THE EVOLUTION OF MASSIVE STARS INITIALLY COMPOSED OF PURE HYDROGEN

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Summary

The object of the present paper is to investigate the influence of carbon abundance on the evolution of the massive stars, initially composed of pure hydrogen, during the pre-main-sequence contraction and the hydrogen burning phase of these stars.

I. Introduction

Equilibrium models of stars composed of pure hydrogen have been considered by Boury (1960) and Ezer (1961) and, as pointed out by these authors, it is to be expected that a certain amount of carbon will be formed in the star during its pre-main-sequence contraction and that, at some time during the evolution of the star, the carbon cycle will become the predominant factor in the nuclear energy generation.

It is the purpose of the present paper to make an evaluation of the amount of carbon formed during the pre-main-sequence contraction of massive stars, initially composed of pure hydrogen, for masses in the range 40 $M_\odot$ to 120 $M_\odot$, to follow the evolution of these stars during their hydrogen burning phase, and to determine the amount of carbon formed during this period.

It will be shown that the evolution of these stars, during the hydrogen burning phase, differs markedly from the evolution of stars, of comparable mass, with a more "normal" composition. Due to the much smaller abundance of carbon in the present models, the central and effective temperatures will be higher and, since the stars considered here have a much higher initial hydrogen abundance, it will take longer to reach the stage of hydrogen depletion in the core.

II. Construction of Models with Non-uniform Composition

Following Schwarzschild and Härm (1958) we have constructed equilibrium models of stars, with non-uniform composition, consisting of three zones:

a convective core which contains almost all the nuclear energy production;

an unstable intermediate zone in convective neutrality, with varying composition;

a radiative envelope composed of pure hydrogen.

If we assume that electron scattering is the main source of opacity, i.e. that the opacity is given by

$$\kappa = 0\cdot 2004(1+X),$$

X being the abundance by weight of hydrogen, then, in order to construct our three-zone models, we have to integrate the following systems of differential equations:

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A. *In the envelope*

$$\frac{\mathrm{d}\bar{m}}{\mathrm{d}x} = 4\pi x^2 \frac{aM_\odot^2}{3\mathscr{R}} \bar{t}^3 \frac{\beta}{1-\beta},$$

$$\frac{\mathrm{d}\bar{t}}{\mathrm{d}x} = -\frac{\beta}{16\pi c\mathscr{R}} \cdot \frac{1}{(1-\beta)M_\odot} \cdot \frac{l_e}{x^2},$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \frac{1}{\mathscr{R}} \frac{\beta(1-\beta)}{\bar{t}} \frac{1}{x^2} \left\{ \frac{l_e}{4\pi c(1-\beta)M_\odot} - \bar{m}G \right\},$$

together with the equation of state

$$\bar{p} = \frac{aM_\odot^2 \bar{t}^4}{3(1-\beta)},$$

where

$$\bar{p} = R^4 \mu_e^4 P / M_\odot^2,$$

$$\bar{t} = R\mu_e T / M_\odot,$$

$$\bar{m} = \mu_e^2 M_r / M_\odot,$$

$$x = r/R,$$

$$l_e = \kappa_e \mu_e^2 L.$$

We have chosen β as dependant variable instead of $\bar{p}$, on account of its smaller variation in the outer layers of the star. μ_e is the molecular weight in the outer layers

$$\mu_e = 1/(1 \cdot 25X_e - 0 \cdot 25Z_e + 0 \cdot 75) = 0 \cdot 5,$$

and

$$\kappa_e = 0 \cdot 2004(1+X_e) = 0 \cdot 4008,$$

since $X_e = 1$ and $Z_e = 0$.

