5	28·1
Hobart	Ho	6·9	4·2	5·1
Kerguelen	Kg	8·5	14·1	14·6
Little America 5*	—	76·0	65·3	79·5
Macquarie I. ..	MI	87·5	86·8	86·0
Mawson	Mw	92·8	92·3	97·4
Melbourne	Me	1·05	0·71	0·75
Roi Baudouin ..	RB	63·3	49·2	60·1
Scott Base ..	SB	66·3	49·0	63·4
Shackleton ..	Sh	31·8	30·2	55·2
South Ice* ..	—	33·3	32·3	72·7
South Pole	SP	97·7	85·1	89·7
Syowa	Sy	90·9	81·3	94·5
Wilkes	Wi	62·0	49·3	67·8

* Data not reliable—see text.

The three estimates p', p'', and P are on the whole reasonably consistent. For the reasons stated, only the estimate P will be considered in the further discussion.

For particular stations certain peculiarities should be noted.

Observations at South Ice were confined, almost wholly, to those taken at intervals of 6 hr ; presence of aurora at other times is likely to have been missed. The estimate P suggests that South Ice is not far from the auroral zone, but, for the reason given this station was excluded from further analysis.

From the geographic location of Little America 5 it is expected that this station is to the south of the auroral zone, and that the diurnal variation curve would display two maxima. During 1958, with a few exceptions visual observa-

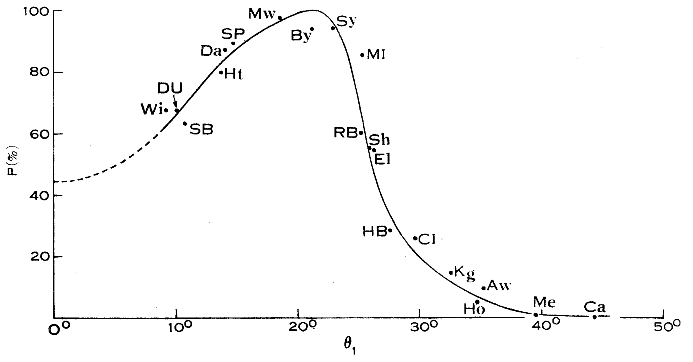

Fig. 1.—Probability of overhead aurora during a night *versus* colatitude θ_1.

tions covered only the intervals 1700–0200 or 1800–0300 h local time. The duration of observations was likely to omit either the evening maximum, or

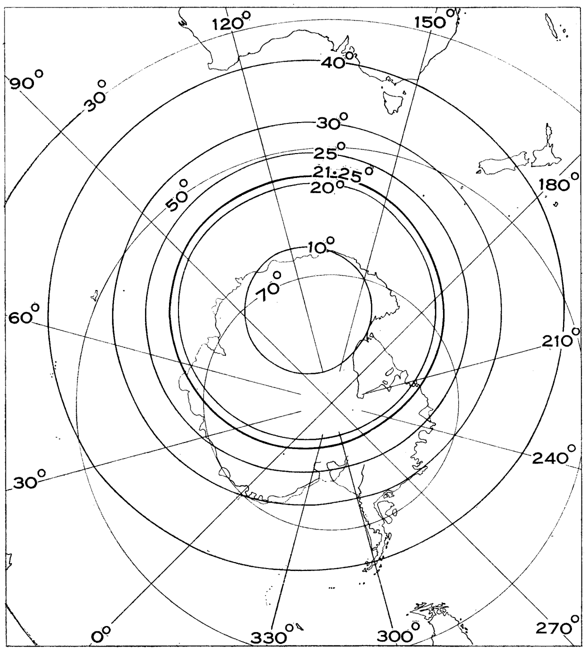

Fig. 2.—Isolines of colatitude θ_1.

the morning maximum, or both. The value obtained for nightly frequency at this station is therefore probably too low and this station was omitted from the subsequent analysis.

Data from Syowa were available for the years 1957 and 1959. These data were used to determine the frequency $F(Kp, t)$ but actual data for 1957 and Kp-time values for 1958 were used in estimating P.

