EFFECT OF HORIZONTAL AND VERTICAL MAGNETIC FIELDS ON RAYLEIGH–TAYLOR INSTABILITY

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Summary

A general equation studying the combined effect of horizontal and vertical magnetic fields on the stability of two superposed fluids has been obtained. The unstable and stable cases at the interface ($z = 0$) between two uniform fluids, with both the possibilities of real and complex n, have been separately dealt with. Some new results are obtained. In the unstable case with real n, the perturbations are damped or unstable according as $2(k^2 - k_x^2 L^2) - (\alpha_2 - \alpha_1)k$ is $>$ or $<$ 0 under the physical situation (35). In the stable case, the perturbations are stable or unstable according as $2(k^2 - k_x^2 L^2) + (\alpha_1 - \alpha_2)k$ is $>$ or $<$ 0 under the same physical situation (35). The perturbations become unstable if $H_{\|}/H_{\perp}$ ($= L$) is large. Both the cases are also discussed with imaginary n.

I. INTRODUCTION

Hide (1955) considered the effect of a vertical magnetic field on the stability of two superposed fluids, while the effect of a horizontal magnetic field on the Rayleigh–Taylor instability was considered by Kruskal and Schwarzschild (1954). The object of the present paper is to study the combined effect of horizontal and vertical magnetic fields on the Rayleigh–Taylor instability.

After obtaining an equation that describes the effect of the magnetic fields we then suppose that two uniform fluids, of densities ρ_1 and ρ_2, are separated by a horizontal boundary at $z = 0$. The unstable and stable cases for both real and imaginary n are then separately dealt with and discussed.

II. BASIC EQUATIONS

The fluid is considered to be heterogeneous, inviscid, and of zero resistivity. The equations of motion and continuity are

$$\rho\, \mathrm{d}\boldsymbol{q}/\mathrm{d}t = -\nabla p - \rho \boldsymbol{g} + \mu \boldsymbol{j} \times \boldsymbol{H}\,; \tag{1}$$

where $\boldsymbol{q} = (u, v, w)$ is the velocity vector, p the pressure, μ the magnetic permeability, and $\boldsymbol{g}$ the acceleration due to gravity; and

$$\nabla . \boldsymbol{q} = 0\,, \tag{2}$$

as the fluid is considered to be incompressible.

Since the density of a particle moving with the fluid remains constant,

$$\partial\rho/\partial t + (\boldsymbol{q} . \nabla)\rho = 0\,. \tag{3}$$

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Using Maxwell's equations for a perfect conductor ($\eta = 0$),

$$\partial H/\partial t = \nabla \times (\boldsymbol{q} \times \boldsymbol{H}). \tag{4}$$

Let the actual density at any point due to a disturbance be $\rho + \delta\rho$ and let δp denote the corresponding increment in pressure. Further, if $H_\perp$ and $H_\parallel$ denote the vertical and horizontal magnetic fields respectively and $\boldsymbol{h} = (h_x, h_y, h_z)$ is the perturbation in H, we have

$$\rho \frac{\partial u}{\partial t} - \frac{\mu H_\perp}{4\pi}\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right) = -\frac{\partial}{\partial x}.\delta p, \tag{5}$$

$$\rho \frac{\partial v}{\partial t} - \frac{\mu H_\perp}{4\pi}\left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right) - \frac{\mu H_\parallel}{4\pi}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right) = -\frac{\partial}{\partial y}.\delta p, \tag{6}$$

$$\rho \frac{\partial w}{\partial t} + \frac{\mu H_\parallel}{4\pi}\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right) = -\frac{\partial}{\partial z}.\delta p - g.\delta\rho, \tag{7}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \qquad \frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \tag{8}$$

$$\frac{\partial}{\partial t}.\delta\rho = -w\frac{\mathrm{d}\rho}{\mathrm{d}z}, \tag{9}$$

and

$$\frac{\partial h}{\partial t} = \left(H_\parallel . \frac{\partial}{\partial x} + H_\perp . \frac{\partial}{\partial z}\right)\boldsymbol{q}. \tag{10}$$

