ANALYSIS OF (d, pγ) REACTIONS

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Abstract

The reactions ^{24}Mg(d, pγ)^{25}Mg via the 3·40 MeV state, ^{40}Ca(d, pγ)^{41}Ca via the 1·95 MeV state, and ^{28}Si(d, pγ)^{29}Si via the 2·03 MeV state are analysed using the stripping theory developed by Butler, Hewitt, McKellar, and May. The theory is in reasonable agreement with the experimental data. It is pointed out that a study of the correlation as a function of the proton scattering angle is necessary to obtain a test of this theory *vis-à-vis* the standard DWBA theory of stripping reactions.

I. INTRODUCTION

(*a*) *BHMM Theory*

A theory of the deuteron stripping reaction based on a sudden approximation was proposed by Butler, Hewitt, McKellar, and May (1967). This theory (BHMM) constitutes an alternative to the usual distorted wave theory of stripping (DWBA). It was extended by McKellar (1969) to include the effect of antisymmetrization with respect to neutrons.

The BHMM theory is based on the calculations of a matrix element M_s, which is related to the direct reaction matrix element M_c for stripping by

$$M_c = A_0^* M_s/(P-S), \tag{1}$$

where

$$S = |A_0|^2 \tag{2}$$

is the spectroscopic factor and $1-P$ is the probability that the state of the captured neutron is already occupied in the core nucleus. M_s is given by

$$M_s = (N+1)^{\frac{1}{2}} \sum_{\alpha'_n} \int \mathrm{d}\boldsymbol{k}'_n \, \langle \Phi_{0c} \,|\, \psi_{\boldsymbol{k}'_n,\alpha'_n} \rangle \langle \psi_{\boldsymbol{k}'_n,\alpha'_n} \phi_p^- \,|\, \lambda_c V_{np} \,|\, \psi_d^+ \rangle \,. \tag{3}$$

Here $|\psi_d^+\rangle$ is the many-body wavefunction (antisymmetrized with respect to neutrons) for a deuteron incident on the target nucleus; V_{np} is the neutron–proton interaction; λ_c is a projection operator which projects out the direct reaction part of the matrix element; $|\phi_p^-\rangle$ describes a proton scattering from the target nucleus; $|\psi_{\boldsymbol{k}'_n,\alpha'_n}\rangle$ describes a state of the system target nucleus plus a neutron of momentum $\hbar\boldsymbol{k}'_n$, where α'_n represents all the other variables in the wavefunction; $|\Phi_{0c}\rangle$ is the wavefunction for the core nucleus plus neutron in state 0; and N is the number of neutrons.

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In the BHMM theory the expression (3) is evaluated by the use of a sudden approximation to yield

$$M_s = (2\pi)^{-3/2} \int \mathrm{d}\boldsymbol{k}'_p \, \langle \chi_0 \,|\, \chi_{Q'} \rangle \langle \phi_p^- \,|\, \phi_{k'_p}^{+} \rangle G(\boldsymbol{k}'_p, \boldsymbol{k}_d)(E_n - E_{Q'}) \,. \tag{4}$$

In this equation, only single-particle wavefunctions are now involved. Here $|\chi_0\rangle$ describes the neutron in the state into which it is captured, $|\chi_{Q'}\rangle$ a neutron with momentum $\hbar\boldsymbol{Q}'$, and $|\phi_{k'_p}^{+}\rangle$ a proton with momentum $\hbar\boldsymbol{k}'_p$ incident on the target nucleus. $G(\boldsymbol{k}'_p, \boldsymbol{k}_d)$ is the Fourier transform of a plane-wave deuteron wavefunction. For simplicity, the dependence on spins has not been shown in equation (4).

This paper is concerned with examining the predictions of the BHMM theory for (d, pγ) reactions.

(b) (d, pγ) *Angular Correlation*

A (d, pγ) reaction proceeds in two stages: a (d, p) stripping reaction takes place, leaving the residual nucleus in an excited state, and this then subsequently decays to a lower state by emission of a γ-ray. The emission of the γ-ray is not isotropic, the direction depending on the way the intermediate state was formed. An angular correlation experiment involves measurement of the number of γ-rays emitted in various directions for a particular proton direction. Such measurements may be made as a function of deuteron energy and proton direction as well as of the direction of the γ-ray.

The double differential cross section for emission of a proton along $\boldsymbol{k}_p$ and a γ-ray along $\boldsymbol{k}_\gamma$ may be written (Rybicki *et al.* 1970)

$$\mathrm{d}^2\sigma/\mathrm{d}\omega_p\,\mathrm{d}\omega_\gamma = (4\pi)^{-1}(\Gamma^f_{\gamma F}/\Gamma^f)\, W(\theta_\gamma, \phi_\gamma)\, \mathrm{d}\sigma/\mathrm{d}\omega_p \,, \tag{5}$$

where $\Gamma^f_{\gamma F}/\Gamma^f$ is the fraction by which state f decays to state F by γ-emission. The angular correlation W has the form

$$W(\theta_\gamma, \phi_\gamma) = \sum_{kq} \{4\pi/(2k+1)\}^{\frac{1}{2}} A_{kq}\, Y^q_k(\theta_\gamma, \phi_\gamma) \,, \tag{6}$$

where Y^q_k is a spherical harmonic. A_{kq} is the product of two factors,

$$A_{kq} = \rho_{kq}\, G_k \,, \tag{7}$$

which correspond to the two stages of the (d, pγ) reaction. The ρ_{kq} describe the result of the (d, p) stage of the reaction, i.e. they describe the intermediate state, and the G_k are concerned with the γ-emission. The latter factors are independent therefore of the mechanism of the stripping reaction and are functions only of the momenta involved in the decay and the multipole amplitudes (Huby *et al.* 1958; Rybicki *et al.* 1970).

The intermediate state f may be described by the density matrix (McKellar 1968)

$$d_{\mu_f \mu'_f} = \{3(2J_i+1)\}^{-1} \sum_{\mu_i \nu_d \nu_p} \langle \mu_f\, \nu_p \,|\, M \,|\, \mu_i\, \nu_d \rangle \langle \mu'_f\, \nu_p \,|\, M \,|\, \mu_i\, \nu_d \rangle^* \,, \tag{8}$$

where μ_f and μ'_f are projections of the total angular momentum J_f in state f, μ_i is the projection of J_i in state i, ν_d is the projection of the deuteron spin S_d (= 1), and

ν_p is the projection of the scattered proton spin S_p $(=\frac{1}{2})$. The statistical tensors ρ_{kq} are defined in terms of $d_{\mu_f\mu_f'}$ and have the property that they transform under rotation like Y_k^q

$$\rho_{kq} = \sum_{\mu_f\mu'_f} (-1)^{J_f-\mu_f}(2h+1)^{\frac{1}{2}}(2J_f+1)^{\frac{1}{2}}\begin{pmatrix} J_f & J_f & k \\ \mu_f & -\mu_f' & -q \end{pmatrix} d_{\mu_f\mu_f'}. \tag{9}$$

The quantity in parentheses is a 3-j symbol, which is related to a Clebsch–Gordan coefficient (Edmonds 1957). It should be noted that it follows from the definition of ρ_{kq} that ρ_{00} is proportional to the differential cross section.

