CALCULATION OF THE PROMPT GAMMA ENERGY OF INDIVIDUAL FISSION FRAGMENTS FROM THE PROMPT NEUTRON NUMBERS

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Abstract

Approximate calculations of the prompt γ-ray energies of fission fragments and the total kinetic energies of different fragment pairs have been made on the basis of a liquid-drop model. In addition, reliable estimates of the quantum energy for collective quadrupolar vibration have been used to calculate the number of prompt γ-ray quanta emitted by the fragments from the thermal-neutron-induced fission of ^{235}U. There seems to be good agreement with the available experimental data. About 30 pairs of fission fragments from the thermal-neutron-induced fission of ^{233}U, ^{235}U, and ^{239}Pu and the spontaneous fission of ^{252}Cf have been included in this preliminary treatment.

I. Introduction

The total energy E_T liberated in a fission event is given by

$$E_T = E_{K_T}+E_{\gamma_H}+E_{\gamma_L}+E_{\nu_H}+E_{\nu_L}, \tag{1}$$

where E_{K_T} is the total kinetic energy of the two complementary fragments, E_{γ_H} and E_{γ_L} are the prompt γ-ray energies, and E_{ν_H} and E_{ν_L} represent the energies needed to evaporate ν_H and ν_L numbers of prompt neutrons from the heavy and light fragments respectively. On the basis of a simple picture of the scission-point configuration, several authors (Vandenbosch 1963; Terrell 1965; Ferguson and Read 1966) have made quantitative estimates of E_{K_T} and, with much less certainty, of E_{γ_H} and E_{γ_L}. According to this picture, the two complementary fragments at the scission point have nonspherical shapes and the Coulomb energy of repulsion is identical with the total kinetic energy. Also the additional assumption is made that the post-scission excitation energy of a fragment, regarded as the sum of the scission-point deformation energy and any small amount of excitation energy the fragment might possess at the scission point, is completely dissipated by the evaporation of the prompt neutrons and the emission of the prompt γ-rays.

The present calculations of E_{K_T}, E_{γ_H}, and E_{γ_L} are based on the assumptions that: at the scission point the fragments possess no excitation energy; after scission the fragments return to equilibrium shapes that are spherical; and the scission-point deformation energy, which is released by each fragment when it becomes spherical, is solely used up in evaporating the prompt neutrons. On the other hand, we have assumed that the energy of electrostatic interaction between the deformed fragments at the scission point is partly converted into the total kinetic energy of the two fragments and partly into prompt γ-ray energies through the mutual Coulomb excitation of the fragments. Because of their very approximate nature, the present calculations serve merely as a preliminary test of the validity of these assumptions.

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II. Formulation

The scission-point shape of each fragment is assumed to be obtainable by subjecting each spherical fragment to an $\alpha_2 P_2(\cos\theta)$ (i.e. a Legendre polynomial) type of deformation and the scission-point configuration, shown in Figure 1, consists of the two fragments touching at a point on their collinear major axes. The potential energy of this configuration, which can be identified with the total energy E_T released in fission, is given by

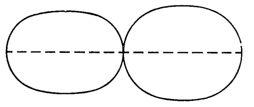

Fig. 1.—Schematic diagram of the scission-point configuration of a fragment pair.

$$E_T = E_{D_H} + E_{D_L} + E_{int}, \tag{2}$$

where E_{D_H} and E_{D_L} are equal in magnitude to the energies required to deform the heavy and the light fragments respectively from the spherical shape to their scission-point shapes, and E_{int} is the electrostatic interaction energy. Since on this picture two highly-charged deformed fragments are envisaged as being very close to each other, mutual and multiple Coulomb excitation is not implausible (Stephens *et al.* 1959). Using a classical approach, we have

$$E_{int} \approx (Z_H + 0\cdot5\, Q_H D^{-2})(Z_L + 0\cdot5\, Q_L D^{-2}) e^2 D^{-1} \tag{3a}$$

$$\approx Z_H Z_L e^2 D^{-1} + 0\cdot5\, Q_H Z_L e^2 D^{-3} + 0\cdot5\, Q_L Z_H e^2 D^{-3}, \tag{3b}$$

where Z_H and Z_L and Q_H and Q_L are the nuclear charge numbers and the classical quadrupole moments of the two fragments respectively, D is the distance between the mass centres, and e is the electronic charge. In the expanded form (3b) of E_{int}, we have neglected the term containing the product of the two quadrupole moments because of its small magnitude. Using the basic formulation of the Coulomb excitation process (Alder *et al.* 1956), we have approximated the Coulomb excitation energy of the heavy and light fragments respectively by the appropriate quadrupole–monopole interaction energy terms

$$0\cdot5\, Q_H Z_L e^2 D^{-3} \qquad \text{and} \qquad 0\cdot5\, Q_L Z_H e^2 D^{-3}.$$