B. *In the intermediate zone and core*

$$\frac{\mathrm{d}\bar{m}}{\mathrm{d}x} = 4\pi x^2 \beta \alpha \frac{\bar{p}}{\mathscr{R}\bar{t}},$$

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} = -\frac{\beta\bar{p}}{\mathscr{R}\bar{t}} \alpha \frac{G\bar{m}}{x^2},$$

$$\frac{\mathrm{d}\bar{t}}{\mathrm{d}x} = -\frac{2(4-3\beta)}{32-3\beta^2-24\beta} \alpha \frac{\beta}{\mathscr{R}} \cdot \frac{G\bar{m}}{x^2},$$

where α is the ratio of molecular weights,

in the core: $\alpha = \alpha_c = \mu_c/\mu_e$,
in the intermediate zone: $\alpha = \mu/\mu_e$,

with

$$\mu = 1/\{1\cdot 25(1+X)-0\cdot 5-0\cdot 25Z\}.$$

The value of $(1+X)$ in the intermediate zone is given by the condition of convective neutrality

$$\left|\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{rad}}\right| = \left|\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}}\right|$$

or

$$\frac{8-6\beta}{32-3\beta^2-24\beta}\cdot\frac{T}{P}\cdot\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{3\kappa}{16\pi ac}\cdot\frac{\rho}{T^3}\cdot\frac{L}{r^2},$$

which, in our notation, reduces to

$$(1+X) = (1+X_e)\frac{32\pi c(4-3\beta)(1-\beta)G\bar{m}M_\odot}{(32-3\beta^2-24\beta)l_e}.$$

It follows from this formula that, at each point of the intermediate zone, the value of α is known in terms of the other variables.

At the surface the boundary condition was taken as $T = 0$, together with the expansions

$$\beta = \beta_e,$$

$$\bar{m}_e = \mu_e^2 M/M_\odot,$$

$$t = \frac{\beta_e G\bar{m}_e}{4\mathscr{R}}\left(\frac{1}{x}-1\right).$$

The condition $\mathrm{d}\beta/\mathrm{d}x = 0$ at the surface then leads to the mass-luminosity relation

$$l_e = 4\pi Gc\bar{m}_e(1-\beta_e).$$

Starting with a trial value of β_e and the expansions mentioned above, the integration was then continued till the boundary between the envelope and the intermediate region was reached, i.e. when the condition

$$(32-3\beta^2-24\beta)l_e = 32\pi c(4-3\beta)(1-\beta)G\bar{m}M_\odot$$

was satisfied.

The integrations were then continued throughout the intermediate layer with the appropriate value of α.

The boundary between intermediate zone and core is reached when $\alpha = \alpha_c$, and the integration proceeds then with this constant value of α_c.

Throughout the integration in the core a constant check was kept on the value of $\mathrm{d}\beta/\mathrm{d}x$ and $\mathrm{d}^2\beta/\mathrm{d}x^2$. Both these quantities should be positive.

If $\mathrm{d}\beta/\mathrm{d}x$ becomes negative it means that the integration was started with too large a value of β_e. If $\mathrm{d}^2\beta/\mathrm{d}x^2$ is negative the initial β_e is too small.

As soon as a point x_p was reached, where either $\mathrm{d}\beta/\mathrm{d}x$ or $\mathrm{d}^2\beta/\mathrm{d}x^2$ was negative, a rough estimate was made of the value of β at the centre, using the formula

$$\beta_\mathrm{c} = \beta_\mathrm{p} - \tfrac{1}{2}x_\mathrm{p}\left(\frac{\mathrm{d}\beta}{\mathrm{d}x}\right)_\mathrm{p},$$

where

$$\left(\frac{\mathrm{d}\beta}{\mathrm{d}x}\right)_\mathrm{p} = \frac{3G\alpha_\mathrm{c}\beta_\mathrm{p}^3(1-\beta_\mathrm{p})\bar{m}_\mathrm{p}}{\mathscr{R}\bar{t}_\mathrm{p}x_\mathrm{p}^2(32-3\beta_\mathrm{p}^2-24\beta_\mathrm{p})}.$$

With this rough estimate of β_c a series expansion of β was constructed, of the form

$$\beta = \beta_\mathrm{c} + \beta_2' x^2 + \beta_4' x^4 + hx^6,$$

where

$$\beta_2' = \frac{4\pi G}{\mathscr{R}} \cdot \frac{1}{\Delta t_\mathrm{c}} \rho_\mathrm{c}' \beta_\mathrm{c}^3 (1-\beta_\mathrm{c}),$$

$$\beta_4' = \frac{\pi G}{\mathscr{R}} \cdot \frac{1}{t_\mathrm{c}\Delta}\left\{bc+ad+ac\left(f-\frac{t_2}{t_\mathrm{c}}\right)\right\},$$

