SEA-LEVEL MOMENTUM SPECTRA OF MUONS

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Summary

The sea-level muon momentum spectrum in the range 0·7–30 GeV/c has been measured at Nottingham, England, at zenith angles of 0°, 30° W., and 45 °W., based on 1052, 1010, and 1033 particles respectively. Agreement is found at 0° with the results of Owen and Wilson (1955) and disagreement with those of Caro, Parry, and Rathgeber (1951). The same form of disagreement occurs with the results of Moroney and Parry (1954) at 30° obtained using the same instrument as Caro, Parry, and Rathgeber. Within the statistical accuracy of the experiment satisfactory agreement is found with the theories based on either single level or extended production of the muons.

I. INTRODUCTION

The primary component of the cosmic radiation, 85% of which consists of protons, interacts mainly in the first 100 g/cm^2 of the atmosphere to produce π-mesons, recoil nuclei, and a small fraction of heavy mesons and hyperons. The π-mesons are removed by the competing processes of muon decay and nuclear interaction which may produce further π-mesons. The heavy mesons and hyperons are removed by similar competing processes, the process of K-μ-decay contributing to the muon spectrum at high energies. The π- and K-mesons travelling in the more tenuous regions of the atmosphere have a higher probability of decay than those of the same energy moving in denser regions; which may offset the overall relativistic effect of increasing probability of interaction with energy. As muons have a relatively long life time of $2{\cdot}25\times10^{-6}$ sec and a small cross section for nuclear interaction compared with π- and K-mesons, they have a high probability of survival to sea-level.

Measurements of the momentum spectrum of muons at sea-level can provide information on the production spectrum of the parent particles. Measurements made at increasing zenith angles contribute information concerning π- and K-mesons of progressively higher energies (Stern 1960).

The present paper is concerned with muon spectrum measurements which were carried out at Nottingham, England, at zenith angles of 0°, 30° W., and 45° W., at sea-level.

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II. Experimental Method

The spectrograph is of the type described by Ashton, Nash, and Wolfendale (1959) and is illustrated in Figure 1. It utilizes neon flash tube arrays each consisting of two trays (O, P, and Q, R), in association with a permanent magnet (M) of

$$\int_{-\infty}^{+\infty} H\mathrm{d}l = 107 \text{ kG cm}$$

(Cousins and Nash 1959). The flash tubes, the coordinates of whose centres are accurately known with respect to the Cartesian axes shown, are triggered by a fourfold coincidence of the Geiger counter trays A, B, C, and D which

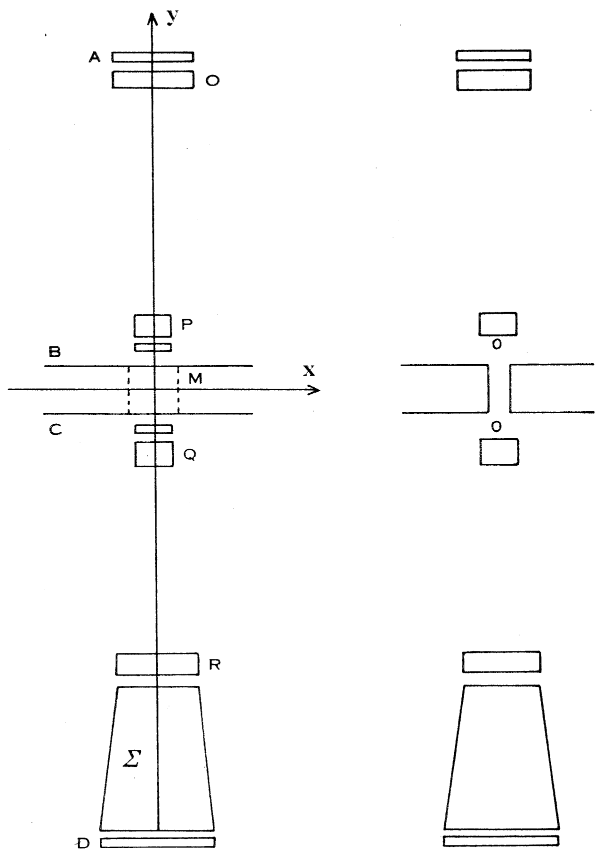


Fig. 1.—The spectrograph.

define the solid angle of acceptance of the instrument, and permit use only of a sensibly unifo m region of the magnetic field. A layer of lead, Σ, 47·3 cm thick between the tube tray R and the counter tray D imposes a momentum cut-off of 0·7 GeV/c of the particles accepted.

The flashes associated with the particle trajectories were photographed, and the data recorded by re-projection onto a scale diagram of the arrays. The coordinates of the tubes which had flashed were noted, and if ambiguity occurred because of the flashing of more than one tube in a layer, the event was rejected. However, many of the events in which multiple flashing occurred could be interpreted unambiguously, and were accepted. As the probability of knock-on electron or shower production is not significantly momentum-sensitive in the range under investigation (Fowler and Wolfendale 1958), it is unlikely that bias is introduced by this arbitrary acceptance condition.

III. Analysis of, and Corrections to the Experimental Results

(*a*) *Analysis of the Results*

The analysis of the data was carried out by the method fully described by Bull *et al.* (1962). It uses a knowledge of the distribution function $P(\theta)$, which gives the probability of a tube flashing, when a particle traverses the tube a distance θ from its centre. The most probable inclination φ, to the y-axis of a trajectory producing the observed set of flashes and blanks is computed, and the particle's deflection in the magnetic field found. Trajectories showing an intercept difference >10 mm at the x-axis were regarded as unassociated. In all, 1052, 1010, and 1033 particles were accepted at zenith angles of 45° W., 30° W., and 0° respectively.

(*b*) *Corrections for Instrumental Bias*

In arriving at the sea-level muon momentum spectrum, the procedure adopted was to calculate a theoretical deflection spectrum, correct it for instrumental bias, and compare it with the measured spectrum using the χ^2 test. The corrections applied were for:

(i) Magnetic deflection and multiple scattering out of the solid angle of the instrument of particles which would otherwise have been accepted.