Analysing the disturbances into normal modes, we seek solutions whose dependence on x, y, and t is given by

$$\exp(\mathrm{i}k_x.x + \mathrm{i}k_y.y + n.t), \tag{11}$$

where k_x, k_y, and n are constants, k_x being the wave number along the x direction, k_y the wave number along the y direction, and k the resultant wave number. Using this perturbation, equations (5)–(10) become

$$\rho n u - (\mu H_\perp/4\pi)(\mathrm{D}h_x - \mathrm{i}k_x.h_z) = -\mathrm{i}k_x.\delta p, \tag{12}$$

$$\rho n v - (\mu \mathrm{H}_\perp/4\pi)(\mathrm{D}h_y - \mathrm{i}k_y.h_z) - (\mu H_\parallel/4\pi)(ik_x.h_y - \mathrm{i}k_y.h_x) = -\mathrm{i}k_y.\delta p, \tag{13}$$

$$\rho n w + (\mu H_\parallel/4\pi)(\mathrm{D}h_x - ik_x.h_z) = -\mathrm{D}.\delta p + (g/n)(\mathrm{D}\rho)w \tag{14}$$

(on substituting for $\delta\rho$ from (9)),

$$\mathrm{i}k_x.u + \mathrm{i}k_y.v = -\mathrm{D}w, \qquad \mathrm{i}k_x.h_x + \mathrm{i}k_y.h_y = -\mathrm{D}h_z, \tag{15}$$

and

$$h_x = (H_\perp/n)\mathrm{D}u + (H_\parallel/n)\mathrm{i}k_x.u, \tag{16a}$$

$$h_y = (H_\perp/n)\mathrm{D}v + (H_\parallel/n)\mathrm{i}k_x.v, \tag{16b}$$

$$h_z = (H_\perp/n)\mathrm{D}w + (H_\parallel/n)\mathrm{i}k_x.w. \tag{16c}$$

Multiplying (12) by $-\mathrm{i}k_x$ and (13) by $-\mathrm{i}k_y$ and adding, we get

$$\rho \mathrm{D}w - \frac{\mu H_\perp}{4\pi n}(\mathrm{D}^2 - k^2)h_z + \frac{\mu H_\parallel}{4\pi n}k_y(k_y.h_x - k_x.h_y) = -\frac{k^2}{n}\delta p. \tag{17}$$

Substituting the values of h_x, h_y, and h_z, equation (17) becomes

$$\rho \mathrm{D}w + \mathrm{i}\frac{\mu H_{\parallel} H_{\perp}}{4\pi n^2}\left(k_y.\mathrm{D}\zeta - k_x(\mathrm{D}^2-k^2)w\right) - \frac{\mu H_{\perp}^2}{4\pi n^2}(\mathrm{D}^2-k^2)\mathrm{D}w - \frac{\mu H_{\parallel}^2}{4\pi n^2}k_x k_y \zeta = -\frac{k^2}{n}\delta p\,, \tag{18}$$

where ζ, the z component of vorticity, is given by

$$\zeta = \partial v/\partial x - \partial u/\partial y = \mathrm{i}k_x.v - \mathrm{i}k_y.u\,. \tag{19}$$

Since equation (19) and $-\mathrm{D}w = \mathrm{i}k_x.u + \mathrm{i}k_y.v$ hold,

$$u = k^{-2}(\mathrm{i}k_y.\zeta + \mathrm{i}k_x.\mathrm{D}w)\,.$$

Again, from equation (16a)

$$h_x = \left(\frac{H_{\perp}}{n}\mathrm{D} + \frac{\mathrm{i}k_x}{n}H_{\parallel}\right)u,$$

so that

$$\mathrm{D}h_x = \frac{D}{k^2}\left(\frac{H_{\perp}}{n}\mathrm{D} + \frac{\mathrm{i}k_x}{n}H_{\parallel}\right)\left(\mathrm{i}k_y.\zeta + \mathrm{i}k_x.\mathrm{D}w\right), \tag{20}$$

which is obtained on substituting the value of u. Eliminating δp between (14) and (18), we obtain

$$\{\mathrm{D}(\rho \mathrm{D}w) - k^2\rho w\} - 2\mathrm{i}k_x.\frac{\mu H_{\parallel} H_{\perp}}{4\pi n^2}(\mathrm{D}^2-k^2)\mathrm{D}w - \frac{\mu H_{\perp}^2}{4\pi n^2}(\mathrm{D}^2-k^2)\mathrm{D}^2 w + \frac{\mu H_{\parallel}^2 . k_x^2}{4\pi n^2}(\mathrm{D}^2-k^2)w = -\frac{gk^2}{n^2}(\mathrm{D}\rho)w\,. \tag{21}$$

