FINITE REST MASSES OF WAVE QUANTA IN MATERIAL MEDIA

By K. D. Cole*

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Abstract

The equivalence of a dispersion relationship and Einstein's mass–energy relationship leads to the specification of a particle in a vacuum which is equivalent to a "photon" in a medium. The invariance of the rest mass of this particle leads to a formula for the Doppler effect which is good for all forms of waves whose quanta can be described by $E = \hbar\omega$ and $\boldsymbol{p} = \hbar\boldsymbol{k}$. Applying dynamical equations to the equivalent particle in the case of a radiofrequency photon in a plasma around a star, a new gravitational redshift formula is deduced which reduces to the well-known expression in the appropriate limit. A new form of bending of photon trajectories in a gravitational field is also described. At frequencies near the plasma frequency ν_{p} the bending is $\nu_{\mathrm{p}}/4(\nu-\nu_{\mathrm{p}})$ times that for light in a vacuum.

I. Introduction

For the purpose of this paper the word photon will be used in a general sense to connote a quantum of any form of wave energy which can be described by the two equations

$$E = \hbar\omega\,, \qquad p = \hbar k\,, \tag{1,2}$$

where E and p are the energy and momentum of the quantum respectively, ω is the angular frequency, k is the wave number, and $\hbar$ is the usual Planck's constant divided by 2π. From the equations (1) and (2)

$$E = c_{\mathrm{ph}}\,p\,, \tag{3}$$

where c_{ph} is the phase velocity given by

$$c_{\mathrm{ph}} = \omega/k\,. \tag{4}$$

By the theory of special relativity the relationship

$$E^2 = p^2c^2 + m_{\mathrm{p}}^2\,c^4 \tag{5}$$

is valid for a particle of rest mass m_{p} (c being the speed of light).

The present paper is concerned with some of the consequences of asserting that the relations (3) and (5) may be used to describe the same quantum. In the first place it is noted that (3) contains information about the medium, whereas (5), as normally used, contains no such information and refers to a particle without reference to other matter. Asserting the equivalence of (3) and (5) means that a photon in a medium may be considered as a particle in a vacuum and that this

* Division of Space Physics, La Trobe University, Bundoora, Vic. 3083.

particle has a "rest mass" derived from the bulk propagation characteristics of the medium through which it moves. It is found in the present paper that this approach leads to the conventional expressions for the Doppler effect and the gravitational redshift, thus giving some confidence in this fundamental assertion. The assertion is shown here to predict effects which are as yet not tested by experiment. With the equivalence of (3) and (5) assumed and the implication of "rest mass" as stated above, the inverted commas are omitted hereinafter.

It follows from the equivalence of (3) and (5) that a photon, be it an electromagnetic one in a plasma, a sound one in an elastic medium, or any other "photon" describable by (1) and (2), may be considered to be a particle whose rest mass is

$$m_{\mathrm{p}} = pc^{-2}(c_{\mathrm{ph}}^2 - c^2)^{\frac{1}{2}} = pc^{-1}(n^{-2}-1)^{\frac{1}{2}} \tag{6a}$$

$$= \hbar\omega c^{-1}(c^{-2}-c_{\mathrm{ph}}^{-2})^{\frac{1}{2}} = \hbar\omega c^{-2}(1-n^2)^{\frac{1}{2}}, \tag{6b}$$

where $n = c/c_{\mathrm{ph}}$ is the refractive index. For a detectable photon (real p) m_{p} is real or imaginary depending on whether $c_{\mathrm{ph}} \gtrless c$, so that, as examples, electromagnetic waves in a simple plasma (see Section III(a)) have real m_{p}. However, some photons, e.g. phonons in air, have imaginary rest mass (see Section IV), as also do hydromagnetic photons in a magnetic plasma. Electromagnetic photons in a vacuum, possessing a velocity c, have $m_{\mathrm{p}} = 0$, as is well accepted.