Now, in order to calculate the angular correlation W the ρ_{kq} have to be calculated. This is done for a specific stripping model via equation (9), using in equation (8) the matrix elements calculated by the model. This calculation is straightforward in principle but gives expressions involving summation over many 3-j symbols. Several restrictions apply to ρ_{kq}. The allowable values of k are even because the states f and F have definite parity and the direction only of the γ-ray is observed. The maximum value of k is given by equation (A13) of Appendix II, and in practice is 0, 2, or 4. Since the density matrix is Hermitian, the relation

$$\rho_{kq}(J_f J_f) = (-1)^q \rho^*_{k\,-q}(J_f J_f) \tag{10}$$

must be satisfied. Other relationships hold in specific frames of reference. In particular, in the BHMM frame (z axis along $\boldsymbol{k}_d$, y axis along $\boldsymbol{k}_d \times \boldsymbol{k}_p$) the ρ_{kq} are all real.

The calculation of ρ_{kq} for BHMM theory has been done by McKellar (1968), who reduced the sums over angular momenta by the use of relations linking the 3-j, 6-j, and 9-j symbols. The resulting expression is given below. Since no absolute normalization of angular correlations is carried out, we have removed from the expression factors which only affect normalization.

$$\begin{aligned}
\rho_{kq} \propto & \left(\frac{2k+1}{4\pi}\right)^{\frac{1}{2}} \begin{Bmatrix} j_n & j_n & k \\ J_f & J_f & J_i \end{Bmatrix} \\
& \times \sum (-1)^{J_f+J_i+j_n+L+l_n+l_p'+j_p'+\frac{1}{2}+q}(2j_p+1)(2j_p'+1) \\
& \qquad \times (2a+1)(2b+1)(2c+1)(2l_p'+1)^{\frac{1}{2}}(2l_p+1)^{\frac{1}{2}}(2L+1)^{\frac{1}{2}} \\
& \qquad \times \begin{pmatrix} l_p & l_p' & L \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} j_p' & j_p & L \\ l_p & l_p' & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & b \\ \frac{1}{2} & \frac{1}{2} & 1 \end{Bmatrix} \begin{Bmatrix} l_p & l_p' & c \\ \frac{1}{2} & \frac{1}{2} & b \\ j_p & j_p' & L \end{Bmatrix} \begin{Bmatrix} l_n & l_n & a \\ \frac{1}{2} & \frac{1}{2} & b \\ j_n & j_n & k \end{Bmatrix} \\
& \qquad \times \begin{pmatrix} l_p & l_p' & c \\ -\lambda_n & \lambda_n' & \lambda_n-\lambda_n' \end{pmatrix} \begin{pmatrix} l_n & l_n & a \\ \lambda_n' & -\lambda_n & \lambda_n-\lambda_n' \end{pmatrix} \begin{pmatrix} a & b & k \\ \lambda_n-\lambda_n' & q+\lambda_n'-\lambda_n & -q \end{pmatrix} \\
& \qquad \times \begin{pmatrix} c & b & L \\ \lambda_n-\lambda_n' & q+\lambda_n'-\lambda_n & -q \end{pmatrix} Y_L^{-q}(\theta_p, \phi_p) \\
& \qquad \times \eta_{j_p l_p}(\boldsymbol{k}_p) \overset{*}{\eta}_{j_p' l_p'}(\boldsymbol{k}_p) I_{l_p\lambda_n}(j_n l_n) \overset{*}{I}_{l_p'\lambda_n'}(j_n l_n).
\end{aligned} \tag{11}$$

The summation is over values of j_p, l_p, j'_p, l'_p, λ_n, λ'_n, a, b, c, and L. The quantities j_n and l_n are the neutron total angular momentum and orbital angular momentum respectively. The quantities $\eta_{j_p l_p}$ and $I_{l_p \lambda_n}(j_n\, l_n)$ occur in the evaluation of the matrix element M_s in BHMM theory (Butler *et al.* 1967; McKellar 1968).

TABLE 1

OPTICAL PARAMETERS USED IN CALCULATIONS

The parameter sets are: R, Rosen *et al.* (1965); KM, King and McKellar (1970); ST, Satchler and Tobocman (1960)

Parameter	R (all nuclei)		KM (^{40}Ca)	Fitted parameters (^{28}Si)			ST (^{40}Ca)
	n	p	p	n	p	d	n, p
V (MeV)	$49\cdot3-0\cdot33E_n$	$53\cdot8-0\cdot33E_p$	49·3	44·0	62·1	93·8	60·0
W_s (MeV)	5·75	7·5	5·2	7·0	3·0	29·0	0
W_v (MeV)	0	0	0	2·0	3·0	0	10·0
r_R (f)	1·25	1·25	1·25	1·25	1·06	1·20	1·21
r_I (f)	1·25	1·25	1·25	1·25	1·41	1·50	1·21
a_R (f)	0·65	0·65	0·65	0·70	0·61	0·82	0·40
a_I (f)	0·70	0·70	0·65	0·44	0·55	0·45	0·40
V_{so} (MeV)	5·5	5·5	5·0	6·0	5·3	0	0
r_{so} (f)	1·25	1·25	1·25	1·25	1·06	1·20	—
a_{so} (f)	0·65	0·65	0·37	0·70	0·61	0·82	—
r_C (f)	1·25	1·25	1·25	1·25	1·06	1·25	1·21

(*c*) *Calculations*

In this paper we consider the results of (d, pγ) calculations for three nuclei using the BHMM theory. The calculations were carried out by applying the formulae (6) and (11) in a straightforward fashion. Further details and a more extensive display of the results are given by Steketee (1971).

For the present calculations, the theory was mainly applied using optical parameters obtained from interpolations rather than detailed fits to the elastic scattering data. The parameters used for bound neutrons, scattered neutrons, and protons were normally the averaged optical parameters of Rosen *et al.* (1965), so that the BHMM calculations were done exactly as described in the original BHMM paper. In addition, some calculations were also carried out using optical model parameters fitted to appropriate elastic scattering data.

