THE AKHIESER–POMERANCHUK–BLAIR MODEL ANALYSIS FOR ELASTIC SCATTERING OF DEUTERONS*

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Summary

Angular distributions of deuterons elastically scattered from Mg, Fe, Cu, and Zn have been analysed in terms of the smoothed Akhieser–Pomeranchuk–Blair model and reasonable agreement was obtained in each case up to about 90°. The interaction radius R and the surface thickness ΔR were obtained from the best-fit parameter values.

I. Introduction

The phenomenon of elastic scattering of a strongly absorbing particle can be described without detailed knowledge of the exact absorption mechanism. The small-angle diffraction model proposed by Akhieser and Pomeranchuk (1945) for the elastic scattering of high energy charged particles was extended by Blair (1954) to explain the experimental α-scattering data from a number of medium weight and heavy nuclei. The model is termed the Akhieser–Pomeranchuk–Blair (APB) model. In this semiclassical approach, the nuclear interaction zone is represented by a totally absorbing spherical region with a well-defined boundary. According to the model all incident waves up to a certain critical orbital angular momentum l_A are completely absorbed while those with higher momenta suffer scattering that is characteristic of the Coulomb potential of the target nucleus. These assumptions, however, underwent various modifications in subsequent years (McIntyre, Wang, and Becker 1960; Frahn and Venter 1963, 1964).

McIntyre, Wang, and Becker (1960) replaced the sharp cutoff assumption in angular momentum by a gradual transition from complete absorption to no absorption over a width Δl_A in l-space, corresponding to the diffuseness of the nuclear surface, and further introduced a real nuclear phase shift.

The APB model thus improved upon has been used in analyses of elastic scattering of α-particles and to some extent of heavy ions; the present authors applied the model to other projectiles such as deuterons (Sen Gupta, Rahman, and Khan 1967; henceforth referred to as paper I). Since these preliminary works gave quite

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encouraging results, further analyses were carried out on elastic scattering of deuterons. Results are presented here for scattering from Mg at 21·6 MeV, Fe at 11·8 and 13·6 MeV, Cu at 15·0 and 21·0 MeV, and Zn at 21·6 MeV. The various experimental data were taken from Yntema (1959), Cindro and Wall (1960), Gofman and Nemets (1961), Igo, Lorenz, and Schmidt-Rohr (1961), and Jolly, Lin, and Cohen (1963).

II. Theoretical Calculations

Details of the analyses and procedure of calculations have been given in paper I. The adjustable parameters of the model are l_A, Δl_A, δ, and Δl_δ, where l_A is the critical angular momentum, Δl_A defines the width of the transition region (as mentioned above), δ is the real nuclear phase shift, and Δl_δ carries a meaning similar to Δl_A. The parameters are adjusted systematically one after another in order to minimize the root-mean-square difference between the measured cross sections and the calculated ones at the respective angles.

Table 1

PARAMETERS OF THE APB MODEL

The parameters for the nuclei Ni, Zr, Ag, and Er are taken from paper I

Target Nucleus	Beam Energy (MeV)	Parameter l_A	Δl_A	δ (rad)	Δl_δ	R (fm)	ΔR (fm)
Mg	21·6	6·7	0·6	0·3	2·2	5·87	0·47
Fe	11·8	5·9	0·4	0		8·40	0·39
Fe	13·6	5·9	0·35	0		7·69	0·32
Ni	12·8	6·0	0·4	0·1	2·0	7·90	0·36
Cu	15·0	7·2	0·5	0		8·21	0·42
Cu	21·6	9·1	0·7	0·1	0·5	8·34	0·47
Zn	21·6	9·1	0·6	0·3	1·0	8·43	0·40
Zr	11·8	4·75	0·3	0·1	1·0	8·11	0·27
Ag	21·6	9·3	0·6	0·1	1·0	8·67	0·43
Er	15·0	7·0	0·5	0		10·44	0·38

III. Results

The best-fit parameter values of the model are given in Table 1 and the corresponding results are shown in Figures 1(*a*)–1(*c*). Agreement is reasonably good in all cases up to about 90° and in some cases beyond this angle. The predicted curves at large angles show a well-defined diffraction oscillation in contrast to the rather damped structure of the experimental angular distribution. This disagreement should not, however, be counted against the APB model since this is a small-angle approximation and is expected to be particularly valid in medium weight and heavy nuclei (Blair 1954; Khan, Rahman, and Sen Gupta 1967). Further, the deuteron data at large angles (i.e. for small impact parameters) are somewhat difficult to analyse in view of the loosely bound structure of the deuteron.