The speed of the particle associated with the photon is given from (6a) by

$$v = (k/\omega)c^2. \tag{7}$$

It is convenient for present purposes to define the vector velocity with reference to the medium as

$$\boldsymbol{v} = (\boldsymbol{k}/\omega)c^2. \tag{8}$$

The speed v may be greater than c, consistent with the possibility that m_{p} is imaginary (see Section IV). However, the physically observable properties of the photon are E and p, or alternatively ω and k. It should be noted that, in general, $E = pc^2/v$.

When discussing the propagation of electromagnetic waves in a material medium it is conventional to describe it in terms of the dielectric constant ϵ and magnetic permeability μ of the medium. Thus (Born and Wolf 1965) $c_{\mathrm{ph}} = c/(\epsilon\mu)^{\frac{1}{2}}$ in which both ϵ and μ may be functions of frequency. In these terms the parameter n of equations (6a) and (6b) is given by $(\epsilon\mu)^{\frac{1}{2}}$. (CGS units are used throughout this paper.)

II. Doppler Effect

Consider a frame of reference at rest in a uniform medium in which photons are propagating. In the case of photons in a vacuum any frame of reference is as good as any other for this purpose, as will be noted, *a posteriori*. In the case of plasmons or phonons the rest frame is taken to be an inertial frame fixed with respect to the centre of mass of the medium. Let the source of the photons be moving with velocity $\boldsymbol{v}_{\mathrm{s}}$ and the observer with velocity $\boldsymbol{v}_0$, both measured with respect to the rest frame in the medium. Instead of considering the photon we consider the particle to which it is equivalent. The invariance of rest mass of this particle under Lorentz transformation implies that

$$h^2\nu_0^2 - \frac{m_p^2 v^{*2} c^2}{1-v^{*2}/c^2} = h^2\nu^2 - \frac{m_p^2 v^2 c^2}{1-v^2/c^2}, \tag{9}$$

where ν_0 and ν are the frequencies in the observer's frame and in the medium frame respectively and $\boldsymbol{v}_1^* = (v_1^*, v_2^*, 0)$ is the velocity of the photon in the observer's frame, with

$$v_1^* = \frac{\omega^{-1} k \cos\theta_0 c^2 - v_0}{1-\omega^{-1} k \cos\theta_0 v_0}, \qquad v_2^* = \frac{\omega^{-1} k \sin\theta_0 c^2 (1-v_0^2/c^2)^{\frac{1}{2}}}{1-\omega^{-1} k \cos\theta_0 v_0}, \tag{10a, b}$$

θ_0 being the angle between $\boldsymbol{k}$ and $\boldsymbol{v}_0$.

It follows from (9) and (10a) after some algebra that

$$\frac{\nu_0^2}{\nu^2} = \frac{\{1-(v_0\cos\theta_0)/c_{\mathrm{ph}}\}^2}{1-v_0^2/c^2}. \tag{11}$$

Therefore

$$\frac{\nu_0^2}{\nu_s^2} = \frac{\{1-(v_0\cos\theta_0)/c_{\mathrm{ph}}\}^2}{1-v_0^2/c^2} \frac{1-v_s^2/c^2}{\{1-(v_s\cos\theta_s)/c_{\mathrm{ph}}\}^2}. \tag{12}$$

The formula (12) is valid for light, sound, or any other form of wave energy whose quanta can be described by (1) and (2). To illustrate that (12) contains the presently known Doppler effects, a few important special cases are now considered.

When $\boldsymbol{k}$, $\boldsymbol{v}_0$, and $\boldsymbol{v}_s$ are all parallel, equation (12) yields

$$\frac{\nu_0}{\nu_s} = \frac{1 \pm v_0/c_{\mathrm{ph}}}{1 \pm v_s/c_{\mathrm{ph}}} \left(\frac{1-v_s^2/c^2}{1-v_0^2/c^2}\right)^{\frac{1}{2}}, \tag{13}$$

where the plus or minus sign applies according as $\boldsymbol{k} \cdot \boldsymbol{v}_0$ or $\boldsymbol{k} \cdot \boldsymbol{v}_s$ is negative or positive. Without the terms under the radical sign, formula (13) is the conventional Doppler expression as used, for example, in sound.