The angular correlation experiments with which we compare our results are: (1) ^{24}Mg(d, pγ)^{25}Mg at a deuteron energy of 15 MeV, via the 3·40 MeV excited state of ^{25}Mg (Martin *et al.* 1960); (2) ^{40}Ca(d, pγ)^{41}Ca at 7·78 MeV via the 1·95 MeV state of ^{41}Ca (Taylor 1959); and (3) ^{28}Si(d, pγ)^{29}Si at 4–9 MeV via the 2·03 MeV state of ^{29}Si (Kuehner *et al.* 1960; Hausman *et al.* 1966).

II. ^{24}Mg REACTION

The reaction with ^{24}Mg is a $2p_{3/2}$ reaction, going to the $3/2^-$ state of ^{25}Mg at 3·40 MeV. The γ-decay may be to the $5/2^+$ ground state ($E_\gamma = 3\cdot40$ MeV) or to the 0·58 MeV $1/2^+$ level (2·82 MeV), both of which are E1 multipole decays. The

γ-rays from the two decays could not be separated in the experiment, so that a mixture of the correlations to the two states was observed. This presents no serious problem, since the branching ratio and the relative detector efficiency were known. However, the uncertainty in the branching ratio does introduce an uncertainty into the calculated correlation.

TABLE 2

(d, pγ) ANGULAR CORRELATION PARAMETERS

Origin	A_2^0	A_2^2	α_{22} (degrees)
^{24}Mg			
Experimental	$0\cdot180\pm0\cdot021$	$-0\cdot072\pm0\cdot014$	-62 ± 3
BHMM	$0\cdot210\pm0\cdot016$	$-0\cdot068\pm0\cdot006$	-49
Plane wave	$0\cdot210\pm0\cdot016$	$-0\cdot105\pm0\cdot008$	-33
^{40}Ca			
Experimental	$-0\cdot085\pm0\cdot03$	$0\cdot043\pm0\cdot02$	-62 ± 15
BHMM (R set*)	$-0\cdot071$	$0\cdot035$	-66
BHMM (KM set*)	$-0\cdot072$	$0\cdot035$	-57
Plane wave	$-0\cdot071$	$0\cdot036$	-59

* Optical parameter set as defined in Table 1.

(*a*) *Angular Correlations*

The experiments were done at a deuteron laboratory energy of 15 MeV. The results are at a proton angle of 15°(lab.) and in two planes, the reaction plane and an azimuthal plane. The azimuthal plane taken was the plane which contains the vectors $\boldsymbol{k}_\mathrm{d}$ and $\boldsymbol{k}_\mathrm{d}\times\boldsymbol{k}_\mathrm{p}$; these planes are shown in Figures 1(*a*) and 1(*b*) respectively. The optical parameters used for the BHMM calculations were those of Rosen *et al.* (1965), the values (designated R set) being given in Table 1.

The results of the calculations are shown in two forms: tabulations of the parameters of the correlation and diagrams. The correlation for the ^{24}Mg nucleus takes the form (equation (A17), Appendix II)

$$W \propto 1+A_2^0P_2^0(\cos\theta)+A_2^2P_2^2(\cos\theta)\cos2(\phi-\alpha_{22}) \qquad (12)$$

in the usual DWBA frame of reference ($\boldsymbol{k}_\mathrm{d}$ along the positive x axis, $\boldsymbol{k}_\mathrm{d}\times\boldsymbol{k}_\mathrm{p}$ along the positive z axis, and a right-handed xyz frame). Table 2 gives the parameters in this form. We have included in this table also the values of the parameters as calculated from the plane-wave or Butler theory. As noted above, the uncertainties in theoretical values come about because of the uncertainty in the proportion of the two γ-decays measured. Figures 1(*a*) and 1(*b*) show the measured correlations compared with the correlations calculated from BHMM theory.*

We see that the BHMM results for A_2^0 and A_2^2 agree with the experimental results, to within the error range, but that the values of α_{22} do not agree, the largest difference being 13°$\pm$3°. We conclude that the BHMM results show satisfactory

* No comparison of absolute values of the correlation is made. In all correlation diagrams here the relative normalizations have been adjusted to allow the various curves and experimental points to be compared. We have also shown the correlation obtained from the plane-wave theory, being the only other result available.

agreement, taking into account the fact that the calculations are done with averaged optical parameters. On the whole, these results are considerably better than the plane-wave results, which is encouraging, though hardly surprising.

We note here the effect of spin–orbit forces on the BHMM results. For $l = 1$ reactions, a general result of direct reaction theory without spin–orbit forces is that A_2^0 is unchanged from the plane-wave value (Huby *et al.* 1958). The results of Table 2 suggest that spin–orbit forces are unimportant to the BHMM results at 15°. We see also that the experimental results are consistent with a no spin–orbit direct reaction theory.

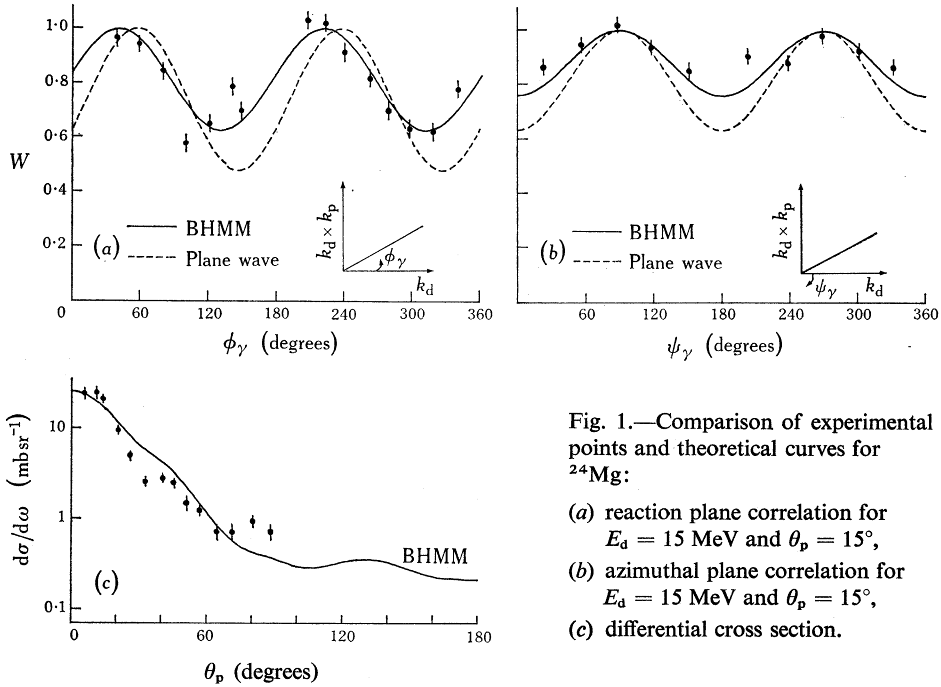

Fig. 1.—Comparison of experimental points and theoretical curves for ^{24}Mg:

(*a*) reaction plane correlation for $E_d = 15$ MeV and $\theta_p = 15°$,

(*b*) azimuthal plane correlation for $E_d = 15$ MeV and $\theta_p = 15°$,

(*c*) differential cross section.