(ii) Multiple scattering in the spectrograph causing spurious particle deflection.

(iii) The errors in angular deflection measurement by the spectrograph. These are due firstly to the range of trajectories having a finite probability of producing the same combination of flashed and unflashed tubes; and secondly to random errors in location of tubes, and random variations in tube diameters. Both contributions have been considered in detail by Bull *et al.* (1962).

IV. Derivation of "Best Fit" Spectra

Following the method of Moroney and Parry (1954), theoretical deflection spectra were calculated for muons arriving at zenith angles of 30° and 45°. The survival probability,

$$w(x_0,p_0,\theta)=\exp\left(\frac{-1}{\tau c}\int_0^{x_0}\frac{\mathrm{d}x}{p(x)}\right) \tag{1}$$

represents the probability that a muon will survive to sea-level with a momentum p_0 at zenith angle θ after production at a distance x_0 cm from sea-level, measured along its path. $p(x)$ represents the momentum in units of μc a distance x cm along the path, τ is the proper mean life of the muon, and μ its rest mass (207 electron masses).

In the evaluation of the survival probabilities, the variation of pressure with height was assumed to be that for the International Standard Atmosphere (Admiralty Weather Manual 1953, p. 145), additional high altitude data being taken from Spencer and Dow (1954). The rate of energy loss in the atmosphere was assumed to be $2 \cdot 2$ MeV g^{-1} cm^{-2} which is a good approximation in the momentum range 3–40 GeV/c, and the integral (1) was evaluated graphically for seven values of p_0 between $0 \cdot 7$ and 40 GeV/c. In this way a set of curves representing the variation of survival probability with production depth was constructed.

As a first approximation, the muon momentum distribution at production given by Moroney and Parry was assumed, that is,

$$N(p)\mathrm{d}p = Kp^{-3} \exp(-r/125)\mathrm{d}p,$$

where K is a constant and r is the depth in g cm^{-2} from the top of the atmosphere measured along the path. This assumes that muon production varies with depth in a similar way to the absorption of the primary protons. Then the sea-level spectra were evaluated by integrating numerically through the atmosphere the product of survival probability and production spectrum. The resulting spectra were converted into deflection distributions, and normalized to a total of 1000 particles in the sea-level momentum range $0 \cdot 7$ GeV/c $-\infty$.

To determine whether there is a significant difference between the above process of extended muon production, and a simpler process involving production at $r = 100$ g cm^{-2}, sea-level spectra were evaluated on the second assumption, using a momentum distribution at production of the form

$$N(p)\mathrm{d}p = K^1 p^{-3}\mathrm{d}p.$$

These spectra were normalized as before, and the sea-level deflection distributions at zenith angles of 30° and 45° are compared in Figures 2 (*a*) and 2 (*b*). It is apparent that there is good agreement between the distributions calculated assuming either production process. It is also known that the distribution for muons arriving vertically is adequately explained by production at a single level in the momentum range considered (Janossy and Wilson 1946), and as the assumption of this process considerably simplifies the calculation of theoretical deflection distributions for comparison with experimental results, it was employed throughout the following work.

Deflection distributions at zenith angles of 0°, 30°, and 45° were calculated for muon production spectrum exponents, γ, in the range $2 \cdot 5$–$3 \cdot 3$. These were multiplied by the overall correction factor (Fig. 6), broadened for the known errors in deflection measurement including those due to scattering, and compared with the experimental measurements using the χ^2 test (Fig. 3 (*a*)). In Tables 1, 2, and 3 the theoretical and experimental results are compared, for the relevant " best fit " value of γ. In addition, at 0° the experimental results are compared with those of Owen and Wilson (1955), which are now regarded as standard in the range 1–20 GeV/c. The associated value of χ^2 would be exceeded for 54% of random samples of 1000 events drawn from the Owen and Wilson distribution. This is taken to indicate good agreement between the present results and the standard vertical spectrum.