In the further special case of $c_{\mathrm{ph}} = c$, equation (13) yields

$$\frac{\nu_0}{\nu_s} = \frac{1 \pm v_0/c}{1 \pm v_s/c} \left(\frac{1-v_s^2/c^2}{1-v_0^2/c^2}\right)^{\frac{1}{2}}, \tag{14}$$

with the same sign convention as for (13). Equation (14) is the relativistic Doppler formula which is usually written

$$\frac{\nu_0}{\nu_s} = \frac{1 \pm v_r/c}{1-v_r^2/c^2}, \tag{15}$$

where the plus or minus sign is taken depending on whether the source and observer are approaching one another or receding from one another and

$$v_r = \frac{|v_0 - v_s|}{1-v_0 v_s/c^2} \tag{16}$$

is the relative velocity of source and observer.

It is readily shown that equations (14) and (15) are identical in all physically relevant situations. Equation (15) shows that when the phase speed is c the choice of frame of reference in the medium is irrelevant. It is evident that this happens because the relativistic formula for addition of velocities says that when one of the velocities is c the relativistic sum is also c. This is another way of saying that experiments done with photons of phase velocity c yield no information about the motion of the medium. Such experiments do not prove that the medium (recall the "ether") does not exist. They simply say that it is irrelevant in the circumstances. The involvement of the medium is clearly relevant in equation (13), when in general $c_{\mathrm{ph}} \neq c$.

Photon–Photon Interaction

In the special case where $\boldsymbol{v}_0$ is the velocity of the particle associated with a photon (equation (8)), equation (11) yields

$$\frac{\nu_0^2}{\nu^2} = \frac{\{1-(c^2\cos\theta)/c_{\mathrm{ph}}^2\}^2}{1-c^2/c_{\mathrm{ph}}^2}. \tag{17}$$

This equation shows that, for real ν, ν_0 is imaginary if $c_{\mathrm{ph}} < c$. This can be interpreted as meaning that photon–photon interaction is possible only if $c_{\mathrm{ph}} > c$.

III. Photons in a Plasma

As one special case of the more general situation, photons in the radiofrequency range in a simple plasma (without magnetic field) are now considered here. Other special cases will be discussed in future papers. After deriving the rest mass of radiofrequency photons in a plasma the gravitational redshift of these photons emitted by a star enveloped by plasma is calculated. This is done by replacing the photon in the plasma by a particle in vacuum, by invoking the equivalence of equations (3) and (5), and then applying dynamical considerations to these particles in a gravitational field.

(*a*) *Rest Mass of Radiofrequency Photons*

The usual manner of calculating the gravitational redshift is to consider a null-geodesic in Schwarzschild space–time from the source to the observer. This consideration is associated with the fact that a photon in vacuum has zero rest mass or, alternatively, that its assumed velocity is c. In this section a photon in a plasma is considered as a particle in a vacuum on an ordinary geodesic in space–time rather than on a null-geodesic. The recovery of the conventional gravitational redshift (see subsection (*b*) below) shows that the approach is consistent with accepted physics and serves further to suggest acceptance of the finite rest mass of the photon in material media. In subsection (*c*) below a new phenomenon of the bending of the trajectories of photons near the plasma frequency in a gravitational field is explained.