(*b*) *Differential Cross Section*

Although our primary concern is with the angular correlation, we note here the results of some cross section calculations. Martin *et al.* (1960) give the cross section for the 3·40 MeV state at a deuteron energy of 15 MeV. The comparison of BHMM results with experiment is shown in Figure 1(*c*). The overall shape is satisfactory in that the fall of cross section with angle is reproduced, although the detailed structure of the experimental results is not obtained. The spectroscopic factor used to normalize the BHMM cross section was 0·20.

III. ^{40}Ca Reaction

The ^{40}Ca reaction is $2p_{3/2}$, and the γ-decay is an E2 transition to the $7/2^-$ ground state of ^{41}Ca. Two sets of optical parameters have been used for the BHMM calculation. As well as the Rosen parameters, another set of parameters fitted to the elastic scattering of protons on ^{41}Ca was available. This set was obtained from King and McKellar (1970), and its use in BHMM calculations for stripping to the ground state of ^{41}Ca is studied there. The particular parameters we have used are the 14·5 MeV proton parameters of King and McKellar and we have extrapolated in the same way to the actual proton energy (11·9 MeV), by giving the real well depth the energy dependence

$$V(E_p) = V(0) - 0\cdot 33\, E_p. \tag{13}$$

These parameters are shown in Table 1 (designated set KM). For the neutron parameters, the standard Rosen *et al.* set was used, following King and McKellar (1970).

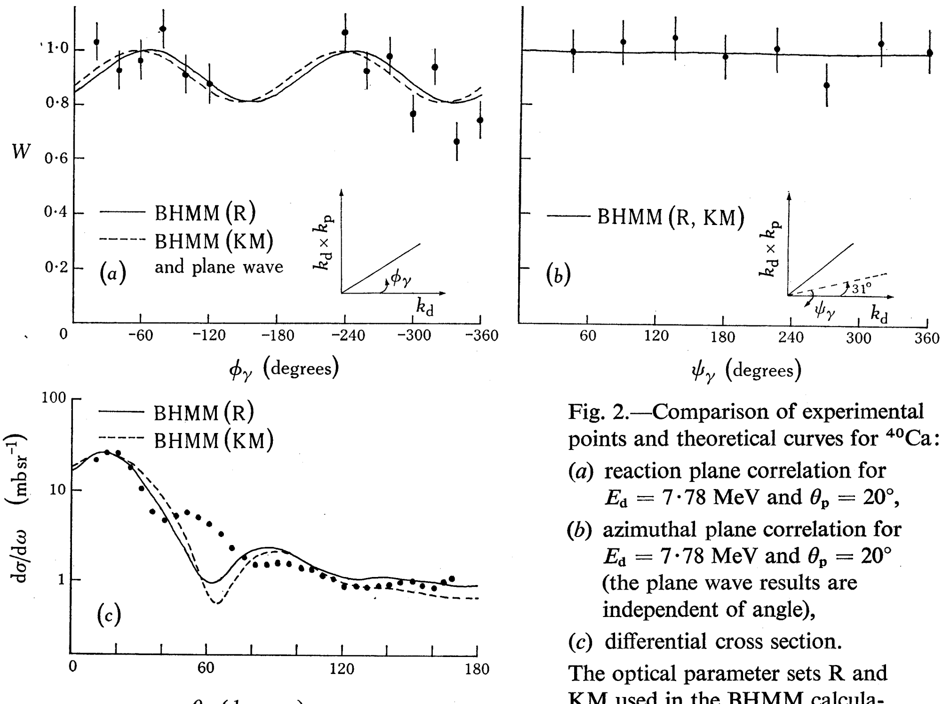

Fig. 2.—Comparison of experimental points and theoretical curves for ^{40}Ca:
(*a*) reaction plane correlation for $E_d = 7\cdot 78$ MeV and $\theta_p = 20°$,
(*b*) azimuthal plane correlation for $E_d = 7\cdot 78$ MeV and $\theta_p = 20°$ (the plane wave results are independent of angle),
(*c*) differential cross section.
The optical parameter sets R and KM used in the BHMM calculations are defined in Table 1.

(*a*) *Angular Correlations*

The form of the correlation is again given by equation (12). Gamma measurements were taken for one proton angle (20°, i.e. near the stripping maximum) and in an azimuthal plane as well as the reaction plane. The correlation parameters deduced from experiment and from BHMM theory are shown in Table 2, together with a comparison with plane-wave theory. The corresponding graphs are given in Figures 2(*a*) and 2(*b*). Figure 2(*b*) includes a diagram indicating the azimuthal plane used. This plane is at 31° to the deuteron axis and is perpendicular to the reaction plane.

Both calculations show reasonable agreement with experimental results. However, in view of the rather large experimental errors in this experiment, it does not constitute a very powerful test of stripping theories.

We note also that as far as the effect of spin–orbit forces is concerned, similar remarks hold as for ^{24}Mg at 15°. It appears (as might be expected near the stripping peak) that spin–orbit forces do not play a significant role.

(*b*) *Differential Cross Section*

Figure 2(*c*) shows the results of differential cross section calculations for stripping to the 1·95 MeV state of ^{41}Ca at the energy for which the (d, pγ) calculations have been done. The experimental points for 8·0 MeV have been taken from Lee *et al.* (1964). The results show fair agreement with experiment, as might be expected at this energy. The spectroscopic factor extracted from these calculations is 0·46 with Rosen *et al.* parameters and 0·41 using the King and McKellar parameter set.

IV. ^{28}Si Reaction

Angular correlation measurements have been done for two ^{28}Si(d, pγ) reactions: stripping reactions to the first and second states (p_1 and p_2 reactions) with subsequent decay to the ground state. All the relevant quantities, including the multipole mixing ratio for the γ-decay from the first excited state, are known. For the p_2 reaction, we may write the correlation as (equation (A17), Appendix II)

$$W \propto 1 + A_2^0 P_2^0(\cos\theta) + A_2^2 P_2^2(\cos\theta)\cos 2(\phi-\alpha_{22}) + A_4^0 P_4^0(\cos\theta) + A_4^2 P_4^2(\cos\theta)\cos 2(\phi-\alpha_{42}) + A_4^4 P_4^4(\cos\theta)\cos 4(\phi-\alpha_{44}) \quad (14)$$

in the usual DWBA reference frame.

The results obviously depend on the angle at which the proton is detected. This was not the same in the different experimental results: Hausman *et al.* (1966) used 40°(lab.) which is about 41° in the centre of mass system at these energies, while Kuehner *et al.* (1960) used the position of the stripping maximum. The BHMM calculations have been done at the appropriate proton angle for each case, namely 41° and the position of the BHMM stripping maximum respectively.