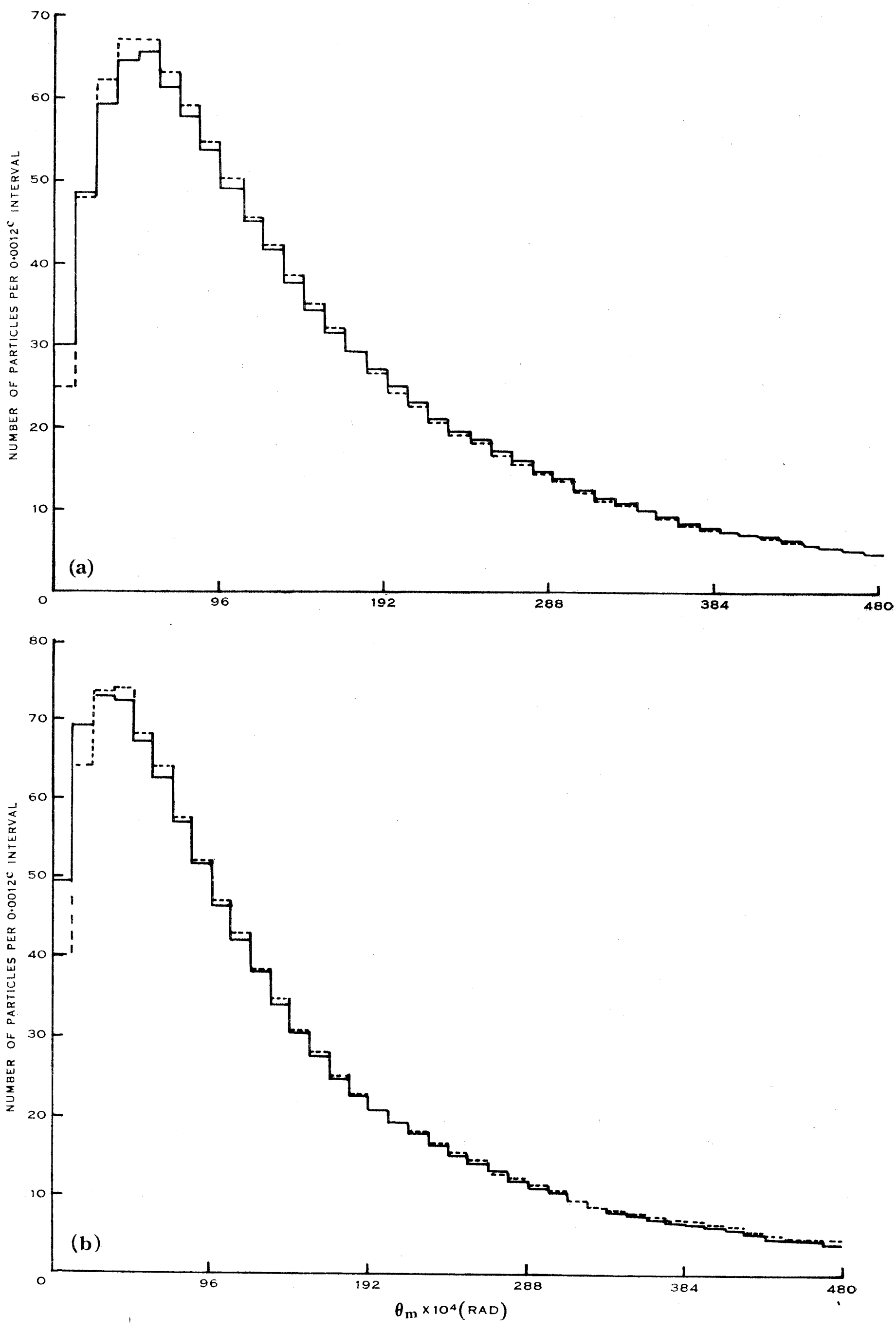


Fig. 2.—Comparison of theoretical deflection spectra.
(*a*) At 30° for production spectrum exponent = 3.
(*b*) At 45° for production spectrum exponent = 3.
——— Extended production. - - - - - 100 mB production.

On the arbitrary assumption that the value of χ^2 associated with the 25% level is unlikely to be exceeded, the values of γ at 0°, 30°, and 45° are $2\cdot 88^{+0\cdot 16}_{-0\cdot 30}$, $3\cdot 03^{+0\cdot 15}_{-0\cdot 11}$, and $2\cdot 75^{+0\cdot 17}_{-0\cdot 13}$ respectively. These results are taken to justify the assumption of a common production spectrum for muons arriving at zenith angles up to 45°, and for each value of γ, $\sum_{0°,\,30°,\,45°} \chi^2$ was evaluated (Fig. 3 (*b*)) and was

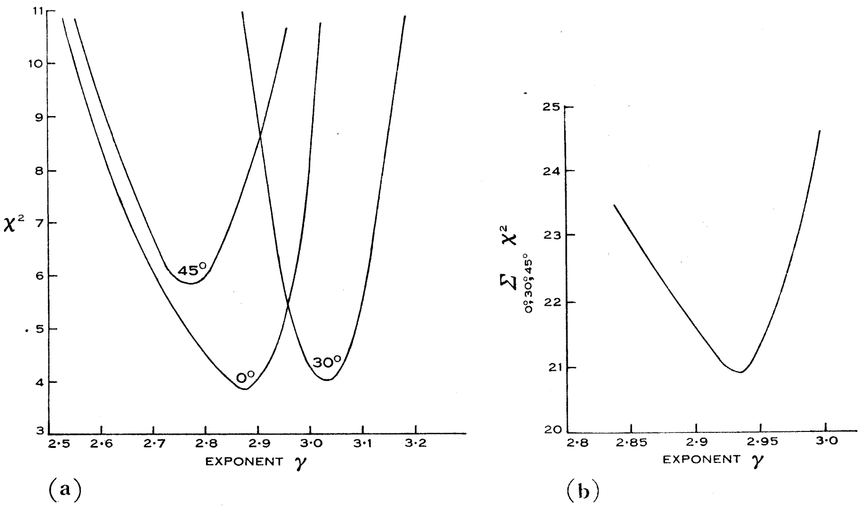


Fig. 3.—(*a*) χ^2 as a function of the exponent of the muon production spectrum. (*b*) $\sum_{0°,\,30°,\,45°}$ as a function of the exponent of the muon production spectrum.

found to have a minimum value of 21·0 for $\gamma=2\cdot 94$ with 20 degrees of freedom, the probability of exceeding this value is 35% and the limits on the value of γ are 2·98 and 2·86 ; giving the " best fit " value of $\gamma=2\cdot 94^{+0\cdot 04}_{-0\cdot 08}$.

The " best fit " momentum distribution of muons at production given by the present work was obtained by fitting the experimental results to the absolute

Table 1

0°, $\gamma=2\cdot 88$

Deflection Intervals (rad × 10^4)	Momentum Interval (GeV/c)	Observed	Expected: Owen and Wilson	Expected: $\gamma=2\cdot 88$
0– 12	27·25 –∞	44·57	43·47	40·55
12– 48	6·81 –27·25	207·6	228·0	226·3
48– 84	3·89 – 6·81	159·3	158·8	158·9
84–120	2·725– 3·89	129·1	125·9	113·1
120–156	2·1 – 2·725	101·0	99·72	105·1
156–204	1·6 – 2·1	101·7	104·0	98·29
204–288	1·14 – 1·6	115·1	125·5	118·9
288–432	0·76 – 1·14	97·22	105·37	96·01

TABLE 2

30°,* $\gamma=3\cdot03$

Deflection Interval (rad × 10⁴)	Momentum Interval (GeV/c)	Observed	Expected
0– 12	27·25–∞	34·73	32·01
12– 48	6·81–27·25	252·0	246·0
48– 96	3·41– 6·81	222·0	225·6
96–132	2·48– 3·41	73·66	79·81
132–168	1·95– 2·48	91·6	94·31
168–204	1·60– 1·95	61·91	69·34
204–276	1·18– 1·60	95·61	87·90
324–468	0·70– 1·01	79·33	73·16

* It will be noted that in the evaluation of χ^2 for the 30° spectrum a deflection group making an abnormal contribution to χ^2 has not been considered. This is statistically justified by the procedure adopted by Davis and Nelson (1937) and Wilson (1952).