The dielectric constant of a plasma is a function of frequency, so that the dispersion relationship of radio waves in a simple cold plasma is (Ratcliffe 1959)

$$\epsilon\mu = n^2 = c^2k^2/\omega^2 = 1-\omega_{\mathrm{p}}^2/\omega^2, \tag{18}$$

where $\omega_p^2 = 4\pi n_e e^2/m_e$ is the plasma angular frequency, n_e being the electron density and e and m_e the electronic charge and mass. Equation (18) is a valid approximation only for $\omega < c/n_e^{\frac{1}{3}}$, that is, the frequency associated with the transmission for electromagnetic radiation in vacuum across a distance comparable to the average electron spacing. In each medium and in each frequency range, appropriate values of the pairs ϵ and μ, or alternatively n, must be found in order to apply the relations (6) to electromagnetic waves. In anisotropic media the quantities have to be specified for each wave mode and direction. For present purposes we consider the case of a simple cold plasma to illustrate some basic physics associated with the equivalence principle enunciated in Section II.

It is of interest to note that a dispersion relationship similar to (18) holds for electromagnetic waves in all substances composed of neutral atoms (Landau and Lifshitz 1960). In this case n_e is equal to the number of electrons in all the atoms in unit volume of the substance and it is assumed that $\omega \ll c/a$, where a is an atomic dimension.

It follows from equations (1), (2), and (18) that for radiofrequency photons

$$\begin{aligned} m_p = \hbar\omega_p/c^2 &= (4\pi n_e \alpha \lambda_e^3)^{\frac{1}{2}} m_e \\ &= 1\cdot 02\times 10^{-43} n_e = 7\cdot 36\times 10^{-48}\nu_p\,, \end{aligned} \tag{19}$$

where α is the fine structure constant, λ_e is the Compton wavelength of an electron, and $\nu_p = \omega_p/2\pi$ is the plasma frequency.

In the case of a simple plasma, equations (7) and (18) yield $v = \partial\omega/\partial k$, the group velocity of the waves. However, this is not true of media in general.

Let us consider (18) in two ranges of interest:

(1) $\omega > \omega_p$. In this case k is real, $c_{ph} > c$, $v < c$, and $m > m_p$.

(2) $0 < \omega < \omega_p$. For this situation k is imaginary, i.e. the wave is evanescent, and $0 < m < m_p$.

Characteristic rest masses of radiofrequency photons in five plasmas of interest are set out below.

Ionosphere	Photon rest mass = 10^{-40} g
Solar corona (base)	10^{-39} g
Solid state plasma	10^{-32} g
Solar interior	10^{-31} g
Very dense star	$>10^{-27}$ g (?)

The possibility exists that in some very dense stars the rest mass of such a photon may exceed that of an electron.

(b) *Gravitational Redshift*

Given a Schwarzschild metric in space–time, the speed V of a particle moving radially out from a star at a distance r from its centre is (see e.g. McVittie 1965)

$$V = V_\infty\{1+(2m'/r)(c^2/V_\infty^2-1)\}^{\frac{1}{2}}, \tag{20}$$

where V_∞ is the speed of the particle at infinity and $m' = GM/c^2$, M being the mass of the star and G the constant of gravitation. In the present paper it is assumed that $m'/r \ll 1$; this assumption will be removed in a future paper. Suppose now that the star has a plasma envelope from which it is radiating, and assume that (18) is a valid local relationship in this envelope in a local inertial frame of reference. This latter assumption appears to be a mild one, for if the plasma were grossly in diffusive equilibrium the only change necessary in (18) would be to allow for collisions of electrons with ions. At a high enough frequency this correction can be small enough to be negligible. More complicated dispersion relations than (18) will be considered in future papers.

Considering electromagnetic radiation as particles, equation (5) shows that

$$h^2\nu^2 = E^2 = m^2v^2c^2 + m_{\mathrm{p}}^2 c^4. \tag{21}$$

The velocity v in (21) is now identified with the velocity V in (20).

If waves are treated as particles then it is noted that the velocity V of a particle is determined by two different effects. According to (20) it is changed by motion in the gravitational field and according to (5) it is changed by passing through regions of changing m_{p}. In the latter changes, E remains constant but, in the former, E does not. It is in the former changes only, that changes of frequency occur.