$$h = \frac{\left(\frac{\mathrm{d}\beta}{\mathrm{d}x}\right)_\mathrm{p} - 2\beta_2' x_\mathrm{p} - 4\beta_4' x_\mathrm{p}^3}{6x_\mathrm{p}^5},$$

and

$$t_\mathrm{c} = \bar{t}_\mathrm{c} M_\odot,$$

$$\rho_\mathrm{c}' = \frac{a}{3\mathscr{R}} t_\mathrm{c}^3 \frac{\beta_\mathrm{c}}{1-\beta_\mathrm{c}},$$

$$\Delta = 32 - 3\beta_\mathrm{c}^2 - 24\beta_\mathrm{c},$$

$$t_2 = \frac{-(4-3\beta_\mathrm{c})}{\Delta} \cdot \frac{\beta_\mathrm{c}}{\mathscr{R}} \cdot \frac{4\pi G \rho_\mathrm{c}'}{3},$$

$$\rho_2' = \frac{a}{3\mathscr{R}} \cdot \frac{t_\mathrm{c}^3}{1-\beta_\mathrm{c}}\left\{\frac{\beta_2'}{1-\beta_\mathrm{c}} + 3\beta_\mathrm{c}\frac{t_2}{t_\mathrm{c}}\right\},$$

$$a = \beta_\mathrm{c}(1-\beta_\mathrm{c}),$$

$$b = \beta_2'(1-2\beta_\mathrm{c}),$$

$$c = \rho_\mathrm{c}'\beta_\mathrm{c}^2,$$

$$d = \beta_\mathrm{c}(2\rho_\mathrm{c}'\beta_2' + \tfrac{3}{5}\rho_2'\beta_\mathrm{c}),$$

$$f = 6\beta_2'(\beta_\mathrm{c}+4)/\Delta.$$

This series expansion was used to compute the value of β_p(int.) at the point x_p and this value was then compared with the value of β_p(ext.) obtained from the integration of the differential equations.

The value of β_c was then adjusted in order to make the difference between these two values of β_p smaller than a given number (e.g. 10^{-5}).

Using this value of β_c it is then possible to estimate the mass $\bar{m}_p$(int.) at the point x_p, using the formula

$$\bar{m}_p = \frac{4\pi a\alpha_c}{3\mathscr{R}M_\odot^4}\int_0^{x_p}\frac{\beta}{1-\beta}x^2\bar{t}^3\,dx,$$

where (Van der Borght and Meggitt 1963)

$$\bar{t} = \bar{t}_c\left\{\frac{\beta_c \exp(-4/\beta_c)}{1-\beta_c}\cdot\frac{1-\beta}{\beta\exp(-4/\beta)}\right\}^{2/3}.$$

The value of

$$\bar{M} = |\bar{m}_p(\text{int.})-\bar{m}_p(\text{ext.})|,$$

where $\bar{m}_p$(ext.) is the value of $\bar{m}_p$ obtained from the integration of the differential equations, was then used as a measure of the accuracy of the trial value of β_e.

Once two values of β_e have been found, one too large and the other too low, it is easy to make the procedure quite automatic and by using the weight factor $\bar{M}$ to construct an iterative method which converges quite rapidly to the right value of β_e.

Using the above method, the system of differential equations was integrated on the IBM 1620 Computer of the Research School of Physical Sciences for values of $\bar{m}_e = 10, 15, 20, 30$ for various values of α_c ranging from 1 to $2\cdot4$ corresponding to hydrogen abundances X_f in the core, in the range 1 to $0\cdot067$.

The main results of these computations are given in Tables 1–4.

III. Pre-main-sequence Contraction

The carbon abundance at the end of the pre-main-sequence contraction has been computed by considering the homologous contraction of the stars, i.e. by assuming that the distributions of the quantities x, $\bar{m}$, $\bar{p}$, $\bar{t}$, and β remain similar, throughout the contraction, to those derived for the uniform main-sequence model.

Since the aim of the present section is to derive an estimate of the carbon abundance when the stars have reached the main sequence, it is to be expected that the method of homologous contraction will yield fairly accurate results, especially during the later stages of the contraction when most of the carbon is formed.