The expressions for the total kinetic energy and the prompt γ-ray energies have been obtained by assuming that: (1) the whole of the Coulomb excitation energy, which must be in the form of vibrational energy as the proposed symmetrical configuration precludes rotation of the fragments, is dissipated by each fragment in the form of prompt γ-rays; and (2) the monopole–monopole interaction energy at the scission point is converted into the total kinetic energy. Briefly, these assumptions are

$$E_{K_T} = Z_H Z_L e^2 D^{-1}, \qquad E_{\gamma_H} \approx 0\cdot5\, Q_H Z_L e^2 D^{-3}, \qquad E_{\gamma_L} \approx 0\cdot5\, Q_L Z_H e^2 D^{-3}. \tag{4a, b, c}$$

Furthermore, for conservation of energy and consistency between equations (1) and (2), we have made the additional assumptions

$$E_{\nu_H} = E_{D_H} \qquad \text{and} \qquad E_{\nu_L} = E_{D_L}. \tag{5}$$

III. Calculations

(a) Deformation Parameter α_2

If a nucleus, considered as a charged incompressible liquid drop, is so deformed that the distance $R(\theta)$ between its centre of mass and any arbitrary point on its surface is given by

$$R(\theta) = R_0\lambda^{-1}\{1+\alpha_2 P_2(\cos\theta)\} \tag{6}$$

then the energy required to cause such a deformation is (see Swiatecki 1958)

$$\begin{aligned} E_D = {} & a_s A^{2/3}(0\cdot4000\,\alpha_2^2-0\cdot0381\,\alpha_2^3-0\cdot3771\,\alpha_2^4+0\cdot1011\,\alpha_2^5+0\cdot3091\,\alpha_2^6) \\ & -a_c Z^2 A^{-1/3}(0\cdot2000\,\alpha_2^2+0\cdot0381\,\alpha_2^3-0\cdot2082\,\alpha_2^4-0\cdot0465\,\alpha_2^5+0\cdot1827\,\alpha_2^6)\ldots, \end{aligned} \tag{7}$$

where a_s and a_c are the coefficients of the surface and Coulomb energy terms in Weizsäcker's mass formula, R_0 is the radius of the spherical nucleus, A and Z are the mass and charge numbers of the nucleus, λ is a volume-preserving scale factor, and α_2 is a constant characterizing the deformation. On the assumptions (5), E_{ν_H} or E_{ν_L} can be expressed by the expansion (7) with parameters A, Z, and α_2 appropriate to the heavy or light fragments respectively (i.e. with A_H, Z_H, α_{2_H} or A_L, Z_L, α_{2_L}).

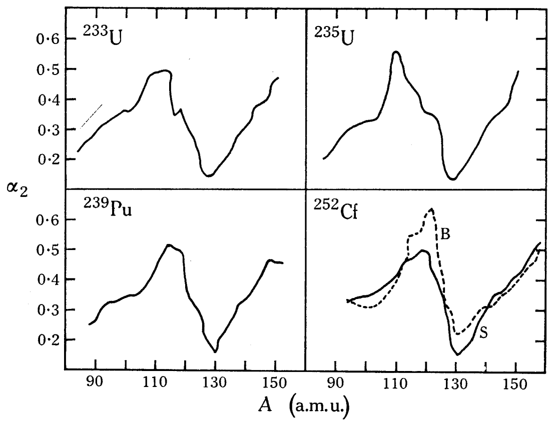

Fig. 2.—Calculated dependences of the deformation parameter α_2 on the mass number A of either fragment from the fission of the indicated nuclei. The curves for ^{252}Cf are based on prompt neutron numbers from:
B, Bowman *et al.* (1963);
S, Schmitt *et al.* (1968).