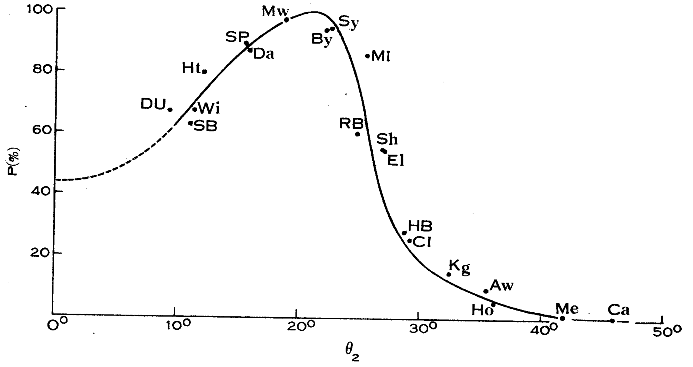

Fig. 3.—Probability of overhead aurora during a night *versus* colatitude θ_2.

At the time of writing the only data available from the region of the centre of the auroral polar cap are those quoted by Fel'dstein (1960) for July 1959

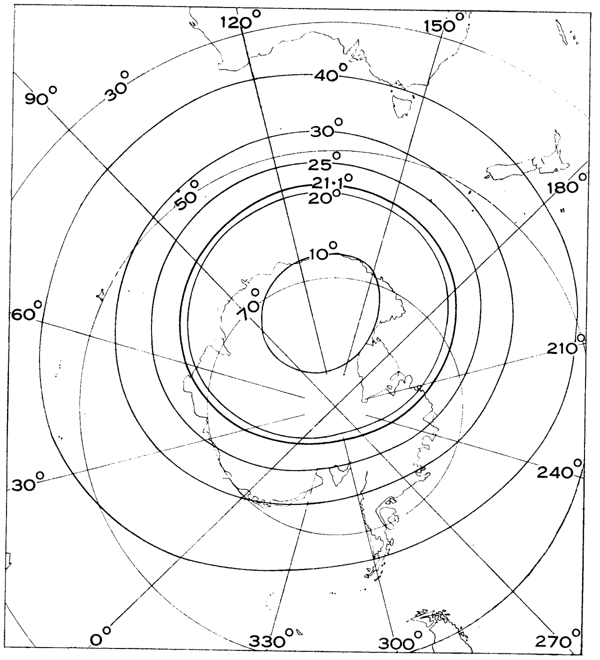

Fig. 4.—Isolines of colatitude θ_2.

from Vostok. These data indicate a nightly frequency of overhead aurora of roughly 44%. This value has been used as a guide in drawing the dashed portions of Figures 1, 3, and 5.

III. Representation of the Data

Gartlein and Sprague (1960) suggest that the southern hemisphere isoaurores parallel the isoclines. A plot of P against magnetic inclination for the several stations gives no indication of any simple relationship. The annotated map given by Gartlein and Sprague (loc. cit.) shows Mawson and Melbourne on the same isoaurore. Reference to Table 1 indicates that this is not the case. Further, a plot of P against inclination at a height of 100 km shows no sign of any simple relationship.

A plot of P against the geomagnetic latitude for each station again shows no sign of any simple relationship.

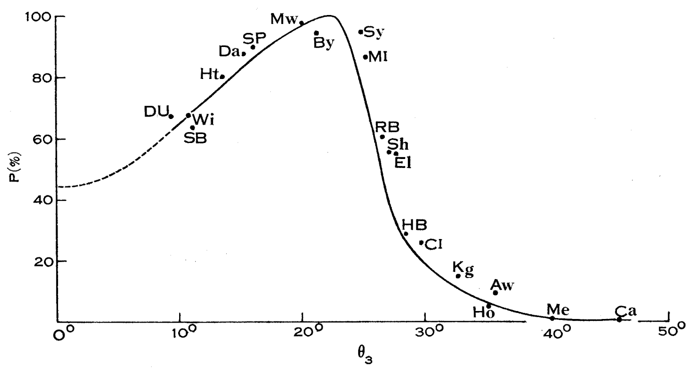

Fig. 5.—Probability of overhead aurora during a night *versus* colatitude θ_3.