(*a*) *Angular Correlations*

The BHMM calculations have been done using the standard parameter set of Rosen *et al.* (Table 1). Some comparison calculations have also been done using specially fitted parameters at the highest energy (8·96 MeV). These calculations have been carried out at every energy for which experimental data are available but not all the results are presented here. The graphs in Figure 3 are, however, representative of the results and demonstrate most of the characteristics we wish to discuss.

Figures 3(*a*) and 3(*b*) (5·8 MeV) are typical of the results obtained for the p_2 reaction. The reaction-plane agreement is reasonable but that in the azimuthal plane is not.

Figure 3(*c*) shows the results for the p_2 reaction in the reaction plane at 8·96 MeV, the highest energy for which experimental results are available. As is to be expected, the agreement with experiment is considerably better at this energy. This figure also illustrates the effect of using different optical parameters, these fitted parameters being more fully discussed in subsection (*c*) below. As can be seen, the effect is quite small. This is generally true of BHMM angular correlation calculations, especially near the stripping maximum.

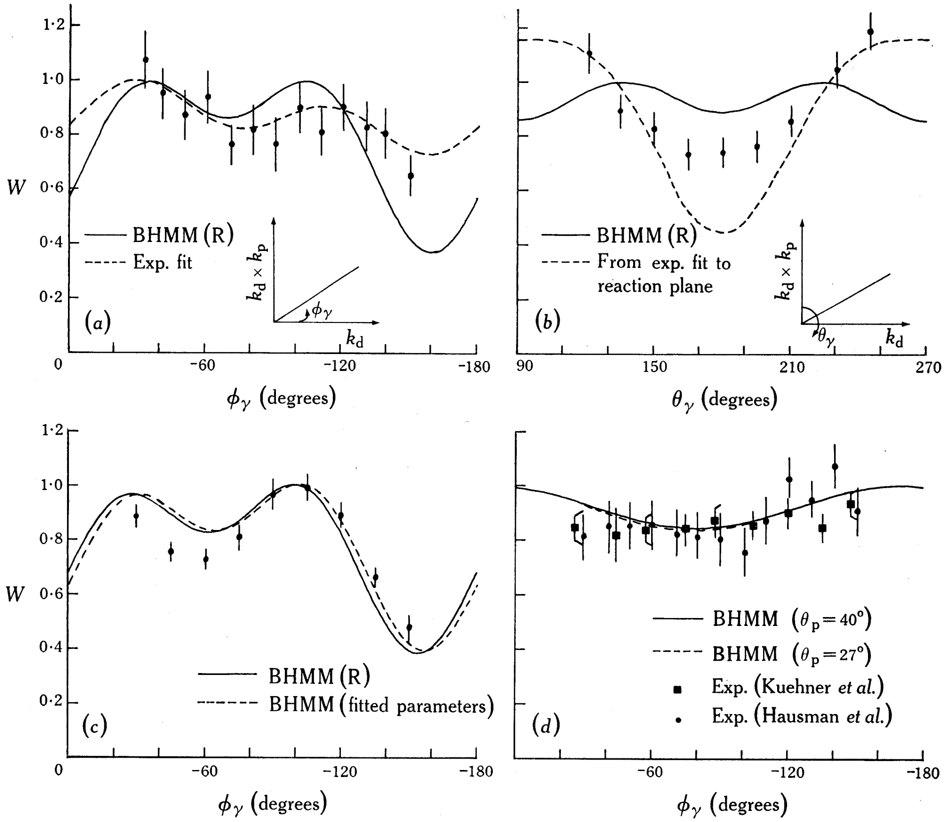

Fig. 3.—Comparison of experimental and theoretical correlations for ^{28}Si:
(*a*) p_2 reaction plane for $E_d = 5 \cdot 8$ MeV, (*b*) p_2 azimuthal plane for $E_d = 5 \cdot 8$ MeV,
(*c*) p_2 reaction plane for $E_d = 8 \cdot 96$ MeV, (*d*) p_1 reaction plane for $E_d = 6 \cdot 07$ MeV.

Figure 3(*d*) is for the p_1 reaction in the reaction plane at $6 \cdot 07$ MeV. This energy is common to the experiments of Kuehner *et al.* (1960) and Hausman *et al.* (1966). It can be seen that there is a considerable difference between the two experiments, possibly because the protons were detected at different angles. The BHMM calculations do not show very much dependence on proton angle here. The agreement with experiment is fair and is representative of the p_1 results.

On the average, our calculations show reasonable agreement with the experimental correlations, the agreement on the whole becoming better with increasing energy. Especially at the lower energies, the experimental results show a considerable random variation from one energy to the next. This variation is both in shape and in magnitude (the magnitude variation is not shown in the diagrams, in which the normalization is arbitrary). Such variation is obviously not present in the BHMM or any other direct reaction theory, and indicates a significant compound nucleus contribution to the stripping process.

The results in the azimuthal plane are mostly not good. Only that at the highest energy for which such measurements were taken (6·07 MeV) shows some similarity to experiment. Again the suggestion arises that the failure is due to the low energies used, but it is not possible to be definite about this because there are unfortunately no results at the higher energies.

There is an independent pointer to the importance of compound nucleus contributions. We note in Appendix II that the assumption of a direct reaction theory with no spin–orbit interactions for the scattered particles allows the parameters of the p_2 reaction to be obtained from the reaction-plane correlation only. This allows comparison of the azimuthal-plane measurements with the predictions made by this theory. The results of this comparison are quite poor in all cases for which it can be made. The results at 5·8 MeV (Figs. 3(*a*) and 3(*b*)) are typical. It must be concluded then either that spin–orbit forces play an important role in angular correlation theory or that the results are not wholly described by direct reaction assumptions. In BHMM theory, however, the inclusion of spin–orbit forces has virtually no effect on the correlation involving protons near the stripping maximum, and a large effect at no proton angle. There is no reason to suppose that DWBA is any different in this respect. The likely conclusion is once again that there are compound nucleus contributions to this reaction, large enough to have significant effect on $(d, p\gamma)$ measurements even near the stripping peak.

In summary, the main problem with the ^{28}Si experiments is the low energy at which they have been performed, as this leads to significant compound nuclear contributions to the reaction. In addition, BHMM theory cannot be expected to give good results for the direct (d, p) reaction at low energies, since it is based on a sudden approximation. The other point that should be mentioned is that, since ^{28}Si is a deformed nucleus, calculations using spherical models are only an approximation to those that ought to be carried out for an exacting test of stripping theories.