Equation (21) is thus a general equation formulating the effects of both the horizontal and vertical magnetic fields on the Rayleigh–Taylor instability. If we put $H_{\parallel} = 0$ in (21) we get an equation that is the same as the corresponding equation in the presence of a vertical magnetic field, as given in Chandrasekhar (1961, p. 458). Also, if we put $H_{\perp} = 0$ in (21) we get the corresponding equation in the presence of a horizontal magnetic field (Chandrasekhar 1961, p. 465).

III. Boundary Conditions

We suppose that the two uniform and perfectly conducting fluids are separated by a horizontal boundary at $z = 0$. Then, at the interface,

$$w \text{ and } h_z \text{ are continuous}, \tag{22}$$

and from equation (16c), the continuities of w and h_z imply that

$$\mathrm{D}w \text{ is continuous}. \tag{23}$$

Also at the interface between two uniform fluids $e^{\pm kz}$ is a solution of equation (21). Since w and $\mathrm{D}w$ are continuous at the interface, we infer from equation (21) that

$$\mathrm{D}^2 w \text{ is also continuous}. \tag{24}$$

Integrating equation (21) across the interface, a further boundary condition can be obtained, namely

$$\Delta_s\left(\rho\,Dw-\frac{\mu H_\perp^2}{4\pi n^2}(D^2-k^2)Dw+\frac{\mu H_\parallel^2 . k_x^2}{4\pi n^2}Dw-2ik_x.\frac{\mu H_\perp H_\parallel}{4\pi n^2}(D^2-k^2)w\right)$$

$$=-\frac{gk^2}{n^2}\Delta_s(\rho)\,w_s\,. \tag{25}$$

IV. At the Interface between Two Uniform Fluids

We suppose that the two uniform fluids of densities ρ_1 and ρ_2 are separated by a horizontal boundary at $z=0$ and define a dimensionless parameter $L=H_\parallel/H_\perp$, so that equation (21) reduces to

$$(D^2-k^2)D^2w+2ik_x.L(D^2-k^2)Dw-(4\pi\rho n^2/\mu H_\perp^2+k_x^2L^2)(D^2-k^2)w=0\,. \tag{26}$$

The solution of (26) is a linear combination of $e^{\pm kz}$ and $e^{\pm qz}$, where

$$(q+ik_x L)^2=4\pi\rho n^2/\mu H_\perp^2\,.$$

We now consider the unstable and stable cases separately.

(a) *Unstable Case*

If n is real then the unstable case requires $n^2>0$, and if n is complex it is supposed that the real part of n^2 is positive. Further we assume that

$$q=-ik_x L+n(4\pi\rho/\mu H_\perp^2)^{\frac{1}{2}} \qquad \text{and} \qquad \mathrm{Re}(n)>0\,.$$

Since w must be bounded when $z\to+\infty$ (in the upper fluid) and $z\to-\infty$ (in the lower fluid), the solutions of equation (26) can be written as

$$w_1=A_1\exp(+kz)+B_1\exp(+q_1z)\,, \qquad z<0\,, \tag{27a}$$

$$w_2=A_2\exp(-kz)+B_2\exp(-q_2z) \qquad z>0\,, \tag{27b}$$

where A_1, B_1, A_2, and B_2 are constants of integration,

$$q_1+ik_xL=n(4\pi\rho_1/\mu H_\perp^2)^{\frac{1}{2}}\,, \qquad \text{and} \qquad q_2+ik_xL=n(4\pi\rho_2/\mu H_\perp^2)^{\frac{1}{2}}\,. \tag{28}$$

In writing the solutions for w in the two regions $z<0$ and $z>0$ in the manner (27), we have assumed that q_1 and q_2 are so defined that their real parts are positive.

