GRAVITATION AND THE ABUNDANCE OF HYDROGEN

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[*Manuscript received December* 20, 1965]

Summary

The effect of gravitation on the physical constants has been used in an investigation of the formation of elements in the Universe. It has been shown that there should be an abundance of hydrogen.

I. Introduction

Many theories have been proposed for the explanation of different phenomena in cosmology, but little attention has been paid to the well-known abundance of hydrogen in the whole cosmos. The question that is posed is why should there be an excess of one element, when all elements are formed from the same fundamental particles? In the present paper we propose an explanation of this abundance of hydrogen.

The explanation is based upon the results of Infeld (1963)

$$\left.\begin{aligned} h &= h_0/g_{00} \\ \mu &= \mu_0/g_{00}, \end{aligned}\right\} \tag{1}$$

where h_0 and μ_0 are values of Planck's constant and the inertial mass at a place where $g_{00} = 1$ (the Earth can be assumed to be such a place). g_{00} is taken from the static metric

$$\mathrm{d}s^2 = g_{00}\,\mathrm{d}t^2 + g_{kl}\,\mathrm{d}x^k\,\mathrm{d}x^l,$$

where g_{00} and g_{kl} are independent of time.

The second consideration arises from the result of Durgapal and Gehlot (1965)

$$V' = g_{00}\,V\,, \tag{2}$$

where V' is the effective potential for a particle in a static gravitational field. This result was applied to collapsing stars. When the stars shrink and try to attain a minimum size, the value of g_{00} is reduced, giving a reduced potential barrier. Under their initial energy E (energy when $g_{00} \approx 1$) and the reduced barrier, the particles are released from the nucleus and become cosmic particles. We now consider the result (2) in relation to the formation of matter.

II. Case of an Initially Low Value of g_{00}

To investigate the formation of matter we must know the various energy levels of the atoms, the energies of the nucleons, and the potentials. These quantities must be evaluated for a place where the initial value of g_{00} is small.

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Aust. J. Phys., 1966, **19**, 531–4

(a) Energy States of an Atom

The energy of the nth state of an atom is given by

$$E_n = 2\pi^2 \mu e^4 Z^2 / h^2 n^2,$$

where Z is the proton number and e is the charge of an electron. Substituting the values of h and μ from (1), we obtain

$$E_n = g_{00}(2\pi^2 \mu_0 e^4 Z^2 / h_0^2 n^2)$$

$$= g_{00} \times E_n^{(0)}, \tag{3}$$

where $E_n^{(0)}$ is the energy of the nth state of an atom on Earth.

Hence, if the formation of an atom starts at a place where the value of g_{00} is small, then the energy levels in an atom will be reduced as well as the Coulomb potential (equation (2)). This reduction in energy together with potential allows the formation of an atom, though the electron may be loosely bound (thus increasing the size of the atom). The Bohr radius becomes

$$a = \hbar^2/\mu e^2 = \hbar_0^2/g_{00}\, \mu_0\, e^2 = a_0/g_{00}\,.$$

(b) Radius of the Nucleus

The Yukawa potential is given by

$$V = -g^2 \exp(-kr)/r\,,$$

where g is the strength of the nuclear source and k is a quantity having the dimension of $(\text{length})^{-1}$, with $1/k$ deciding the range of the nuclear forces and hence the radius of the nucleus. The value of k is given by

$$k = \mu\,.\,c/h\,,$$

where μ is the meson mass and c is the velocity of light. As the velocity of light is $g_{00}^{1/2}$ (Infeld 1963), using (1) we obtain

$$k = g_{00}^{1/2}\, \mu_0/h_0 = k_0\, g_{00}^{1/2}\,. \tag{4}$$

As the radius of the nucleus R is given by $1/k$, we have from (4)

$$R = R_0/g_{00}^{1/2}\,,$$

where R_0 is the nuclear radius on Earth. Hence the volume of the nucleus Ω is given by

$$\Omega = \Omega_0/g_{00}^{3/2}\,. \tag{5}$$

From equations (2) and (4) we see that under the changed value of g_{00} the Yukawa potential to be used in a Schrödinger equation becomes

$$V' = -g_{00} \times g^2 \exp(-k_0\, g_{00}^{1/2}\, r)/r\,. \tag{6}$$

(c) *Energy of the Nucleus*

Fermi (1951) has shown that energies of neutrons and protons in a nucleus are given by

$$\left.\begin{aligned} E_{\text{neutron}} &= \left\{\frac{\pi^2}{72}\right\}^{\frac{1}{3}} \times \frac{\Omega M}{\hbar^2}\left\{\frac{A-Z}{\Omega}\right\}^{\frac{2}{3}}(kT)^2, \\ E_{\text{proton}} &= \left\{\frac{\pi^2}{72}\right\}^{\frac{1}{3}} \times \frac{\Omega M}{\hbar^2}\left\{\frac{Z}{\Omega}\right\}^{\frac{2}{3}}(kT)^2, \end{aligned}\right\} \tag{7}$$

where k is the Boltzmann constant, T is the absolute temperature, Ω is the volume of the nucleus, A is the mass number, and Z is the proton number.