(*b*) *Cross Sections and Polarizations*

We now consider some results of the cross section and polarization calculations for ^{28}Si. For the purpose of comparison, the calculations were carried out in the DWBA as well as in the BHMM theory. For this reason, it was thought worth while to use optical parameters fitted to the appropriate elastic scattering data. The parameters used are included in Table 1:

^{28}Si neutron elastic scattering cross sections were obtained from Clarke and Cross (1964) for 14·1 MeV and from Petitt *et al.* (1966) for 2·45 to 5·8 MeV. The neutron parameters, which give satisfactory results for elastic scattering cross sections over this energy range, come directly from the former reference.

^{28}Si(d, d) elastic scattering cross sections have been measured, and best fit parameters calculated, by Lacek and Strohbusch (1970). The parameters used here are those obtained by these authors for a deuteron laboratory energy of 10·9 MeV.

^{28}Si (p, p) elastic scattering cross sections and polarizations have been measured at 17·8 MeV by Baugh *et al.* (1965), and results are also available at lower energies (Greenlees *et al.* 1958; Rosen *et al.* 1965). The proton parameters used in our calculations were obtained with the help of the SEEK program (M. A. Melkanoff, T. Sawada, and J. Raynal, personal communication), and reproduce the cross section and polarization data satisfactorily, with best results at the higher energies. DWBA calculations were done with the DWUCK program (P. D. Kunz, personal communication).

A further discussion of the elastic scattering fits and the effect of variation in optical parameters is given by Steketee (1971).

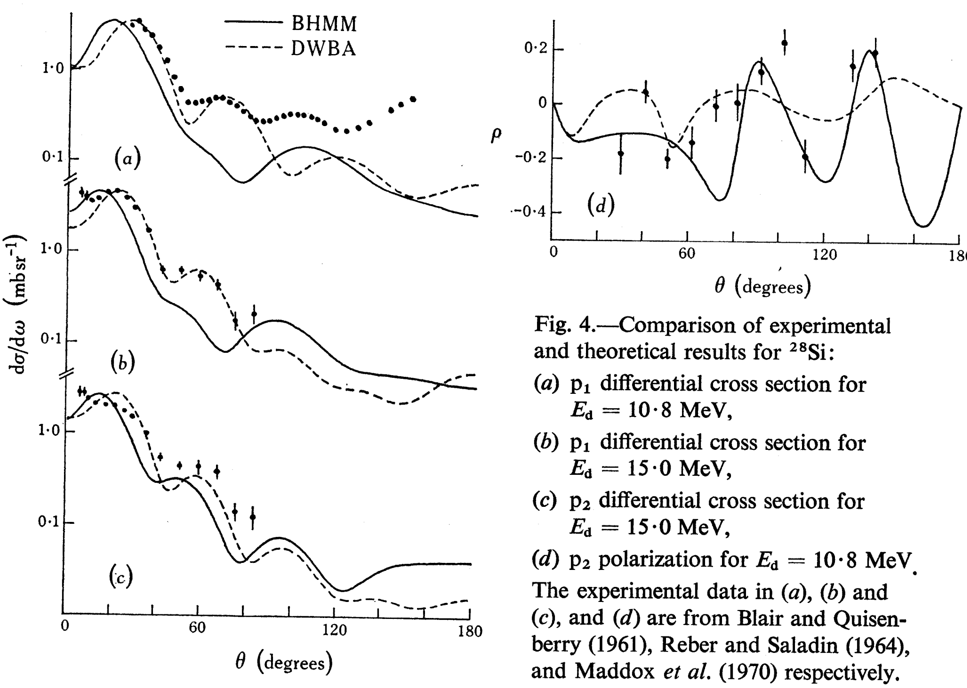

Fig. 4.—Comparison of experimental and theoretical results for ^{28}Si:
(*a*) p_1 differential cross section for $E_d = 10 \cdot 8$ MeV,
(*b*) p_1 differential cross section for $E_d = 15 \cdot 0$ MeV,
(*c*) p_2 differential cross section for $E_d = 15 \cdot 0$ MeV,
(*d*) p_2 polarization for $E_d = 10 \cdot 8$ MeV.
The experimental data in (*a*), (*b*) and (*c*), and (*d*) are from Blair and Quisenberry (1961), Reber and Saladin (1964), and Maddox *et al.* (1970) respectively.

The graphs in Figure 4 are representative of the results obtained for BHMM and DWBA (d, p) cross section and polarization calculations. On the whole, the theories give reasonable account of the results and are comparable in their agreement with experiment, with the expected improvement towards higher energies. The results shown are for the p_1 and p_2 reactions, although calculations have also been carried out for a number of reactions to other states of ^{29}Si.

V. Conclusions

From the above evaluation of the results of the sudden approximation when applied to angular correlation measurements, we have seen that the agreement with experiment was good, and on the whole considerably better than that of the plane-wave theory, which is encouraging. However, we have also seen that the currently available experimental data do not allow a very stringent test of BHMM theory, nor do they allow an effective comparison between competing stripping theories. The causes for this are twofold.

In the first place, experiments have generally been done using one proton direction which is near the stripping peak. This makes it difficult to use the experimental data to distinguish between theories of stripping, since any such reasonable theory should give a good account of results near the stripping peak.

Secondly, several features of the correlation experiments that have been carried out make them strictly unsuitable for analysis by the BHMM theory and, in most cases, by DWBA. One of these features is the low energy at which most of the experiments have been performed. This affects both the applicability of BHMM, because it is a sudden approximation, and also DWBA, since at low energies compound nucleus processes play an important part which is difficult to take into account in $(d, p\gamma)$ calculations. Another feature is the choice of target nuclei. All correlation experiments known to us have involved stripping to states which are deformed. The Mg–Si region is known to be markedly nonspherical, as is at least the first excited state of ^{41}Ca. However, stripping theories have largely been restricted to spherical potentials.

We now consider the first cause in more detail. If the purpose of the experiment is to determine spins or transition multipoles then the use of a fixed proton counter together with a γ-detector which is variable in position is quite suitable. The quantity of interest in this case is the complexity of the correlation, that is, the maximum value of k entering into equation (A8), Appendix II, this value depending on the angular momenta in the manner specified in equation (A13). When the interest is focused on an evaluation of reaction theories, however, as in the present work, the variation with θ_p is of greater interest. A complete angular correlation experiment for this purpose would consist of the measurement of the correlation, in one or more γ-planes, for a range of proton angles. The parameters of the correlation (in the form of the d_{kq} of Appendix II) could thus be extracted and plotted as a function of proton angle. These could then be compared with the theoretical predictions. Such an experiment is obviously a major undertaking, and it may be that it is not feasible to carry it out.

However, there is an alternative experiment available which would appear to be practical, and which would also furnish information as a function of proton direction. In this experiment, measurements are made using a fixed γ-detector in coincidence with a variable position proton detector. This yields a linear combination of the d_{kq} as a function of θ_p, if the dependence on cross section is factored out. Again we have a quantity, measured as a function of proton angle and independent of the cross section, which may be compared with the results of various theories.