TABLE 3

45°, $\gamma=2\cdot75$

Deflection Interval (rad × 10⁴)	Momentum Interval (GeV/c)	Observed	Expected
0– 12	27·25 –∞	49·41	60·32
12– 48	6·81 –27·25	238·9	263·2
48– 84	3·89 – 6·81	198·6	183·7
84–120	2·725– 3·89	140·1	136·4
120–156	2·1 – 2·725	119·5	124·6
156–204	1·6 – 2·1	75·93	85·93
204–288	1·14 – 1·6	68·09	71·97
288–432	0·76 – 1·14	81·73	89·16

TABLE 4

COMPARISON OF POSITIVE–NEGATIVE RATIOS

	Present Results	Moroney and Parry (1954)	Owen and Wilson (1955)
0°	1·33±0·10	1·29±0·04	1·32±0·01
30°	1·16±0·11	1·36±0·05	
45°	1·29±0·10		

sea-level rate in the vertical direction at 1 GeV/c, ($2 \cdot 45 \times 10^{-3}$ (sr)$^{-1}$ cm^{-2} sec^{-1} (GeV/c)$^{-1}$), given by Rossi (1948). The resulting production spectrum is

$$N(p)\mathrm{d}p = 0 \cdot 209 p^{-2 \cdot 94} \mathrm{d}p,$$

where p is in GeV/c and the intensity is expressed in units of the number of muons (sr)$^{-1}$ cm^{-2} sec^{-1} (GeV/c)$^{-1}$.

For purposes of comparison with other work, the production spectrum of π-mesons, $F(p_\pi)\mathrm{d}p$, may be derived. Barrett *et al.* (1952) obtained the expression

$$F(p_\pi)\mathrm{d}p = \frac{p_\mu + P_0}{P_0} r . F(p_\mu)\mathrm{d}p,$$

where $F(p_\pi)\mathrm{d}p$ is the π-meson production spectrum giving rise to the muon production spectrum $F(p_\mu)\mathrm{d}p$, and $P_0 = 90$ GeV/c, $r = m_\mu / m_\pi$. The assumptions made are that the primary particles are incident isotropically on the top of the atmosphere, that the π-mesons retain the direction of their producers, that π-mesons of momentum p_π give rise to muons of momentum $r.p_\pi$, and that the absorption mean free paths of the π-mesons and their producers are equal.

The exponent γ_π of the π-meson production spectrum at momentum p_π as given by the present results is then

$$\gamma_\pi = -2 \cdot 94 + p_\pi / (p_\pi + 119).$$

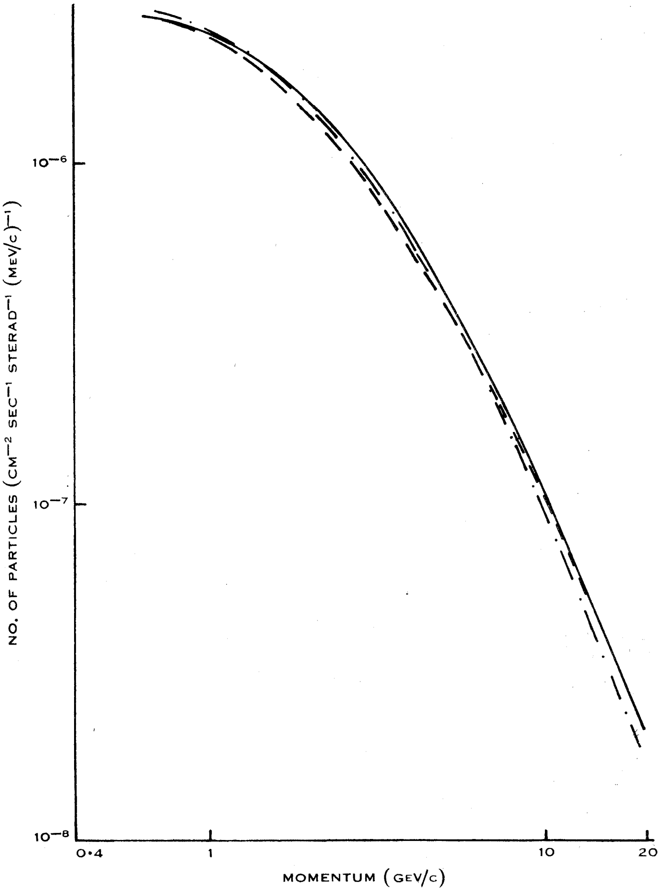
The weighted mean value of p_π for the momentum range of this experiment is $7 \cdot 3$ GeV/c, at which

$$F(p_\pi)\mathrm{d}p = 0 \cdot 325 p_\pi^{-2 \cdot 88} \mathrm{d}p.$$

V. Discussion, Comparison with Other Work, and Conclusions

The " best fit " momentum spectrum of muons at sea-level in the vertical direction is given in Figure 4 (*a*), where it is compared with that of Owen and Wilson and Caro, Parry, and Rathgeber, and in Figure 4 (*b*) the vertical and inclined spectra are given ; the 30° spectrum being compared with that of Moroney and Parry obtained using the same spectrograph as Caro, Parry, and Rathgeber (1951). It is seen that the divergences between the present results and those of Moroney and Parry are similar to those between the Owen and Wilson spectrum and that of Caro, Parry, and Rathgeber, which is high at low momenta, and low at momenta above approximately 6 GeV/c. Doubt has been expressed, in particular by Pine, Davisson, and Greisen (1959), on the accuracy of the Caro, Parry, and Rathgeber (1951) spectrum, and it is not unreasonable to expect that the spectra at various zenith angles are also liable to error. A possible cause of this error lies in the corrections applied for magnetic cut-off and scattering out at low momenta. It may be noted that the most frequently used normalization point at 1 GeV/c lies in the region where errors in the corrections would be important, and where statistical accuracy is relatively poor. These effects are more evident when the best line is drawn through a set of points than when the significance of the agreement between theoretical and experimental distributions is tested statistically.