Substituting (1) and (5) in (7), we obtain

$$\left.\begin{aligned} E_{\text{neutron}} &= E^{(0)}_{\text{neutron}}, \\ E_{\text{proton}} &= E^{(0)}_{\text{proton}}, \end{aligned}\right\} \tag{8}$$

where superscript (0) corresponds to the value on Earth.

Also, the rotational energy levels of nuclei are given by

$$E = (\hbar^2/2I)J(J+1),$$

where I is the moment of inertia of the nucleus. As I depends upon the mass and the square of the radius (i.e. $I \propto MR^2$), we have the relation $I = I_0/g_{00}^2$. Hence

$$E = E^{(0)}$$

Thus the energies of neutrons and protons and also the energy of the nucleus remain constant irrespective of the value of g_{00}.

We can summarize the results of this section as:

(1) the Coulomb potential has the value $V' = g_{00}\, V$;

(2) the nth energy state of an atom is given by $E_n = g_{00} \times E_n^{(0)}$;

(3) with the Yukawa potential, the depth of the potential well decreases but the range increases;

(4) the radius of the nucleus is given by $R = R_0/g_{00}^{1/2}$.

(5) the energies of protons and neutrons inside the nucleus remain constant.

III. The Abundance of Hydrogen

Prior to the formation of elements in the Universe, matter must have existed in the form of elementary particles. If we consider these elementary particles to be confined in a spherical region, then a minimum size of the sphere is given by the Schwarzschild internal solution for spherical bodies, where the value of g_{00} is given by (Wyman 1946)

$$g_{00} = \{2a-2m+m(4a-m)r^2/2a^3\}^2 \times \{(2a+m)(1+mr^2/2a^3)\}^{-2},$$

which is zero at the centre of the sphere when $m = a$. As g_{00} cannot be negative, the minimum size of the sphere occurs when $m = a$.

We now have all elementary particles initially situated in a sphere whose mass is equal to its radius. Before we consider the formation of different nuclei and different atoms we must know the value of g_{00} at different points in the sphere. At $r = a$, $g_{00} \simeq 0 \cdot 1$; at $r = \frac{1}{2}a$, $g_{00} \simeq 0 \cdot 01$; between the centre and $r = \frac{1}{2}a$, the value of g_{00} is $< 0 \cdot 01$ and reduces to zero at the centre. Hence, within seven-eighths of the total volume of the sphere, the value of g_{00} lies between $0 \cdot 01$ and $0 \cdot 1$.

Now at places where $g_{00} < 0 \cdot 1$ the values of E_{neutron} and E_{proton} in a nucleus remain the same as those on Earth, but the depth of the potential well decreases to $< 0 \cdot 1 V$. Under such conditions protons and neutrons will not remain in a nucleus. Thus no formation of nuclei would be possible in the sphere of minimum size, i.e. when $m = a$.

However, for an atom, if the Coulomb potential is reduced then the energies of different states also decrease. Under this condition the electron cannot leave the potential well even though the binding is very weak. The formation of an atom is definitely possible. As the only nuclei available are protons, the formation of atoms of elements other than hydrogen is not possible. For values of $g_{00} < 0 \cdot 01$, even the hydrogen atom so formed will be virtually a free electron and a free proton, because the Bohr radius will be more than 50 a.u. and the ionization potential will be $< 0 \cdot 13$ eV. Hence we can consider formation of matter only in the region in which $g_{00} > 0 \cdot 01$. Thus about seven-eighths of the total volume of the Universe would be initially in the form of hydrogen atoms and neutrons, with the remainder existing as non-interacting neutrons, protons, and electrons.

Thus we see how the Universe would originally contain hydrogen as the only element.

The process of change of the Universe from the minimum-size sphere to the present state is very controversial, but it is definitely known that the value of g_{00} at present is nearly unity at many places. Thus during all this process of change, formation of other nuclei and hence of other elements became possible, but the abundance of hydrogen was maintained.

IV. Conclusion

An exact study of element formation can be made if we are able to formulate the correct metric for our Galaxy and for other nebulae. Once we have found the exact internal solutions for a disk-like body, we can gather extensive theoretical information about the core of our Galaxy and perhaps open the way to much new progress in the study of cosmology.

V. References

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