We argue that there is a need for experimental results to be taken in the above fashion, in order to provide a better method of discriminating between the DWBA and BHMM theories than is currently available. Cross section measurements are not satisfactory for the purpose of discriminating between the theories, since, under suitable conditions, both theories can give a good account of the differential cross section results. The usual alternative measurement has been that of proton polarization.

This, however, is not ideal because of the strong dependence of the results on the values assigned to spin–orbit forces. For BHMM theory this is especially unfortunate, since the neutron spin–orbit force, which is needed over a large range of energies, is difficult to determine.

We consider here one example in support of our contention that (d, pγ) measurements are likely to provide an effective method of discriminating between the two theories. This is the hypothetical $2p_{3/2}$ ^{44}Ca(d, pγ) reaction at 7·01 MeV, in which the intermediate state is the ground state. For this reaction, Satchler and Tobocman (1960) have done some DWBA calculations for the correlation parameters as a function of proton angle. The fact that no γ-ray is emitted is irrelevant for our purposes, since the parameters still describe the state of ^{45}Ca formed by the stripping reaction even though they are not capable of being measured in an angular correlation experiment. Two parameter sets were used for the BHMM calculations: the Rosen *et al.* (1965) set and the Satchler and Tobocman (1960) proton parameters used here for both the proton and neutron potentials, with no energy dependence. These parameters are shown in Table 1.

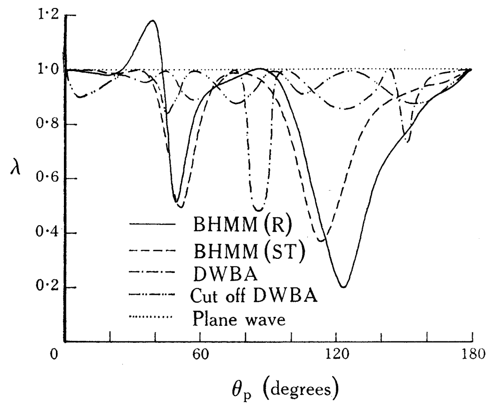

Fig. 5.—Angular dependence of the parameter $\lambda = -2A_2^2/A_2^0$ for the ^{44}Ca(d, p) reaction with $E_d = 7\cdot01$ MeV. The optical parameter sets R and ST used in the BHMM calculations are defined in Table 1.

The results of the calculations are shown in Figure 5. The parameter λ of this figure is equal to $-2A_2^2/A_2^0$ and is independent of the mode of γ-decay. Several features of the diagram are worth noting. Firstly, the dependence (in BHMM) on the parameter set chosen is not large, especially at forward angles. Also the effect of spin–orbit forces is quite small until large backward angles are reached, although this has not been shown on our already cluttered diagram. We have found the same feature in other BHMM calculations of this type that we have done. This is similar to the effect on the cross section of spin–orbit forces, and quite unlike the effect on polarization calculations.

The final and most obvious feature of Figure 5 is the radically different nature of the BHMM, DWBA, and plane-wave results. It seems likely that this will persist when both the BHMM and DWBA calculations are done with properly determined optical parameters. If, moreover, this feature appears in calculations for a wide range of nuclei, angular correlation measurements will offer an effective tool for deciding between the competing theories.

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Appendix I

Optical Model Potentials

The optical model potentials adopted in our calculations for scattered neutron, proton, and deuteron were of the form

$$\begin{aligned} U(r) = & -V(1+\exp x_1)^{-1} - \mathrm{i}W_\mathrm{v}(1+\exp x_2)^{-1} \\ & +4\mathrm{i}W_\mathrm{s}\frac{\mathrm{d}\{(1+\exp x_2)^{-1}\}}{\mathrm{d}x_2} + V_\mathrm{so}\lambda^2\frac{1}{r}\frac{\mathrm{d}\{(1+\exp x_3)^{-1}\}}{\mathrm{d}r}\boldsymbol{\sigma}\,.\,\boldsymbol{l} + V_\mathrm{C}(r)\,, \end{aligned} \tag{A1}$$

where λ is the pion Compton wavelength, taken to be $\sqrt{2}$ f, and $V_\mathrm{C}(r)$ is the Coulomb potential for the particle in the field of a charge Ze distributed uniformly throughout a sphere of radius R_C. We have defined

$$x_1 = \frac{r-R_R}{a_R}, \qquad x_2 = \frac{r-R_I}{a_I}, \qquad x_3 = \frac{r-R_{so}}{a_{so}}, \tag{A2}$$

where the radii have the usual $A^{\frac{1}{3}}$ dependence, so that

$$R_R = r_R A^{\frac{1}{3}}, \quad R_I = r_I A^{\frac{1}{3}}, \quad R_{so} = r_{so} A^{\frac{1}{3}}, \quad R_C = r_C A^{\frac{1}{3}}. \tag{A3}$$

The spin operators were, for the neutron and proton,

$$\boldsymbol{\sigma} = (2/\hbar)\boldsymbol{s} \tag{A4}$$

and, for the deuteron,

$$\boldsymbol{\sigma} = (1/\hbar)\boldsymbol{s}. \tag{A5}$$

For the bound neutron potential, the form (A1) was also used with $W_s = W_v = V_C = 0$. The values of R_R and a_R were the same as those for the scattered neutron. The value of V was chosen to give the correct binding energy for the bound state neutron. The spin–orbit term was chosen to be 25 times the Thomas term (Lee *et al.* 1964), so that for the bound neutron

$$V_{so} = 25(m_\pi^2/4m_p^2)V = 0\cdot138\ V \tag{A6}$$

and

$$r_{so} = r_R, \qquad a_{so} = a_R.$$

It should be noted that we have used a derivative Saxon–Woods form for the surface absorption potential. When it was necessary to convert to or from the Gaussian form $\exp\{-(r-R)^2/b^2\}$, we have used the relation

$$a_I = 0\cdot4b. \tag{A7}$$

DWBA calculations were done without application of a radial cutoff.