No significant tendency has been found for an increased intensity at low momenta at 30° and 45° due to multiple scattering in the atmosphere (Tables 1, 2, and 3). This effect would be expected on the theoretical predictions of Maeda (1960). It should be noted that Moroney and Parry, assuming a muon production spectrum of exponent $-3\cdot0$, indicated that scattering would provide the 50% correction necessary to account for their results at 60°, whereas Maeda also



(a)

Fig. 4(*a*).—The vertical spectrum.
——— Best fit to present results. — — Owen and Wilson (1955).
—·— Caro, Parry, and Rathgeber (1951).

finds agreement using Sands' production spectrum of exponent $-3\cdot58$. Sands' (1950) spectrum predicts sea-level intensities at 30° and 60° some 25% and 40% lower respectively than those predicted at 1 GeV/c by that of exponent $-3\cdot0$.

Other published work on inclined spectra is by Pak *et al.* (1961) and Allen and Apostolakis (1961), who made measurements at 68° and in the range 65–85° respectively. In agreement with the present experiment, both find that the same π-meson production spectrum describes both the vertical and inclined spectra. Pak *et al.* obtained $F(p_\pi)\mathrm{d}p=0\cdot156(p_\pi)^{-2\cdot64}\mathrm{d}p$ in the range $p_\pi=5$ to

100 GeV/c, and Allen and Apostolakis, using the vertical muon spectrum of Ashton *et al.* (1960) for reference, obtained an expression $F(p_\pi)\mathrm{d}p = I_0 p_\pi^{-\gamma_\pi}\mathrm{d}p$, where

$$I_0 = 0\cdot425 - 0\cdot125 \log_{10} p_\pi \text{ and } \gamma_\pi = 3\cdot92 - 0\cdot944/(1 - 0\cdot125 \log_{10} p_\pi).$$

They obtain an approximate expression $F(p_\pi)\mathrm{d}p = 0\cdot15 p_\pi^{-2\cdot55}\mathrm{d}p$ in the range $p_\pi = 6$ to 1000 GeV/c.

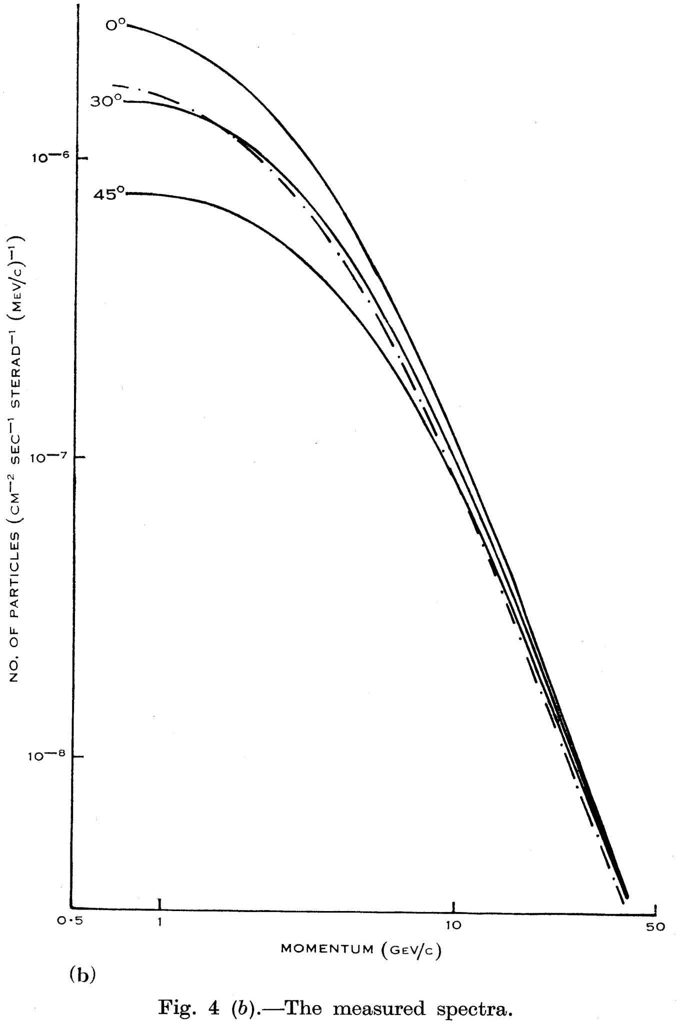


Fig. 4 (*b*).—The measured spectra.
——— Best fit to present results. —·—· Moroney and Parry (30°).

Hayman and Wolfendale (personal communication) have recalculated γ_π, based on additional measurements with the Durham spectrograph, and obtained a value close to $-2\cdot64$ in the range $p_\pi = 16$ to 2000 GeV/c.

The expression for $F(p_\pi)\mathrm{d}p$ given by the present results is strictly only comparable with that of Allen and Apostolakis at $7\cdot3$ GeV/c. The latter give

$0 \cdot 317 p_{\pi}^{-2 \cdot 86} \mathrm{d}p$, which agrees with the present results within the limits of experimental error, and the uncertainty of 10% in the values of I_0 and γ_{π} given by Allen and Apostolakis. It should be noted that values of γ_{π} obtained in cases where σ_{ψ} (see Appendix (*b*)) for the spectrograph is not accurately known, may be in considerable error (Bull, Nash, and Rastin 1962).