Using the boundary conditions (22)–(25) and substituting for w_1 and w_2 from (27), at the interface $z=0$ we get an equation in q_1 and q_2 of the form

$$\Delta(q_1,q_2)=\begin{vmatrix} 1 & 1 & -1 & -1 \\ k & q_1 & +k & +q_2 \\ k^2 & q_1^2 & -k^2 & -q_2^2 \\ C & D & E & F \end{vmatrix}=0\,,$$

where

$$
\begin{aligned}
C &= \tfrac{1}{2}R-\alpha_1-\alpha_1 k_x^2 L^2/(q_1+\mathrm{i}k_x L)^2,\\
D &= \tfrac{1}{2}R-\alpha_1(q_1/k)+\frac{\alpha_1(q_1/k)(q_1^2-k_1)}{(q_1+\mathrm{i}k_x L)^2}-\frac{\alpha_1(q_1/k)k_x^2 L^2}{(q_1+\mathrm{i}k_x L)^2}+\frac{\alpha_1 2\mathrm{i}k_x L(q_1^2-k^2)/k}{(q_1+\mathrm{i}k_x L)^2},\\
E &= \tfrac{1}{2}R-\alpha_2-\alpha_2 k_x^2 L^2/(q_2+\mathrm{i}k_x L)^2,\\
F &= \tfrac{1}{2}R-\alpha_2(q_2/k)+\frac{\alpha_2(q_2/k)(q_2^2-k^2)}{(q_2+\mathrm{i}k_x L)^2}-\frac{\alpha_2(q_2/k)k_x^2 L^2}{(q_2+\mathrm{i}k_x L)^2}-\frac{\alpha_2 2\mathrm{i}k_x L(q_2^2-k^2)/k}{(q_2+\mathrm{i}k_x L)^2},
\end{aligned}
$$

with

$$\alpha_1=\frac{\rho_1}{\rho_1+\rho_2},\qquad \alpha_2=\frac{\rho_2}{\rho_1+\rho_2},\qquad \text{and}\qquad R=\frac{gk}{n^2}(\alpha_2-\alpha_1).$$

The determinant is solved by removing the factors q_1-k and q_2-k (since these become identically zero on substitution of the functions w_1 and w_2, giving the characteristic roots $q_1 = k$ and $q_2 = k$) and expanding the remaining determinant to obtain

$$
\begin{aligned}
&\left\{R-1-L^2k_x^2\left(\frac{\alpha_1}{(q_1+\mathrm{i}k_x L)^2}+\frac{\alpha_2}{(q_2+\mathrm{i}k_x L)^2}\right)\right\}\left(q_1+q_2+2k\right)\\
&\qquad = 2k\left(\frac{\alpha_1 q_1+2\mathrm{i}k_x L\alpha_1}{(q_1+\mathrm{i}k_x L)^2}(q_2+k)+\frac{\alpha_2 q_2-2\mathrm{i}k_x L(\alpha_2/k)(2q_2+k)}{(q_2+\mathrm{i}k_x L)^2}(q_1+k)\right). \qquad (29)
\end{aligned}
$$

We define the Alfven velocity $V_{\mathrm{A}} = \{\mu H_\perp^2/4\pi(\rho_1+\rho_2)\}^{\frac{1}{2}}$, so that

$$q_1 = -\mathrm{i}k_x L+(n/V_{\mathrm{A}})\alpha_1^{\frac{1}{2}} \qquad \text{and} \qquad q_2 = -\mathrm{i}k_x L+(n/V_{\mathrm{A}})\alpha_2^{\frac{1}{2}}.$$