Appendix II

Equations relevant to (d, pγ) *Reactions*

Two coordinate frames have been used in this paper: frame 1 is the BHMM frame, which has the z axis along $\boldsymbol{k}_d$ and the y axis along $\boldsymbol{k}_d \times \boldsymbol{k}_p$; frame 2 is the frame that has usually been used for DWBA calculations, namely the x axis along $\boldsymbol{k}_d$ and the z axis along $\boldsymbol{k}_d \times \boldsymbol{k}_p$. The correlation may be written as (Satchler and Tobocman 1960)

$$W(\theta_\gamma, \phi_\gamma) = \sum_{kq} \{4\pi/(2k+1)\}^{\frac{1}{2}} g_k\, d_{kq}\, Y_k^q(\theta_\gamma, \phi_\gamma), \tag{A8}$$

where

$$g_k = \eta_k(j_n j_n J_i J_f) \sum_{LL'} C_L C_{L'} F_k(LL'J_F J_f), \tag{A9}$$

C_L being the multipole amplitude for a 2^L pole γ-ray. The angular momentum coefficients defined by Biedenharn and Rose (1953) and Satchler (1953) are given by

$$\eta_k(abcd) = \{(2a+1)(2b+1)(2d+1)\}^{\frac{1}{2}}(-1)^{c-d-\frac{1}{2}}\, C(abk;\tfrac{1}{2},-\tfrac{1}{2})\, W(ddab;k,c)\,, \tag{A10a}$$

$$F_k(abcd) = \{(2a+1)(2b+1)(2d+1)\}^{\frac{1}{2}}(-1)^{c-d-1}\, C(abk;1,-1)\, W(ddab;k,c)\,, \tag{A10b}$$

where C is a Clebsch–Gordan coefficient and W a Racah coefficient (Edmonds 1957). We may normalize so that

$$\sum_L C_L^2 = 1\,, \tag{A11}$$

in which case

$$g_0 = 1\,. \tag{A12}$$

The sum in (A8) is over values of k satisfying

$$k \leqslant 2j_{\mathrm{n}}, 2L, 2L', 2J_f \tag{A13}$$

and only even values of k contribute when both the nuclear states involved have definite parity, and the circular polarization of the γ-ray is not observed.

The quantities d_{kq} are normalized statistical tensors which are related to the statistical tensors of equation (7) in Section I(b) by

$$\rho_{kq}(J_f J_f)/\rho_{00}(J_f J_f) = \eta_k(j_{\mathrm{n}} j_{\mathrm{n}} J_i J_f) d_{kq}\,. \tag{A14}$$

Note that $d_{00} = 1$. The d_{kq} satisfy a relation which ensures the reality of W, namely

$$d_{kq}^* = (-1)^q d_{k-q}\,. \tag{A15}$$

In frame 1, the d_{kq} are real; in frame 2 they are complex but, as only even values of q contribute, the number of parameters in the correlation remains the same. In frame 2 we may write

$$d_{kq} = |d_{kq}| \exp(-\mathrm{i}q\alpha_{kq})\,. \tag{A16}$$

The correlation in frame 2 may be expressed as

$$W(\theta_\gamma, \phi_\gamma) \propto 1 + \sum_{k>0} \left(A_k^0 P_k^0(\cos\theta_\gamma) + \sum_{q>0} A_k^q P_k^q(\cos\theta_\gamma) \cos q\,(\phi_\gamma - \alpha_{kq}) \right), \tag{A17}$$

and the A_k^q are related to the d_{kq} by

$$A_k^0 = g_k\, d_{k0}/g_0\, d_{00} \tag{A18a}$$

and for $q > 0$ and even by

$$A_k^q = 2\left(\frac{(k-q)!}{(k+q)!}\right)^{\frac{1}{2}} \frac{|d_{kq}|\, g_k}{d_{00}\, g_0}\,. \tag{A18b}$$

If we assume no spin–orbit forces are present, certain relations appear among the d_{kq} and also between the d_{kq} and the (d, p) proton polarization (Huby *et al.* 1958; Satchler and Tobocman 1960; Hausman *et al.* 1966). In general, the polarization when there are no spin–orbit forces present satisfies

$$|P| \leqslant \tfrac{2}{3}(2j_{\mathrm{n}}+1)^{-1}\,. \tag{A19}$$

For $l_n = 1$ and $j_n = \frac{3}{2}$,

$$d_{20} = -\tfrac{1}{2} \qquad \text{and} \qquad 0 \leqslant |d_{22}| \leqslant \tfrac{1}{4}\sqrt{6}, \tag{A20}$$

that is, $0 \leqslant \lambda \leqslant 1$, where we have defined λ by

$$\lambda = -2A_2^2/A_2^0. \tag{A21}$$

The proton polarization for $l_n = 1$ satisfies

$$(2j_n+1)P = \pm\tfrac{2}{3}(1-\lambda^2)^{\frac{1}{2}}. \tag{A22}$$

For $l_n = 2$, $j_n = 5/2$, the relations are

$$d_{40} = \tfrac{7}{12}+\tfrac{5}{12}d_{20}, \tag{A23a}$$

$$|d_{42}| = (\tfrac{5}{12})^{\frac{1}{2}}|d_{22}|, \tag{A23b}$$

$$\alpha_{42} = \alpha_{22}+\tfrac{1}{2}\pi, \tag{A23c}$$

and

$$\tfrac{72}{35}(1+d_{20})|d_{44}|^2-2(\tfrac{72}{35})^{\frac{1}{2}}|d_{22}|^2|d_{44}|\cos 4(\alpha_{22}-\alpha_{44})$$
$$-\tfrac{1}{4}(1-d_{20})(1-d_{20}^2)+(1-d_{20})|d_{22}|^2 = 0. \tag{A23d}$$

The polarization for $l_n = 2$ satisfies

$$(2j_n+1)P = \pm(\tfrac{4}{15})^{\frac{1}{2}}(\tfrac{5}{3}(1-d_{20})^2-\tfrac{96}{7}|d_{44}|^2)^{\frac{1}{2}}. \tag{A24}$$

It should be noted that in Satchler and Tobocman (1960), this last relation is incorrectly reproduced, but the correct result is obtained by Huby *et al.* (1958).

Lastly, the parameters for the plane-wave or Butler theory may be easily calculated. The correlation is

$$W(\theta_\gamma \phi_\gamma) = \sum_k g_k P_k(\cos\theta'), \tag{A25}$$

where θ' is the angle of γ-emission relative to the recoil axis. We transform to frame 2 by using the well-known relation

$$(2k+1)P_k(\cos\theta') = 4\pi \sum_q Y_k^{q*}(\omega_1)\, Y_k^q(\omega_2), \tag{A26}$$

θ' being the angle between the directions ω_1 and ω_2. This leads to the results in frame 2:

$$d_{k0} = P_k(0), \tag{A27a}$$

$$|d_{kq}| = \left(\frac{(k+q)!}{(k-q)!}\right)^{\frac{1}{2}}(-1)^{(k+q)/2}\, P_k^{-q}(0), \tag{A27b}$$

$$\alpha_{kq} \equiv \phi_0 \bmod(2\pi/q) \qquad \text{for} \qquad (k+q)/2 \text{ even}, \tag{A27c}$$

$$\alpha_{kq} \equiv (\phi_0-\pi/q)\bmod(2\pi/q) \qquad \text{for} \qquad (k+q)/2 \text{ odd}, \tag{A27d}$$

where ϕ_0 is the recoil angle.