The effect of K-μ-decay can be investigated by allowing a fraction of the muons to come from K-decay rather than from π-decay. It was found that if a large fraction of the sea-level muons were assumed to originate from K–decay, the derived production spectra would not be the same at all three zenith angles.

(*a*) *Positive-Negative Ratio*

The positive-negative ratio at sea-level reflects the multiplicity at production, whether it is a property of single collisions or the results of a cascade process.

It would be expected that as the primary energy increases, and therefore the average multiplicity, the positive-negative ratio would decrease. The results of the present experiment are compared with those of Moroney and Parry and Owen and Wilson in Table 4 over the range 0·7–30 GeV/c. The results of Moroney and Parry do not show the expected decrease of the positive-negative ratio with increasing zenith angle. Although the statistics of the present experiment are not good, the tendency of decreasing positive-negative ratio with increasing angle and hence momentum is indicated. This trend is in agreement with the results quoted by Pak *et al.* in their measurements at 68°.

VI. Conclusions

The conclusions of this investigation may be summarized as follows :

(i) The measured momentum spectrum of muons in the vertical direction agrees with that of Owen and Wilson, and disagrees with that of Caro, Parry, and Rathgeber.

(ii) The same form of discrepancy as in (i) occurs between the measured spectrum at 30° W. and that of Moroney and Parry.

(iii) The measured sea-level spectra at 0°, 30°, and 45° can be accounted for by assuming single level production of the muons, the differential π-meson production spectrum being of the form

$$N(p)\mathrm{d}p = 0 \cdot 325 p^{-2 \cdot 88} \mathrm{d}p \quad (\mathrm{cm}^{-2}\ \mathrm{sr}^{-1}\ (\mathrm{GeV/c})^{-1}\ \mathrm{sec}^{-1}),$$

for the primary energy range investigated in this experiment.

(iv) The sea-level muon spectrum for the energy range investigated in the experiment is adequately described in terms of π-μ-decay.

(v) No evidence has been found for an increase in intensity at low momenta due to multiple scattering in the atmosphere. Some doubt must be cast therefore on the calculations by Maeda of the effect of the scattering of muons in traversing the atmosphere.

Further evidence is required regarding (iv) and (v) and this would be provided by spectrum measurements of high statistical precision at zenith angles of greater than 60°, and in particular in the horizontal direction.

VII. Acknowledgments

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Appendix

Calculation of Corrections to the Incident Spectra due to Instrumental Bias

(*a*) *Magnetic and Scattering Correction Factors*

Treating the scattering deflections in the front and back planes as independent, which is justified for the small angles involved, four causes of loss of particles may occur, by:

(i) Magnetic deflection in the front plane beyond the central 36 cm of D (Fig. 5).

(ii) Scattering in the front plane beyond the total length of D by the lead and tray R, assumed to be concentrated at a horizontal line through the centre of gravity of the lead.

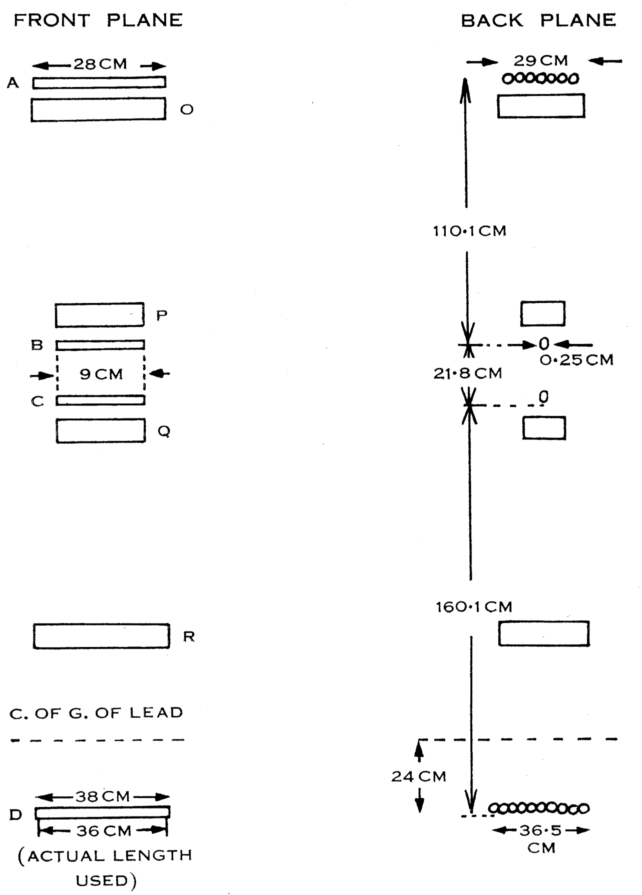


Fig. 5.—Basic geometry of spectrograph.

(iii) Scattering in the front plane beyond the central 36 cm of D by all the material except the lead, assumed to be concentrated at the central line of the magnetic field.

(iv) Scattering in the back plane beyond the limits of D by all the material.

Particles may be deflected into the solid angle by processes like (i), (iii), and (iv).

The probability of acceptance as a function of momentum was determined by computer programmes. For the purposes of computation, lines representing the counters B and C were divided into 50 equal parts in the front plane, and 25 in the back plane. Then the lines joining each combination represent particles