Substituting for q_1, q_2, and R we obtain

$$
\begin{aligned}
&\frac{gk}{n^2}\left(\alpha_2-\alpha_1\right)\left(\frac{n}{kV_{\mathrm{A}}}+\frac{2}{\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}}}\right)=\frac{n}{kV_{\mathrm{A}}}+2(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})+\frac{2kV_{\mathrm{A}}}{n}+\frac{2}{k}k_x^2L^2\frac{V_{\mathrm{A}}}{n}-4\mathrm{i}k_x L\frac{V_{\mathrm{A}}}{n}\\
&\quad-\frac{2\mathrm{i}k_x L}{k(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})}-\frac{4\mathrm{i}k_x L(k^2+k_x^2L^2)}{k(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})}\frac{V_{\mathrm{A}}^2}{n^2}+\frac{2\mathrm{i}k_x L}{k(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})}\frac{gk}{n^2}(\alpha_2-\alpha_1)-4\mathrm{i}k_x L\frac{V_{\mathrm{A}}}{n}\frac{\alpha_1^{\frac{1}{2}}-\alpha_2^{\frac{1}{2}}}{\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}}}\\
&\quad+\frac{8\mathrm{i}k_x L}{k(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})}\left(\frac{k_x^2V_{\mathrm{A}}^2L^2}{n^2}+\mathrm{i}k_x L(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})\frac{V_{\mathrm{A}}}{n}-\alpha_1^{\frac{1}{2}}\alpha_2^{\frac{1}{2}}+\mathrm{i}k_x Lk\frac{V_{\mathrm{A}}^2}{n^2}-k\alpha_2^{\frac{1}{2}}\frac{V_{\mathrm{A}}}{n}\right), \qquad (30)
\end{aligned}
$$

measuring n and k in the units $(g/V_{\mathrm{A}})\,\mathrm{sec}^{-1}$ and $(g/V_{\mathrm{A}}^2)\,\mathrm{cm}^{-1}$ respectively.

Equation (30) in nondimensional form reduces to

$$
\begin{aligned}
&n^3+\left(2k(\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}})-\frac{2\mathrm{i}k_x L}{\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}}}(1+4\alpha_1^{\frac{1}{2}}\alpha_2^{\frac{1}{2}})\right)n^2+k\left(2k+\alpha_1-\alpha_2-\frac{6}{k}k_x^2L^2-8\mathrm{i}k_x L\right)n\\
&\qquad+2k^2(\alpha_1^{\frac{1}{2}}-\alpha_2^{\frac{1}{2}})-\frac{4\mathrm{i}k_x L(k^2+k_x^2L^2)}{\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}}}-2\mathrm{i}k_x L(\alpha_1^{\frac{1}{2}}-\alpha_2^{\frac{1}{2}})k\\
&\qquad+\frac{8\mathrm{i}k_x L}{\alpha_1^{\frac{1}{2}}+\alpha_2^{\frac{1}{2}}}(k_x^2L^2+\mathrm{i}k_x Lk)=0. \qquad (31)
\end{aligned}
$$

This is the general equation for combined horizontal and vertical magnetic fields.

(i) *Real n*

In this case $\alpha_2 > \alpha_1$. If n is real then separating real and imaginary parts of equation (31) we get

$$n^3+2k(\alpha_1^{1/2}+\alpha_2^{1/2})n^2+k\left(2k+\alpha_1-\alpha_2-\frac{6}{k}k_x^2\,L^2\right)n-\frac{8k_x^2L^2k}{\alpha_1^{1/2}+\alpha_2^{1/2}}+2k^2(\alpha_1^{1/2}-\alpha_2^{1/2}) = 0, \quad (32)$$

and

$$(1+4\alpha_1^{1/2}\alpha_2^{1/2})n^2+4k(\alpha_1^{1/2}+\alpha_2^{1/2})n+\{2(k^2-k_x^2\,L^2)+(\alpha_1-\alpha_2)k\} = 0. \quad (33)$$

On solving equation (33) we get

$$n = -2k(\alpha_1^{1/2}+\alpha_2^{1/2})\pm[4k^2(\alpha_1^{1/2}+\alpha_2^{1/2})^2-(1+4\alpha_1^{1/2}\,\alpha_2^{1/2})\{2(k^2-k_x^2\,L^2)+(\alpha_1-\alpha_2)k\}]^{\frac{1}{2}}. \quad (34)$$