A model is taken in which m_{p} is constant in the vicinity of the star in the range $a < r < R$, where a is the radius at which the radiation is generated. Now, from (21)

$$h^2\nu_a^2 = m_a^2 V_a^2 c^2 + m_{\mathrm{p}a}^2 c^4 \tag{22}$$

and

$$h^2\nu_R^2 = m_R^2 V_R^2 c^2 + m_{\mathrm{p}R}^2 c^4, \tag{23}$$

with $m_{\mathrm{p}a} = m_{\mathrm{p}R}$. From equations (22) and (23)

$$h^2(\nu_R^2 - \nu_a^2) = h^2\nu_a^2(V_R^2 - V_a^2)/(c^2 - V_R^2). \tag{24}$$

However, from (20)

$$V_R^2 - V_a^2 = 2m'\left(\frac{1}{R} - \frac{1}{a}\right)\left(c^2 - \frac{V_R^2 - 2m'c^2/R}{1 - 2m'/R}\right). \tag{25}$$

Thus from (24) and (25), since $2m'/R \ll 1$, and $R \gg a$,

$$\nu_R^2 - \nu_a^2 = 2m'\,\nu_a^2\left(\frac{1}{R} - \frac{1}{a}\right)\frac{c^2 - V_R^2 - (2m'/R)V_R^2 + (2m'/R)c^2 + \dots}{c^2 - V_R^2} \tag{26a}$$

$$= -2m'\,\nu_a^2/a. \tag{26b}$$

Equation (26a) then is the full expression for the gravitational redshift of radio photons, while (26b) yields the conventional shift when $\nu_a \approx \nu_R$. Thus

$$\Delta\nu/\nu \approx -m'/a. \tag{27}$$

It is to be noted that the shift is independent of m_{p}, as expected. If, in the region $r > R$, there is effectively a vacuum, the particle changes velocity, but not E, at $r = R$, and continues on a null-geodesic.

The gravitational redshift is seen to be a property not only of a photon of zero rest mass on a null-geodesic, but also of a particle of finite rest mass (associated with a photon) on an ordinary geodesic. Radiofrequency photons in space exist most frequently *not* in vacua but rather in a plasma, however tenuous. This means that electromagnetic waves may be treated as particles of finite rest mass for many purposes.

(c) *Deflection of Photon Trajectories*

The deflection of photon trajectories in a plasma in a gravitational field is now considered. Bending of trajectories is found to occur particularly for photons near the plasma frequency. This is a distinct effect from the well-known bending of light rays in the vicinity of stars.

Consider a massive object surrounded by plasma of constant ν_{p} (variable ν_{p} would introduce refraction effects which we do not wish to consider here). A particle of finite rest mass near a massive object moves on an ordinary geodesic of which the equation is (e.g. McVittie 1965)

$$(\mathrm{d}u/\mathrm{d}\phi)^2 = 2m'u^3-u^2+(2m'c^2/g^2)u-\gamma\,, \tag{28}$$

where $u^{-1} = r$ is the distance of the particle from the centre of the object; $g = r^2\,\mathrm{d}\phi/\mathrm{d}s$ is the constant of integration, $\mathrm{d}s$ being an interval of proper time; ϕ is the azimuth in the plane of the orbit; and

$$\gamma = (c^2/g^2)(1-\beta^2)\,,$$

with

$$\beta = (1-2m'/r)\,\mathrm{d}t/\mathrm{d}s$$

being a constant of integration and t the local time coordinate.