(a) *Colatitude* θ_1

The location of the poles of the eccentric dipole field, and the position of the dipole were taken from Parkinson and Cleary (1958). With the dipole as apex and the line joining the poles as axis, the equation for the cone, having as semi-angle the required colatitude was solved with the equation for the sphere representing the Earth to give a curve delineating eccentric dipole geomagnetic colatitude denoted by θ_1.

A plot of P against θ_1 indicates a simple relationship, as demonstrated in Figure 1.

In drawing the smooth curve associated with the plotted points, account was taken of the following factors:

(i) The term " overhead " applies to aurora within $1\frac{1}{2}°$ great circle distance from the station.

(ii) The probability of occurrence of aurora decreases rapidly as one proceeds from the auroral zone towards the equator. In the region of this rapid decrease the estimate P is in fact representative of the actual overhead frequency at a point nearly $1\frac{1}{2}°$ zonewards of the station. This effect is expected to a lesser extent on the poleward side of the auroral zone.

For these reasons the smooth curve was drawn somewhat inside the plotted points to represent truly the probability of overhead occurrence of auroras.

The maximum probability isoaurore on this representation is read from the curve in Figure 1 as situated at $\theta_1=21\cdot25°$. This value is shown on the map in Figure 2, together with other representative values.

(*b*) *Colatitude* θ_2

A number of authors (e.g. Alfvén 1940) suggest that auroras may be excited by precipitation from solar corpuscular streams, along the geomagnetic lines of force into the atmosphere. On the basis of these theories one would expect the intensity or frequency of occurrence of the precipitation, averaged over long periods of time, to be a function only of magnetic field intensity F at which the interaction of the field with the corpuscular stream takes place. The isoaurores may then be approximated by projection from lines of constant F in the equatorial plane, along the geomagnetic lines of force, into the atmosphere. Further, these lines of constant F may be approximated by circles in the equatorial plane of the centred dipole representation of the field.

Such projections, from circles of radius R_E, have been computed by Hultqvist (1958). Examples of curves given by this projection are shown on the map in Figure 4. The colatitude measure θ_2 is the colatitude of the circle defined by similar projection, from radius R_E, along the lines of force of a centred dipole.

When auroral probabilities P are plotted against θ_2 a smooth curve again results (Fig. 3). In drawing the curve, account was taken of the same factors as for the curve of Figure 1. The value of θ_2 corresponding to the maximum probability isoaurore is $\theta_2=21\cdot1°$.

(*c*) *Colatitude* θ_3

Vestine and Sibley (1960) point out that geomagnetically trapped charged particles drift in longitude in such a manner as to preserve the invariance of the integral

$$I=\int_S^N (1-F/F_m)^{\frac{1}{2}}\mathrm{d}l,$$

where F denotes magnetic field intensity, F_m the field intensity at the particle's " mirror point ", and the integral is evaluated along the line of force l between the southern and northern hemisphere mirror points. These authors present maps showing contours of the invariant I for several stipulated values of F_m. They also adduce evidence indicating that the auroral isochasms parallel these contours of I or, perhaps more accurately, the projections of these contours, along the geomagnetic lines of force, onto the 100 km level. This is to be expected if auroras are excited by precipitation from the trapped radiation by a mechanism, the intensity or frequency of occurrence of which is independent of longitude.

In a dipole field the invariant I is given by

$$I=2\int_{\vartheta_m}^{\frac{1}{2}\pi}\left[1-\frac{\sqrt{(1+3\cos^2\vartheta)}}{\sqrt{(1+3\cos^2\vartheta_m)}}\cdot\frac{\sin^6\vartheta_m}{\sin^6\vartheta}\right]^{\frac{1}{2}}\cdot\left[\frac{a\sqrt{(1+3\cos^2\vartheta_m)}}{F_m}\right]^{\frac{1}{3}}\cdot\frac{\sin\vartheta}{\sin^2\vartheta_m}\sqrt{(1+3\cos^2\vartheta)}\,\mathrm{d}\vartheta,$$

where ϑ denotes colatitude and a the dipole moment.