If the term in square brackets is denoted by T then equation (34) gives the characteristic value of n under the condition

$$f(k) \equiv \pm T^{3/2}-4k(\alpha_1^{1/2}+\alpha_2^{1/2})T\pm\{4k^2(\alpha_1^{1/2}+\alpha_2^{1/2})^2+2k^2+k(\alpha_1-\alpha_2)-6k_x^2\,L^2\}T^{\frac{1}{2}}$$
$$-\left\{2k^2(\alpha_1^{1/2}+\alpha_2^{1/2})\left(2k+\alpha_1-\alpha_2-\frac{6}{k}k_x^2\,L^2\right)-2k^2(\alpha_1^{1/2}-\alpha_2^{1/2})+\frac{8k_x^2\,L^2\,k}{\alpha_1^{1/2}+\alpha_2^{1/2}}\right\} = 0. \quad (35)$$

For n to be real

$$4k^2(\alpha_1^{1/2}+\alpha_2^{1/2})^2-(1+4\alpha_1^{1/2}\,\alpha_2^{1/2})\{2(k^2-k_x^2\,L^2)-(\alpha_2-\alpha_1)k\}$$

must be positive. The perturbations are damped or unstable according as

$$2(k^2-k_x^2\,L^2)-(\alpha_2-\alpha_1)k \qquad \text{is} \qquad > \text{ or } < 0$$

under the condition (35).

(ii) *Complex n*

We suppose that $n = \alpha\pm i\beta$, where α and β are real. Substituting for n in equation (31) and equating real and imaginary parts we obtain,

$$(\alpha^3-3\alpha\beta^2)+\left(2k(\alpha_1^{1/2}+\alpha_2^{1/2})(\alpha^2-\beta^2)+\frac{4k_x\,L(1+4\alpha_1^{1/2}\,\alpha_2^{1/2})}{\alpha_1^{1/2}+\alpha_2^{1/2}}\alpha\beta\right)$$
$$+k\left(2k+\alpha_1-\alpha_2-\frac{6}{k}k_x^2\,L^2+8k_x\,L\,k\beta\right)+2k^2(\alpha_1^{1/2}-\alpha_2^{1/2})-\frac{8k_x^2\,L^2\,k}{\alpha_1^{1/2}+\alpha_2^{1/2}} = 0 \quad (36)$$

and

$$(3\alpha^2\beta-\beta^3)+\left(2k(\alpha_1^{1/2}+\alpha_2^{1/2})2\alpha\beta+\frac{2k_x\,L}{\alpha_1^{1/2}+\alpha_2^{1/2}}(\beta^2-\alpha^2)(1+4\alpha_1^{1/2}\,\alpha_2^{1/2})\right)+\left\{k\beta\left(2k+\alpha_1-\alpha_2-\frac{6}{k}k_x^2\,L^2\right)\right.$$
$$\left.-8k_x\,L\,k\alpha\right\}-\frac{4k_x\,L}{\alpha_1^{1/2}+\alpha_2^{1/2}}(k^2+k_x^2\,L^2)-2k_x\,L\,k(\alpha_1^{1/2}-\alpha_2^{1/2})+\frac{8k_x\,L}{\alpha_1^{1/2}+\alpha_2^{1/2}}k_x^2\,L^2 = 0. \quad (37)$$

Considering the cases $\alpha >, =$, or $< \sqrt{3}\,\beta$, since $\alpha_2 > \alpha_1$ we conclude that, however α and β are related, equation (36) must allow at least one change of sign and hence one positive root, showing that the equilibrium is unstable.

(b) *Stable Case*

(i) *Real n*

In this case $\alpha_2 < \alpha_1$. Again we consider equations (34) and (35). Here the perturbations are stable or unstable according as

$$2(k^2 - k_x^2 L^2) + (\alpha_1 - \alpha_2)k \quad \text{is} \quad > \text{ or } < 0$$

under the physical situation (35). The perturbations become unstable if $H_{\parallel}/H_{\perp} = L$ is large.

(ii) *Complex n*

Consider the three cases $\alpha >, =,$ or $< \sqrt{3}\beta$ in equation (36). If $\alpha \geqslant \sqrt{3}\beta$, equation (36) does not possess any change of sign and hence the equilibrium is stable. Thus $\alpha \geqslant \sqrt{3}\beta$ is the condition for stable equilibrium.

V. References

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