The energy equation

$$\frac{dE}{dt} = \frac{P}{\rho^2}\frac{d\rho}{dt}+\epsilon-\frac{1}{4\pi r^2\rho}\frac{dL_r}{dr},$$

Table 1
$M = 40\ M_\odot$

Characteristic	α_c									
	1·0	1·1	1·2	1·3	1·4	1·6	1·8	2·0	2·2	2·4
β_c	0·794	0·766	0·739	0·716	0·694	0·657	0·625	0·598	0·575	0·553
β_f	0·865	0·832	0·804	0·778	0·754	0·713	0·679	0·648	0·624	0·600
β_1	—	0·844	0·826	0·809	0·794	0·769	0·749	0·732	0·719	0·707
β_R	0·904	0·887	0·872	0·858	0·846	0·824	0·806	0·791	0·779	0·768
x_f	0·392	0·339	0·299	0·266	0·237	0·191	0·156	0·128	0·107	0·088
x_1	—	0·370	0·353	0·338	0·325	0·304	0·288	0·274	0·264	0·255
q_f	0·572	0·517	0·479	0·450	0·425	0·386	0·356	0·331	0·311	0·293
q_1	—	0·593	0·611	0·627	0·641	0·662	0·679	0·693	0·703	0·711
X_f	1·000	0·855	0·733	0·631	0·543	0·400	0·289	0·200	0·127	0·067
$\bar{X}$	1·000	0·921	0·859	0·810	0·769	0·705	0·660	0·626	0·600	0·581
$\log(L/L_\odot)$	5·104	5·173	5·227	5·271	5·308	5·365	5·407	5·440	5·464	5·485
$\log T_e$	4·892	4·842	4·830	4·812	4·802	4·777	4·751	4·724	4·699	4·675
$\log(X_C 10^{10})$	$\bar{3}$·124	0·065	0·380	0·744	0·865	1·152	1·398	1·611	1·810	2·045
τ (10^6 years)	0·176	2·498	4·160	5·325	6·217	7·468	8·245	8·784	9·148	9·417
$\log(R/R_\odot)$	0·288	0·423	0·474	0·531	0·570	0·648	0·723	0·792	0·854	0·913
$T_c 10^{-8}$	1·375	1·143	1·140	1·114	1·131	1·146	1·163	1·186	1·216	1·259

Table 2
$M = 60\ M_\odot$

Characteristic	α_c									
	1·0	1·1	1·2	1·3	1·4	1·6	1·8	2·0	2·2	2·4
β_c	0·714	0·681	0·651	0·625	0·601	0·562	0·529	0·501	0·477	0·456
β_f	0·800	0·757	0·720	0·689	0·661	0·614	0·577	0·546	0·518	0·495
β_1	—	0·773	0·749	0·728	0·709	0·677	0·652	0·631	0·613	0·599
β_R	0·843	0·818	0·796	0·776	0·758	0·728	0·702	0·682	0·664	0·650
x_f	0·435	0·376	0·329	0·292	0·259	0·207	0·167	0·135	0·108	0·088
x_1	—	0·415	0·398	0·383	0·370	0·348	0·330	0·316	0·303	0·293
q_f	0·667	0·606	0·565	0·533	0·506	0·463	0·430	0·402	0·378	0·360
q_1	—	0·693	0·714	0·731	0·746	0·770	0·790	0·800	0·812	0·820
X_f	1·000	0·855	0·733	0·631	0·543	0·400	0·289	0·200	0·127	0·067
$\bar{X}$	1·000	0·908	0·834	0·775	0·726	0·650	0·595	0·554	0·522	0·498
$\log(L/L_\odot)$	5·492	5·555	5·605	5·645	5·679	5·731	5·769	5·798	5·822	5·840
$\log T_e$	4·966	4·885	4·872	4·853	4·841	4·812	4·782	4·752	4·720	4·692
$\log(X_C 10^{10})$	$\bar{3}$·932	0·263	0·560	0·890	1·017	1·290	1·533	1·732	1·945	2·170
τ (10^6 years)	0·093	1·764	2·989	3·876	4·538	5·492	6·109	6·524	6·829	7·043
$\log(R/R_\odot)$	0·334	0·527	0·580	0·637	0·677	0·761	0·841	0·916	0·992	1·056
$T_c 10^{-8}$	1·651	1·192	1·183	1·158	1·171	1·186	1·203	1·229	1·259	1·316