Using a graphical-numerical method and taking $a=8\cdot 1\times 10^{25}$ gauss cm^3, $F_m=0\cdot 45$ gauss, I *versus* ϑ was evaluated.

Vestine and Sibley's I contours for $F_m=0\cdot 45$ were then labelled with values of the " equivalent dipole colatitude " ϑ. These curves were now projected (approximately) along the geomagnetic lines of force to the 100 km level. The new curves were labelled with the numerical value of ϑ, but now denoted θ_3. The projections onto the 100 km level were estimated to the first approximation from the actual surface direction of the field and assuming the height derivative of F as for a dipole field.

In Figure 5 the auroral probabilities P are plotted against θ_3 for each station. The smooth curve was drawn with the same considerations as for Figure 1. In Figure 6 isolines of θ_3 are shown. The value $\theta_3=22\cdot 5°$ corresponds to the maximum of the curve of Figure 5. It will be realized that a different choice of F_m in the range of likely interest makes very little difference to the derived θ_3 isolines.

IV. Discussion

(*a*) *Comparison with Earlier Data*

Whether the auroral probability P be represented in terms of θ_1, θ_2, or θ_3 it is apparent that the maximum frequency isoaurore closely parallels that estimated in Part I of this series (Bond and Jacka 1960) in the regions of the stations considered in that paper. In the region of Macquarie Island the new estimates lie 2 to 3 degrees further south, while in the region of Mawson the new estimates lie about 2 degrees further north.

From the frequency *versus* latitude curves considered in Part I it was possible to estimate the modal latitudes to an accuracy of about ± 1 degree. But further, the reliability of the earlier estimate may be suspected on the grounds of uncertain effects of perspective and of obscuration by cloud in viewing the aurora at relatively great zenith distances. The maxima of the curves of Figures 1, 3, and 5 may also be located with an accuracy of about ± 1 degree. This leaves a possible, slight inconsistency.

However we have no grounds *a priori* for assuming that the maximum probability isoaurore determined from quarter-hourly observations coincides with that determined from nightly observations.

(*b*) *Goodness of Fit of Data*

The smooth curves of Figures 1, 3, and 5 each represent the data rather well considering their relative crudeness. Although it is not feasible to place strict confidence limits on the estimates of P, examination of the tabulations leading to these estimates suggests that they are reliable to within a few per cent. The errors are expected to be randomly distributed. Further the estimates of P refer to the period 1957–58 while the computations of θ_1 and θ_3 are based on geomagnetic data of epoch 1955 and those for θ_2 of epoch 1945. Even for these epochs the uncertainties in our knowledge of the field may give rise to errors in θ_i approaching one degree. Combined, these effects produce systematic errors in θ_i which might amount to rather more than 1 degree.

The factors considered in drawing the smooth curves of Figures 1, 3, and 5 were only partly objective and partly aesthetic. The curves do not represent any particular physical model. It is therefore not meaningful to make strict comparisons of goodness of fit of the data. (Were there very great differences in the goodness of fit one might ascribe physical significance but this is obviously not so in the present case.)

The authors have considered an alternative approach, namely the fitting of a " simple " analytical expression to each of the three sets of data and then comparing goodness of fit. A basic objection to this approach is that without knowing the nature of the physical processes involved one cannot know that the " simple " analytic expression is equally appropriate in each of the three cases.

However, Dr. M. Gadsden (personal communication) has examined this approach. Firstly, he used a cumulative normal transform to give x, where

$$P=\int_{-\infty}^{x} \frac{1}{\sqrt{(2\pi)}} e^{-\frac{1}{2}t^2} dt.$$

He divided each set of data into two groups: those for stations inside the maximum probability isoaurore and those outside. He then fitted " simple " expressions of the form

$$E(\theta_i)=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4,$$

to each group of data. Here $E(\theta_i)$ denotes the expected or predicted value of θ_1, θ_2, or θ_3. The constants were determined to minimize $\Sigma[\theta_i-E(\theta_i)]^2$. (This differs further from the authors' approach in that the curves of Figures 1, 3, and 5 were drawn " within " the points to represent the probability of strictly overhead auroras as functions of θ_i.)