IV. Discussion

From the parameters l_A and Δl_A the interaction radius R and the surface diffuseness ΔR were obtained respectively from the semiclassical relations given by McIntyre, Baker, and Wang (1962), namely

$$l_A(l_A+1) = kR(kR-2\eta) \tag{1}$$

and

$$(2l_A+1)\Delta l_A = 2kR(kR-\eta)\Delta R/R\,, \tag{2}$$

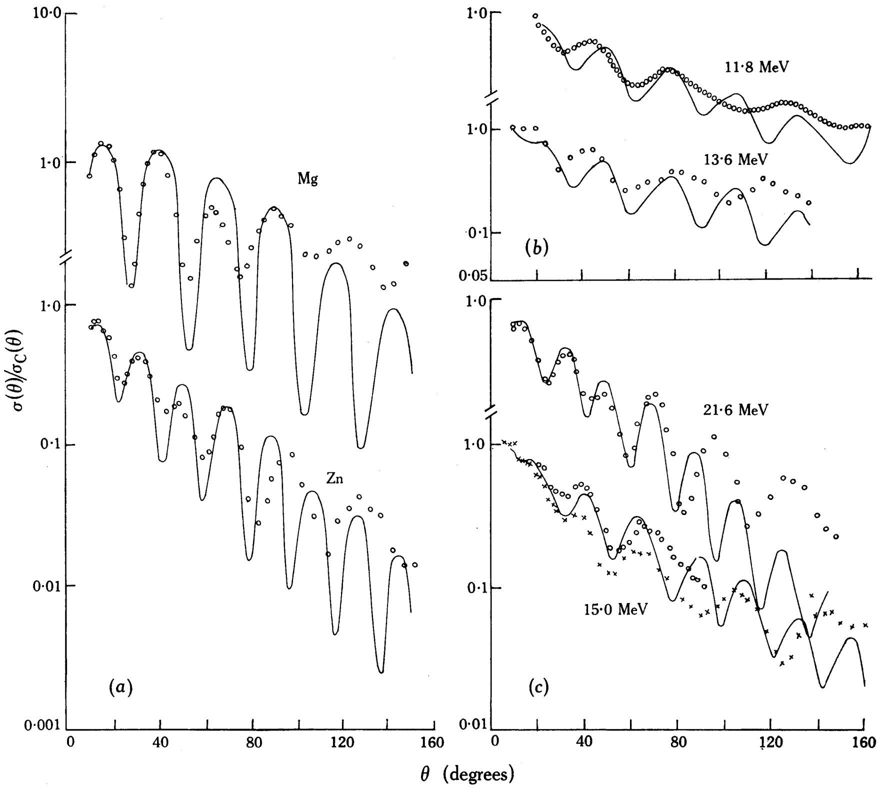

Fig. 1.—Ratio of differential cross section $\sigma(\theta)$ to Coulomb cross section $\sigma_C(\theta)$ plotted against the c.m. angle θ for experimental points and predicted curves from an APB model analysis of elastic scattering of (*a*) 21·6 MeV deuterons from Mg and Zn, (*b*) 11·8 and 13·6 MeV deuterons from Fe, and (*c*) 15·0 and 21·6 MeV deuterons from Cu.