TABLE 3
$M = 80\ M_{\odot}$

Characteristic	α_c									
	1·0	1·1	1·2	1·3	1·4	1·6	1·8	2·0	2·2	2·4
β_c	0·654	0·620	0·589	0·563	0·539	0·500	0·468	0·441	0·417	0·398
β_f	0·747	0·697	0·658	0·624	0·595	0·548	0·510	0·479	0·452	0·431
β_1	—	0·716	0·689	0·666	0·645	0·610	0·583	0·561	0·542	0·527
β_R	0·790	0·760	0·734	0·710	0·690	0·655	0·627	0·603	0·584	0·568
x_f	0·468	0·401	0·351	0·309	0·274	0·216	0·172	0·137	0·109	0·088
x_1	—	0·448	0·431	0·416	0·403	0·380	0·362	0·347	0·333	0·322
q_f	0·731	0·664	0·620	0·585	0·556	0·510	0·472	0·443	0·418	0·398
q_1	—	0·756	0·789	0·794	0·808	0·833	0·845	0·857	0·867	0·874
X_f	1·000	0·855	0·733	0·631	0·543	0·400	0·289	0·200	0·127	0·067
$\bar{X}$	1·000	0·899	0·818	0·754	0·700	0·617	0·557	0·513	0·478	0·453
$\log(L/L_{\odot})$	5·743	5·800	5·846	5·882	5·912	5·959	5·993	6·019	6·040	6·056
$\log T_e$	5·006	4·911	4·896	4·877	4·863	4·832	4·799	4·765	4·730	4·700
$\log(X_C 10^{10})$	$\bar{2}$·225	0·380	0·653	0·981	1·104	1·366	1·610	1·808	2·025	2·263
τ (10^6 years)	0·082	1·453	2·474	3·211	3·772	4·582	5·109	5·470	5·734	5·919
$\log(R/R_{\odot})$	0·378	0·599	0·651	0·708	0·749	0·835	0·919	0·999	1·080	1·148
$T_c 10^{-8}$	1·809	1·221	1·210	1·184	1·197	1·214	1·230	1·258	1·290	1·349

TABLE 4
$M = 120\ M_{\odot}$

Characteristic	α_c									
	1·0	1·1	1·2	1·3	1·4	1·6	1·8	2·0	2·2	2·4
β_c	0·571	0·537	0·507	0·481	0·459	0·421	0·392	0·367	0·346	0·329
β_f	0·666	0·611	0·569	0·535	0·507	0·461	0·425	0·397	0·373	0·355
β_1	—	0·632	0·603	0·578	0·556	0·520	0·492	0·469	0·450	0·436
β_R	0·706	0·671	0·641	0·615	0·592	0·554	0·524	0·499	0·480	0·465
x_f	0·512	0·433	0·376	0·330	0·291	0·227	0·177	0·139	0·108	0·087
x_1	—	0·493	0·477	0·462	0·449	0·426	0·407	0·390	0·377	0·366
q_f	0·808	0·733	0·684	0·646	0·614	0·562	0·522	0·488	0·461	0·440
q_1	—	0·831	0·848	0·863	0·874	0·891	0·903	0·913	0·919	0·924
X_f	1·000	0·855	0·733	0·631	0·543	0·400	0·289	0·200	0·127	0·067
$\bar{X}$	1·000	0·888	0·800	0·729	0·671	0·581	0·517	0·469	0·432	0·405
$\log(L/L_{\odot})$	6·066	6·114	6·152	6·183	6·207	6·246	6·274	6·296	6·313	6·325
$\log T_e$	5·055	4·939	4·924	4·902	4·888	4·853	4·815	4·779	4·739	4·707
$\log(X_C 10^{10})$	$\bar{2}$·496	0·522	0·772	1·097	1·207	1·464	1·719	1·897	2·137	2·362
τ (10^6 years)	0·060	1·160	1·972	2·572	3·028	3·693	4·132	4·440	4·660	4·817
$\log(R/R_{\odot})$	0·444	0·698	0·749	0·807	0·848	0·936	1·027	1·110	1·199	1·269
$T_c 10^{-8}$	2·040	1·259	1·249	1·216	1·232	1·249	1·261	1·297	1·326	1·396