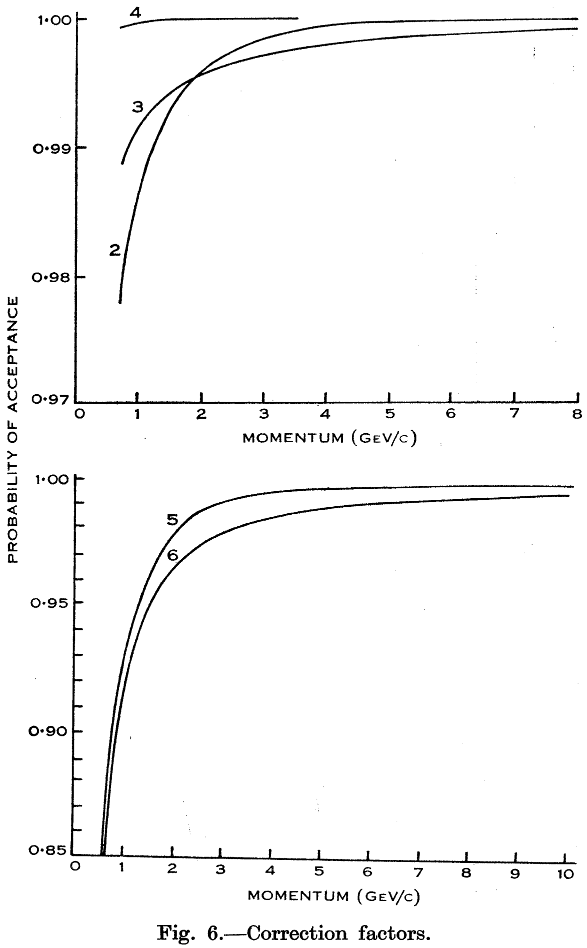


Fig. 6.—Correction factors.

2, Front plane scattering (lead+trays); 3, front plane scattering (trays); 4, back plane scattering (lead+trays); 5, magnetic deflection; 6, overall correction factor to be applied to comparison spectrum.

with $p=\infty$. In calculating the probability, P_0, of scattering out, only trajectories falling inside the solid angle were considered, and assuming a $\cos^2\theta$ variation of intensity with inclination to the vertical, P_0 was determined using the ogive to the normal curve. To calculate the probability, P_{I} of scattering in, only those trajectories falling outside the solid angle were considered, and P_{I} determined in the same way.

The magnetic correction factor was calculated similarly ; and the scattering, magnetic, and resultant correction factors are shown as a function of momentum in Figure 6.

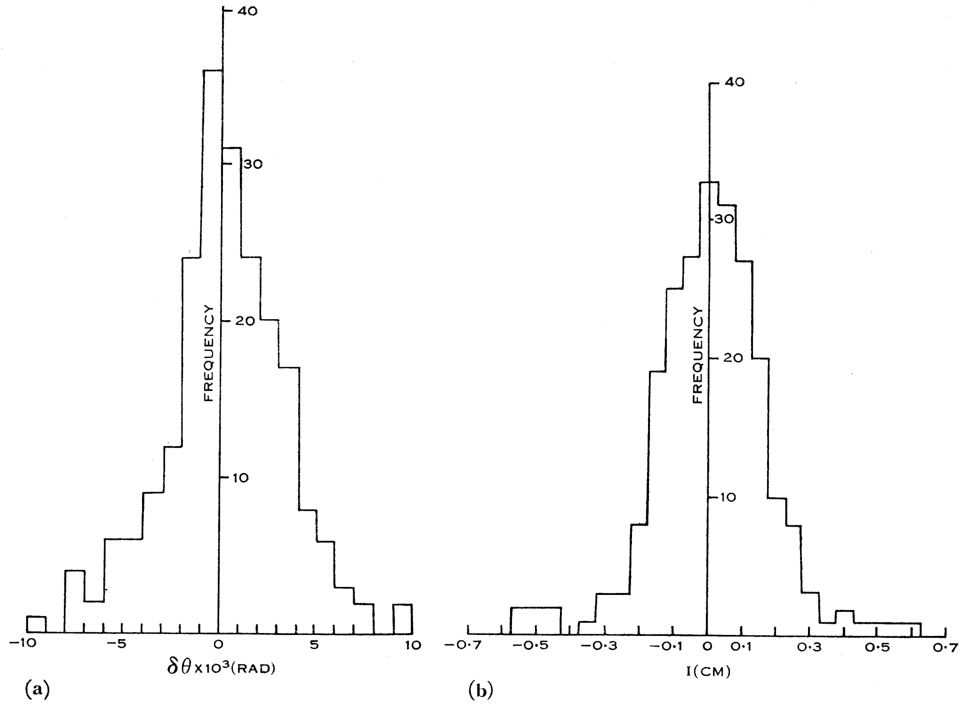


Fig. 7.—No field distribution.
(*a*) Of angular deflections. (*b*) Of intercept differences at x-axis.

(*b*) *Measurement of Errors due to Multiple Scattering*

The magnetic deflection, θ_m, of a particle of momentum p traversing the spectrograph is given by

$$\theta_m=\left(300\int_{-\infty}^{+\infty} Hdl\right)\Big/p\beta \tag{1}$$

and its r.m.s. projected angle of scattering $\langle\theta_s\rangle$ is

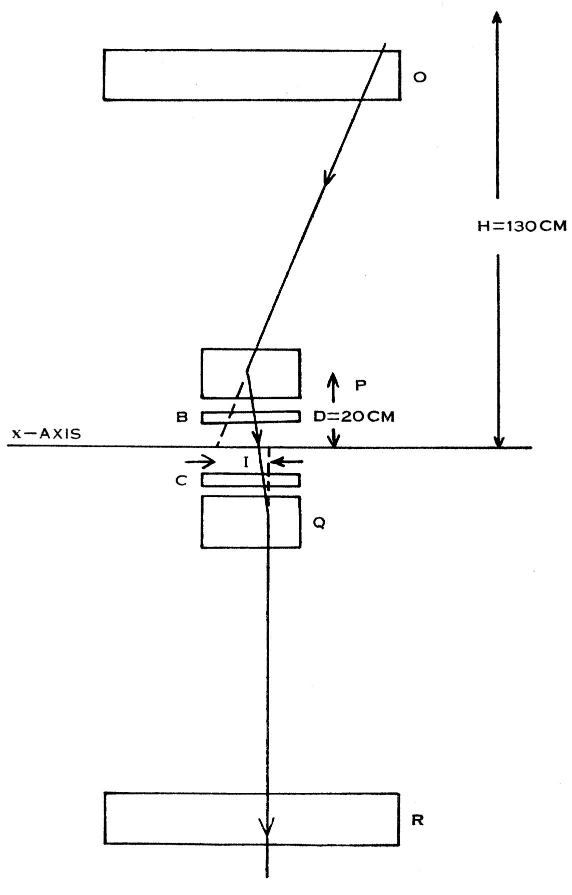
$$\langle\theta_s\rangle=\frac{1}{\sqrt{2}}\cdot\frac{\Sigma Kt^{\frac{1}{2}}}{p\beta}, \tag{2}$$