The term $2m'u^3$ in (28) is usually very small compared with u^2, being in the ratio of $< 2m'/a$, where a is the radius of the star. In this case its presence would cause only a small perturbation on the motion which would be described without it. Omitting then the term $2m'u^3$ from (28), the resultant equation has the solution

$$u = r^{-1} = (1+e\cos\phi)/p\,, \tag{29}$$

where

$$p^{-1} = m'c^2/g^2 \qquad \text{and} \qquad \gamma = (1-e^2)/p^2\,. \tag{30}$$

This is a conic section of eccentricity e and latus rectum $2p$. The curve described by equation (39) will thus be an ellipse, parabola, or hyperbola according as e is less than, equal to, or greater than unity. Now (McVittie 1965)

$$\beta^2 = (1-2m'/r)(1-V^2/c^2)^{-1}\,, \tag{31}$$

where V is the speed of the particle. For very low speeds $V \ll c$, as in the motion of solar planets, β is very close to unity and a little less. This fact is the starting point in the discussion of the advance of the perihelion of planets when the perturbation term $2m'u^3$ is included in the equation of motion (see McVittie 1965). Such solutions are of interest at radiofrequencies because speeds of photons near the plasma frequency are $\ll c$. From (31) we see that

$$\beta \gtreqless 1 \qquad \text{as} \qquad V \gtreqless (2m'/r)^{\frac{1}{2}} c. \tag{32}$$

We identify the speed V with that of the particle associated with a photon.

Using the relationships between frequency and energy and between wavelength and momentum together with the dispersion relationship of a plasma, we have

$$V^2/c^2 = 1 - \nu_p^2/\nu^2. \tag{33}$$

It follows from (32) and (33) that radio waves can move in elliptical orbits (with advancing perihelia) round a massive object provided

$$\nu_p < \nu < \nu_p(1 - 2m'/r)^{-\frac{1}{2}}. \tag{34}$$

Since $2m'/r$ is usually extremely small (e.g. $< 3 \times 10^{-6}$ for the Sun and $< 10^{-8}$ for the Earth), it is not likely that a radio wave of frequency so close to ν_p as prescribed by (34) would progress very far, because damping would be great. However, this effect is likely to be involved in the physics of reflection of radio waves in ionospheres, but it is not intended to pursue this application here.

The more widespread condition occurs when $\beta > 1$, that is, $\nu > \nu_p(1-2m'/r)^{-\frac{1}{2}}$. In this case the trajectories are hyperbolae in the first approximation, i.e. without the term $2m'u^3$ in (28). From equations (30) and (31) the eccentricity of the hyperbola is given by

$$e^2 = 1 - g^2(1-\beta^2)/m'^2c^2. \tag{35}$$

In the approximation $2m'/r \ll 1$, from (31) and (33), $\beta^2 \approx \nu^2/\nu_p^2$ and thus (35) becomes

$$e^2 \approx 1 + g^2(\nu^2/\nu_p^2 - 1)/m'^2c^2. \tag{36}$$

Also $g^2 \approx r^2 V^2 \beta^2 \sin^2 \alpha$ in the same approximation, so that

$$e^2 \approx 1 + (\nu^2/\nu_p^2 - 1)^2 (r^2 \sin^2 \alpha)/m'^2, \tag{37}$$

where α is the angle between the radius vector from the centre of the object and the direction of the trajectory of the particle. For all finite values of α

$$e \approx (\nu^2/\nu_p^2 - 1)(r/m') \sin \alpha, \tag{38}$$

and e can be evaluated in any given case knowing ν, r, and α at, say, the place of origin of the emission of the photon near the object.

The amount of deflection of the trajectory by the gravitational field can now be found from the theory of the hyperbola. The eccentricity implied by (38) is in general very large compared with unity. Under this condition it may be shown that the deflection of the trajectory from the point where it crosses its axis of symmetry (where $\alpha = \frac{1}{2}\pi$) to infinity is e^{-1}. In the present case this is

$$(m'/a)\,\nu_p^2/(\nu^2-\nu_p^2)\,, \tag{39}$$

where a is the radius at which $\alpha = \frac{1}{2}\pi$. The deflections for four values of the ratio ν/ν_p are shown below.

ν/ν_p	e^{-1}
$1\cdot001$	$500\,m'/a$
$1\cdot01$	$50\,m'/a$
$1\cdot1$	$5\,m'/a$
$1\cdot2$	$2\cdot2\,m'/a$

These deflections may be compared with the maximum for light which is about $2m'/a$ (McVittie 1965). For the Sun, with a equal to one solar radius, the unit is approximately $0\cdot87$ sec of arc.