where

$$E = 3\mathscr{R}T/2\mu + aT^4/\rho,$$

reduces, after some lengthy calculations, to

$$l_e = \frac{4\pi M_\odot}{\mathscr{R}}\kappa_e \int_0^{x_f} \epsilon x^2 \frac{\bar{p}\beta}{\bar{t}}\, dx + \frac{6\pi M_\odot^2 \kappa_e}{\mu_e^2} \frac{d}{dt}\left(\frac{1}{R}\right) \int_0^1 x^2 \beta \bar{p}\, dx.$$

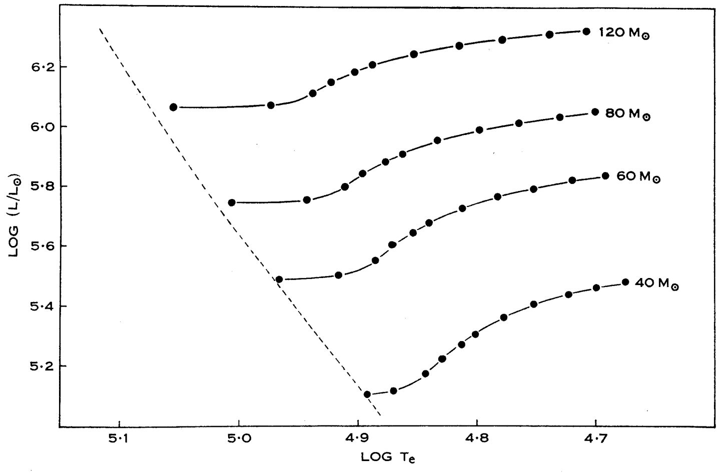

Fig. 1.—Hertzsprung-Russell diagram for massive stars initially composed of pure hydrogen, during their hydrogen burning phase. The dotted line indicates the main sequence for stars composed of pure hydrogen.

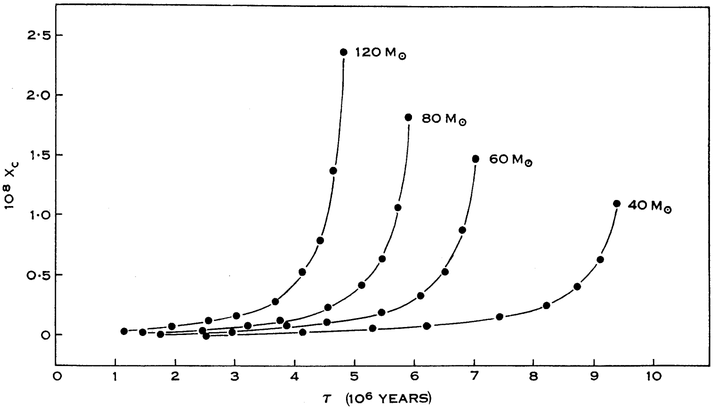

Fig. 2.—Variation of carbon content with time.

Since l_e, κ_e, x_f, μ_e, and the distribution of $\bar{m}$, $\bar{p}$, $\bar{t}$, and β are known for the various models, it is possible to use this equation to estimate the central temperature T_c, the effective temperature T_e, and the radius R at the end of the pre-main-sequence contraction.

These quantities are given in Tables 1 to 4 in the column $\alpha_c = 1 \cdot 0$.

We have also given the time τ taken by the stars to condense, under the assumption of homologous contraction, from an initial radius of 10^{13} cm.