where $K=22$ MeV, and t is the thickness in radiation lengths of material traversed. The sum is taken over the material in the trays P, Q, and the Geiger counters B, C, which contributes to $\langle\theta_s\rangle$. Thus the ratio $\langle\theta_s\rangle/\theta_m$ is

$$\frac{\langle\theta_s\rangle}{\theta_m}=\frac{1}{\sqrt{2}}\cdot\frac{\Sigma Kt^{\frac{1}{2}}}{300\int_{-\infty}^{+\infty} Hdl}\cdot\frac{1}{\beta}, \tag{3}$$

which for $\beta\rightarrow 1$, that is, $p>0\cdot 5$ GeV/c is a constant, independent of momentum.

The value of $\langle\theta_s\rangle$ for the spectrograph was calculated by integrating (2) over the range $0 \cdot 7$ GeV/c$\rightarrow\infty$ for the sea-level muon momentum spectrum corrected for scattering in and out of the instrument (see Appendix (*a*)). The value obtained is $\langle\theta_s\rangle = 2 \cdot 97 \times 10^{-3}$ rad. Combining this with the error in deflection measurement, $\sigma_{\Psi} = 1 \cdot 15 \times 10^{-3}$ rad (Bull *et al.* 1961) gives a value $\sigma = 3 \cdot 18 \times 10^{-3}$ rad to be expected for the " no field " distribution in angular deflection, Ψ. The measured distribution, based on 214 events, is shown in



(a)

Fig. 8 (*a*).—Illustrating origin of intercept differences.

Figure 7 (*a*). It is seen to be approximately normal, and has a standard deviation, $\sigma = (3 \cdot 21 \pm 0 \cdot 27) \times 10^{-3}$ rad. The good agreement with the value predicted is taken to indicate an accurate assessment of the thickness t of scattering material contributing to $\langle\theta_s\rangle$.

Assuming this value of t, a value of $\langle\theta_s\rangle/\theta_m$ was calculated, using equation (3), and found to be $0 \cdot 177$. This value and that of σ_{Ψ} was checked using the method of Ashton, Nash, and Wolfendale (1959). The " no field " distribution in intercept differences at the x-axis of the two halves of a particle trajectory, Figure 7 (*b*), is seen to be approximately normal, justifying the following procedure.

At any momentum

$$\sigma_I^2 = 2D^2\langle\theta^2\rangle + \sigma_0^2,$$

where σ_I is the r.m.s. value of the intercept difference I (Fig. 8) due to errors of deflection measurement and scattering, $\langle\theta\rangle$ is the r.m.s. projected scattering angle

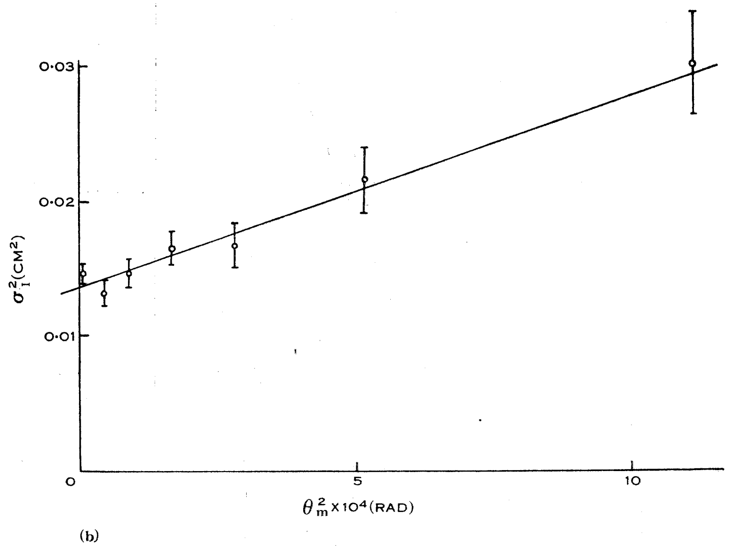


Fig. 8 (*b*).—Illustrating derivation of $\sigma_{\Psi'}$ and $\langle\theta_s\rangle/\theta_m$.

in one-half of the spectrograph, D is the distance between the x-axis and the centre of gravity of the scattering materials (20 cm), and σ_0 is the r.m.s. value of I due only to errors in deflection measurement.

From equation (3)

$$\frac{\langle\theta_s\rangle}{\theta_m} = K_1, \qquad \text{where } \langle\theta_s\rangle = \sqrt{2}\langle\theta\rangle,$$

that is,

$$\sigma_I^2 = K_1^2 D^2 \theta_m^2 + \sigma_0^2.$$

A graph of σ_I^2 versus θ_m^2 is shown in Figure 8 (*b*). The gradient gives $K_1 = 0\cdot19 \pm 0\cdot02$, and the intercept $\sigma_0 = (1\cdot34 \pm 0\cdot20) \times 10^{-2}$ cm^2, giving

$$\sigma_{\Psi'} = \sqrt{2}\sigma_0/H = (1\cdot25 \pm 0\cdot10) \times 10^{-3} \text{ rad}.$$

The agreement with theoretical calculations is good, and the values $\langle\theta_s\rangle/\theta_m = 0\cdot177$ and $\sigma_{\Psi'} = 1\cdot15 \times 10^{-3}$ rad have been employed throughout. Defining the maximum detectable momentum of the spectrograph as that momentum at which the r.m.s. error in angular deflection measurement equals the magnetic deflection of the particle, its value is $28\cdot5$ GeV/c.