Gadsden finds that, in terms of $x(P)$, θ_2 is predicted better than θ_1 and θ_1 better than θ_3, but only the statement " θ_2 is predicted better than θ_3 " is significant at the 10% level.

The maxima of $x(P)$ occur at $\theta_1=19°$, $\theta_2=19°$, and $\theta_3=20\cdot5°$ while the maxima of the curves of Figures 1, 3, and 5 occur at $\theta_1=21\cdot25°$, $\theta_2=21\cdot1°$, and $\theta_3=22\cdot5°$. The latter values are admittedly somewhat arbitrary. But so also are the former as these are strongly dependent on the form of the analytic expressions and, especially, the manner of division of the data into two groups for curve fitting.

Gadsden further divided the data into two groups according to longitude and found that for each θ_i there is no significant difference between the two groups as regards goodness of fit of the " simple " expression.

These findings are meaningful and valuable as commentaries on the " simple " analytic relations between θ_1, θ_2, θ_3, and $x(P)$. But for the reason stated above the authors do not consider that they can be taken to infer that θ_2 has greater *physical* significance than has θ_3.

Finally it may be remarked that, for purposes of description of the distribution of auroras, the curve of Figure 1 has the advantage of simplicity of concept.

(c) *Physical Models*

It was pointed out in Section III (c) that if auroras are excited by precipitation from geomagnetically trapped radiation and if the precipitation mechanism is independent of longitude, one would expect the isoaurores to parallel the isolines of θ_3. A possible precipitation mechanism is coulomb scattering in the atmosphere but brief consideration indicates that this mechanism is not independent of longitude and does not offer a tenable explanation of the observations.

Coulomb scattering would occur mainly in the region of the mirror points of the trapped particles which lie on surfaces of constant magnetic field intensity above the θ_3 isolines. The intensity of precipitation or, considering the great

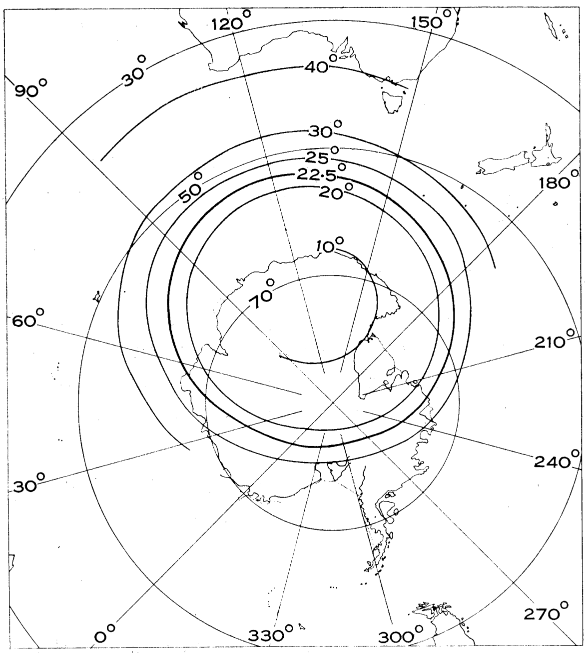

Fig. 6.—Isolines of colatitude θ_3.

variability of flux within the trapped radiation, the frequency of occurrence of intense precipitation, would depend on atmospheric density in the region of the mirror points. Now the height of the $F=0\cdot45$ gauss surface above, say, the $\theta_3=22\cdot5°$ isoline varies with longitude by more than 500 km (Vestine and Sibley loc. cit.). The atmospheric density then varies over several orders of magnitude. This would give rise to very large variations of probability of occurrence of auroras along the θ_3 isolines. The small dispersion of points about the curve of Figure 5 clearly indicates that this is not so.