where k is the c.m. wave number of the system and η is the Coulomb parameter. These quantities were calculated also for the nuclei Ni, Zr, Ag, and Er from the parameter values given in paper I and are included in Table 1. Figure 2 gives a plot of R against $A^{\frac{1}{3}}$, using the values obtained from equation (1). The errors shown in this figure are not exact but are estimated from the range in which l_A appears to lie; the straight line is the least-squares fit given by the relation

$$R = r_0 A^{\frac{1}{3}} + R_{\mathrm{d}}\,,$$

with $r_0 = 1 \cdot 53$ fm and $R_{\mathrm{d}} = 1 \cdot 84$ fm, which are quite reasonable values. The surface diffuseness obtained for the nuclei studied lies around $0 \cdot 4$ fm, being slightly less than that given by the optical model. This difference may well be attributed to the slightly different ways in which ΔR is defined in the two models. The value of δ is small in all cases whereas Δl_δ does not show any systematic change with target variation (the effect of this parameter is small, however; see paper I).

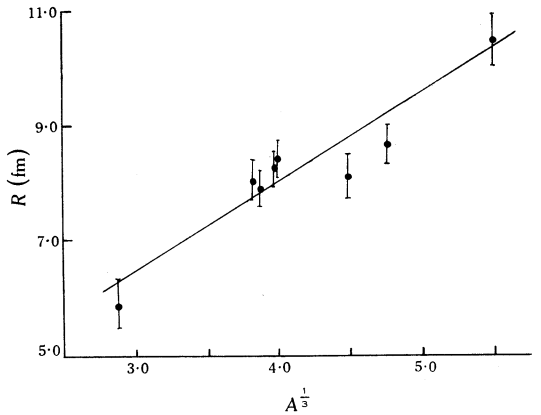

Fig. 2.—Plot of interaction radius R against $A^{\frac{1}{3}}$. The straight line is the least-squares fit given by the relation

$$R = 1 \cdot 53\, A^{\frac{1}{3}} + 1 \cdot 84 .$$

We conclude by saying that the APB model, originally proposed for α-particles, can equally describe the elastic scattering process of deuterons and the parameters of the model seem to be more or less uniquely given. Agreement with this four-parameter model (sometimes only two parameters l_A and Δl_A are sufficient; Khan, Rahman, and Sen Gupta 1967) is of comparable quality to that of the complex potential optical model. In addition to the fact that deuterons are likely to be lost in the elastic channel as a result of being broken up, particularly for large-angle scattering, reflection of deuterons from the nuclear interaction zone (i.e. from the so-called "illuminated" part of the nuclei) should also be incorporated in the scattering process; this is expected to fill up the valleys in the backward-angle diffraction oscillations, as suggested by Dar (1964).

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VI. References

Akhieser, A., and Pomeranchuk, I. (1945).—*Fiz. Zh.* **9**, 471.
Blair, J. S. (1954).—*Phys. Rev.* **95**, 1218.
Cindro, N., and Wall, N. S. (1960).—*Phys. Rev.* **119**, 1340.
Dar, A. (1964).—*Nucl. Phys.* **55**, 305.
Frahn, W. E., and Venter, R. H. (1963).—*Ann. Phys.* **24**, 243.
Frahn, W. E., and Venter, R. H. (1964).—*Ann. Phys.* **27**, 135.

Gofman, Y. V., and Nemets, O. F. (1961).—*Soviet Phys. JETP* **12,** 1035.
Igo, G., Lorenz, W., and Schmidt-Rohr, U. (1961).—*Phys. Rev.* **124,** 832.
Jolly, R. K., Lin, E. K., and Cohen, B. L. (1963).—*Phys. Rev.* **130,** 2391.
Khan, A. H., Rahman, M., and Sen Gupta, H. M. (1967).—*Nuovo Cim.* B **48,** 13.
McIntyre, J. A., Baker, S. D., and Wang, K. H. (1962).—*Phys. Rev.* **125,** 584.
McIntyre, J. A., Wang, K. H., and Becker, L. C. (1960).—*Phys. Rev.* **117,** 1337.
Sen Gupta, H. M., Rahman, M., and Khan, A. H. (1967).—*Aust. J. Phys.* **20,** 265.
Yntema, J. L. (1959).—*Phys. Rev.* **113,** 261.