If L_{pp}, L_{CNO}, and $L_{3\alpha}$ are the energy in ergs per second, liberated by the proton-proton, carbon, and triple-alpha cycles, the rate of change of the helium abundance was derived from

$$\frac{dY}{dt} = \left(\frac{L_{pp}}{E_{pp}} + \frac{L_{CNO}}{E_{CC}} - \frac{L_{3\alpha}}{E_{3\alpha}}\right) \bigg/ (q_f M),$$

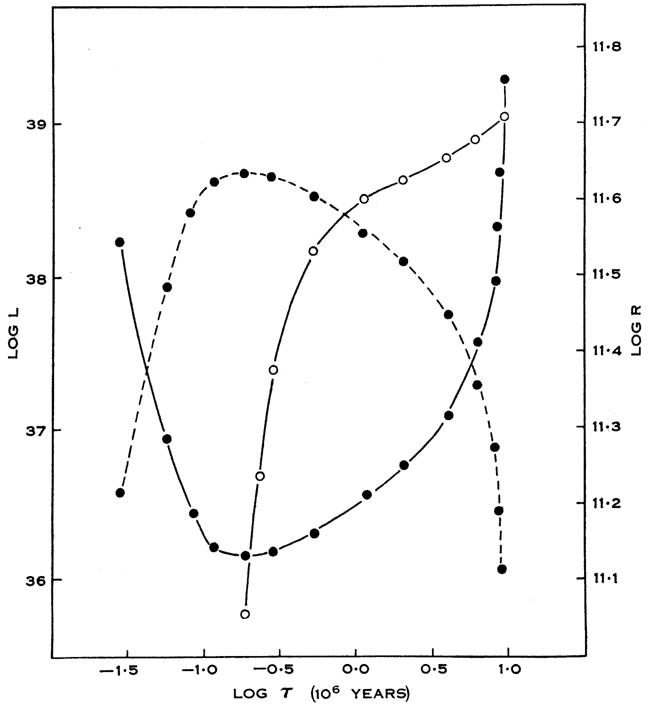

Fig. 3.—Variation with time in a star of mass 40 $M_\odot$ of:
the radius ——●——●——,
luminosity due to the proton-proton cycle - - - -●- - - -●- - - -,
luminosity due to the carbon cycle ——○——○——.

where (Schwarzschild 1958)

$$E_{pp} = 6 \cdot 3 \times 10^{18} \text{ erg/g},$$
$$E_{CC} = 6 \cdot 0 \times 10^{18} \text{ erg/g},$$
$$E_{3\alpha} = 6 \cdot 0 \times 10^{17} \text{ erg/g},$$

q_f being the fraction of mass in the core and M the mass of the star.

The rate of change of carbon abundance per unit mass was computed by the formula

$$\frac{dX_c}{dt} = \frac{L_{3\alpha}}{1 \cdot 17 \times 10^{-5}} \times 12 \times 1 \cdot 672 \times 10^{-24} / (q_f M).$$

The luminosities for the various nuclear reactions were obtained from the rates of energy generation as given by Ledoux (1961) (Van der Borght and Meggitt 1963).

At temperatures greater than $1 \cdot 2 \times 10^8$ the energy generation by the carbon cycle has practically no temperature sensitivity. In order to take this into account we have computed L_{CNO} at these temperatures by assuming that the rate of energy generation is equal to the one computed for a temperature of $1 \cdot 2 \times 10^8$.

IV. Evolution during the Hydrogen Burning Stage

The luminosity of the star during this stage is given by the condition of thermal equilibrium

$$\frac{l_{\mathrm{e}}}{\kappa_{\mathrm{e}}} = \frac{4\pi t_{\mathrm{c}}^3}{\mu T_{\mathrm{c}}^3} \int_0^{x_{\mathrm{f}}} x^2 \rho \epsilon \,\mathrm{d}x,$$

where $t_{\mathrm{c}} = \mu_{\mathrm{e}} R T_{\mathrm{e}}$, and ρ and x_{f} are known from the integrations in the second paragraph.

This formula can be used to derive the central temperature and hence the radii and effective temperatures of these stars for various values of α_{c}.

The age was computed from the formula

$$\Delta t = 6 \cdot 0 \times 10^{18} \Delta \overline{X} \, . \, M/L,$$

and the main results of these computations are also given in Tables 1 to 4.

The positions of the stars in the Hertzsprung-Russell diagram are given in Figure 1 and the variation of their carbon content with time in Figure 2.

In Figure 3 we have shown the variation with time of the radius and compared the contribution to the energy generation of the proton-proton and carbon cycles during the pre-main-sequence contraction and hydrogen burning stage of a star of mass $M = 40\ \mathrm{M}_{\odot}$.

V. References

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