This process has been examined in greater detail by Loughnan (1961), who considered also the variation with latitude of the flux of trapped radiation. His results are here replotted in Figure 7 in which φ measures the expected intensity of particle precipitation. The auroral probabilities P plotted against φ for those stations covered by Loughnan's calculations do not indicate any simple relationship.

Cole (1962*a*, 1962*b*) has suggested that intense precipitation may take place above those ionospheric electric currents which produce geomagnetic perturbations. The electric currents cause joule heating and considerable expansion of the atmosphere up the geomagnetic lines of force. This treatment gives primacy to the electric currents. Without detailed knowledge of their distribution and the distribution of ionospheric conductivity it is not possible to formulate a model with which to compare the auroral observations.

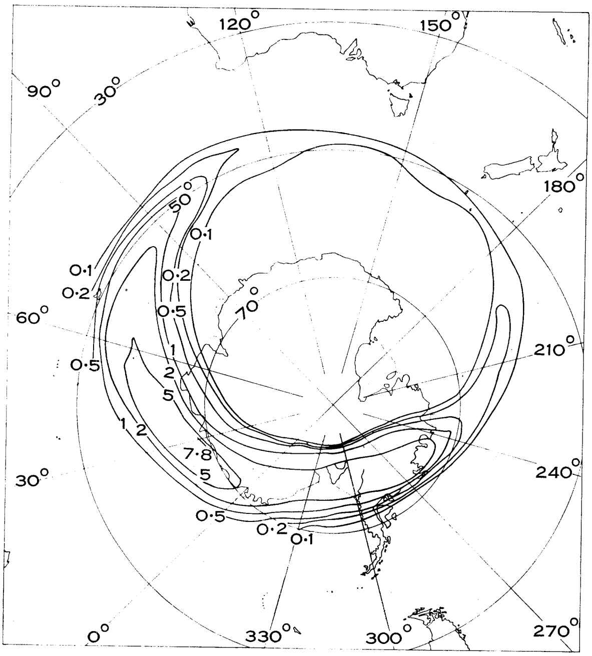

Fig. 7.—Isolines of φ denoting expected intensity of precipitation due to atmospheric scattering from the geomagnetically trapped radiation—replotted from data of Loughnan (1961).

As noted earlier (Cole and Jacka 1961), basic difficulties are met in accounting for the occurrence of auroras over the polar cap in terms of any model involving precipitation from trapped radiation. Such radiation does not appear to exist on the geomagnetic field lines connecting to the polar caps. One might however speculate on the possibility of more or less continuous feeding into quasi-periodic orbits of particles from the solar or interplanetary plasma. These particles may be precipitated by scattering in the atmosphere stimulated by a process such as that suggested by Cole (loc. cit.).

V. Conclusions

The distribution of probability P of occurrence of auroras during a night can be well represented in terms of θ_1, θ_2, or θ_3. For descriptive purposes θ_1

has the advantage of simplicity, but no special physical significance attaches to this representation.

Representation of P in terms of θ_2 is relevant to theories such as that of Alfvén (1940), but these theories are not developed in sufficient detail to permit of comparison with the present data.

Representation of P in terms of θ_3 is relevant to precipitation from geomagnetically trapped radiation by longitude-independent processes. Because of the asymmetry of the geomagnetic field scattering in a longitude-independent quiet atmosphere is not such a process. Analysis of the process of scattering in a disturbed atmosphere such as that suggested by Cole (1962*a*, 1962*b*) is not at present tractable.

The basic data, classified according to form of the aurora, are now being analysed on an hourly basis. It is expected that the results of this analysis may be more amenable to comparison with prediction from theory.

VI. Acknowledgments

We are grateful to our various international colleagues who provided the data used in this investigation. Discussions with Mr. K. D. Cole of this Division and with Dr. M. Gadsden of Dominion Physical Laboratory, New Zealand, have provided valuable stimulus; the latter's contribution to the analysis of the data is appreciated. It would be ungallant not to acknowledge also the considerable and valuable support of our computing and secretarial assistants.

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