A test of the present theory would be to observe the occultation of a strong celestial radio source (e.g. Jupiter) by the solar corona at frequencies near the plasma frequencies that exist in the corona. There should be considerable bending of radio wave paths (twice those given by (39)) near the occultation condition. Another application of the theory may be to the calculation of trajectories of radio waves in the plasma envelopes of very massive and small celestial objects, i.e. the cases where $m'/r \sim 1$. In these examples the exact equation (28) should be integrated rather than the approximate form. Numerical computations for these cases will be presented in a future paper.

IV. Discussion

The assumed equivalence of a dispersion relationship and the Einstein energy–mass relationship leads to the consideration of a particle in a vacuum instead of a photon in a medium. The rest mass of this particle embodies the properties of the medium. The way in which these properties are embodied may be a function of frequency in the rest frame of the medium. This is so in the case of a plasma in a magnetic field, which will be reported in a future paper. Depending upon the medium and the frequency of the photon, the velocity of the associated particle may be less than or greater than the speed of light, i.e. its rest mass may be real or imaginary. In the latter possibility the particles discussed here would fall into the mathematical category of tachyons whose existence has been debated in the last decade (for reviews of this topic see Bilaniuk and Sudarshan 1969; Bilaniuk *et al.* 1969). Although the mathematical possibility of tachyons has always been implicit in Einstein's equations, the physical arguments for their existence differentiate between various physical theories of them. It is apparent that in the existing literature on tachyons the search has been for particles with speeds in excess of c and to construct the appropriate quantum mechanics for these particles (see Bilaniuk and Sudarshan 1969). This approach then is conventionally employing the de Broglie conversion from particle to wave and constructing an appropriate quantum mechanics. This is in contrast to the approach in the present paper which is that of inverting the de Broglie approach and finding the particles associated with waves whose mathematical descriptions are already well known.

It is therefore incidental that some tachyons have emerged as a subgroup of the total group of particles implied by the theory presented here. It may be that these tachyons do not exhaust the possibilities of the tachyon group of particles. In the physical, as distinct from the mathematical, discussion of tachyons, the present work which involves them only incidentally is different from that previously reported.

Greenberg and Greenberg (1968) have speculated on the rest mass of photons in media in a spirit similar to the present paper. However, their speculation is incorrect for they choose, on the basis of a faulty energy argument, $v = c_{\mathrm{ph}}$ for the speed of the associated particle. Their work could not lead to the results found in this paper, which employs $v = c^2/c_{\mathrm{ph}}$. It may be noted in passing that this value of v leads to an action principle which is consistent with Fermat's principle.

V. Conclusions

By invoking the equivalence of a dispersion relationship and Einstein's energy–mass relationship, a rest mass may be ascribed to any photon whose phase velocity does not equal c. For many purposes a photon in a medium can then be considered to be a particle in a vacuum with this rest mass. In particular, a consistent and general description of the Doppler effect can be generated using this concept.

In addition here a formula for the gravitational redshift of radio photons in a plasma has been found which reduces to the well-known expression in the appropriate circumstances. Also a new effect involving the bending of radio photon trajectories in a plasma in a gravitational field has been predicted. In the former effect the result is the same as that known previously for light in a vacuum. However, in the latter effect the theoretical result is new and of a different origin from the previous result for light. It is perhaps important to note that though these two results have been developed and presented in the format of general relativity it is clear, *a posteriori*, that in the approximation $m'/r \ll 1$ they need only special relativity and Newtonian gravitation theory for their explanation. However, if this condition does not apply, general relativity is required.

The implications of the present approach could reach far beyond the particular examples discussed here. It means that dynamical equations appropriate to particles could be used to discuss a range of problems of photons in media. Some such problems will be discussed in future papers.

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