Using experimental values of the prompt neutron numbers, E_{ν_H} and E_{ν_L} have been calculated from the formulae

$$E_{\nu_H} = \sum_{\nu=0}^{\nu=\nu_H}(B_\nu+E_{K_\nu}) = B(A_H,Z_H)-B(A_H-\nu_H,Z_H)+1\cdot2\,\nu_H, \tag{8a}$$

$$E_{\nu_L} = \sum_{\nu=0}^{\nu=\nu_L}(B_\nu+E_{K_\nu}) = B(A_L,Z_L)-B(A_L-\nu_L,Z_L)+1\cdot2\,\nu_L, \tag{8b}$$

where B_ν is the binding energy of the νth prompt neutron, E_{K_ν} is the centre-of-mass kinetic energy of the prompt neutrons for which a constant value of 1·2 MeV has been used (Milton and Fraser 1962), and $B(A,Z)$ represents the binding energy of a nucleus of mass and charge numbers A and Z respectively. For Z_H and Z_L we have used the values of the most probable charges associated with mass numbers A_H and A_L as given by Mukherji (1969):

$$Z_H = A_H/2\cdot 587 \qquad \text{and} \qquad Z_L = (Z_c - Z_H), \tag{9}$$

where Z_c is the charge number of the parent fissile nucleus. The binding energies $B(A,Z)$ have been calculated by means of Weizsäcker's mass formula with Fowler's values (Green 1955) for the coefficients a_s and a_c, for reasons mentioned below in subsection (*c*). The experimental prompt-neutron numbers have been taken from Apalin *et al.* (1965) in the cases of the thermal-neutron fission of ^{233}U, ^{235}U, and ^{239}Pu and from both Bowman *et al.* (1963) and Schmitt *et al.* (1968) in the case of the spontaneous fission of ^{252}Cf. On evaluating the equations (8) and substituting for E_{ν_H} and E_{ν_L} in the form of the expansion (7) with Fowler's values for the coefficients a_s and a_c, we obtain α_{2_H} and α_{2_L} for different fragment pairs. Plots of α_2 against A obtained in this way are shown in Figure 2.

(*b*) *Energies* E_{K_T}, E_{γ_H}, *and* E_{γ_L}

With the values of the deformation parameters already obtained, the total kinetic energy and the prompt γ-ray energies have been evaluated in the following manner. The distance D between the mass centres of the complementary fragments at the scission point is given by

$$D = r_0\{A_H^{1/3}\lambda_H^{-1}(1+\alpha_{2_H}) + A_L^{1/3}\lambda_L^{-1}(1+\alpha_{2_L})\}, \tag{10}$$

where r_0 is the unit radius parameter and the values of the scale factor λ have been given in terms of α_2 by Swiatecki (1956). The classical quadrupole moments Q_H and Q_L are given by

$$Q_H \approx 0\cdot 4\, Z_H(a_H^2 - b_H^2) \qquad \text{and} \qquad Q_L \approx 0\cdot 4\, Z_L(a_L^2 - b_L^2), \tag{11a, b}$$

where a and b are the semi-major and semi-minor axes respectively. In the case of an $\alpha_2\,P_2(\cos\theta)$ type of deformation, we have

$$a = r_0\lambda^{-1}A^{1/3}(1+\alpha_2) \qquad \text{and} \qquad b = r_0\lambda^{-1}A^{1/3}(1-0\cdot 5\,\alpha_2). \tag{12}$$

Since the "nuclear force" radius is larger than the "charge distribution" radius, we have followed the suggestion of Rasmussen *et al.* (1969) and chosen two different values of r_0:

(1) $r_0 = 1\cdot 37$ fm to evaluate D as a function of A_H from equation (10). These results are plotted in Figure 3. Thence, by means of equations (4a) and (9), E_{K_T} has been evaluated as a function of A_H to give the results plotted in Figure 4. This figure includes, for comparison, the corresponding experimental data of Schmitt *et al.* (1966), Neiler *et al.* (1966), and Pleasonton (1968).

(2) $r_0 = 1\cdot 16$ fm, which is consistent with Fowler's value (Green 1955) of $0\cdot 741$ for a_c, to calculate a and b from equations (12) and the quadrupole moments from (11). The prompt γ-ray energies, then obtained from equations (4b) and (4c), are plotted as functions of the fragment mass in Figure 5. In the case of ^{252}Cf a comparison has been made with the corresponding experimental values of Johansson (1964). Since the experimental values were given on relative scales, we have

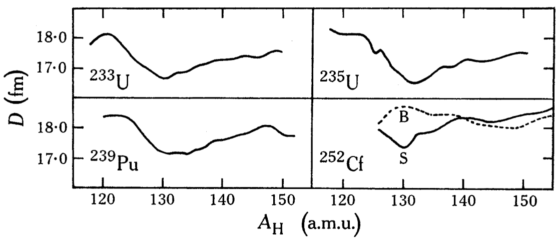

Fig. 3.—Calculated dependences of the distance D between mass centres of complementary fragments on the mass number A_H of the heavy fragment from the fission of the indicated nuclei. The curves for ^{252}Cf are based on prompt neutron numbers from:

B, Bowman *et al.* (1963);

S, Schmitt *et al.* (1968).

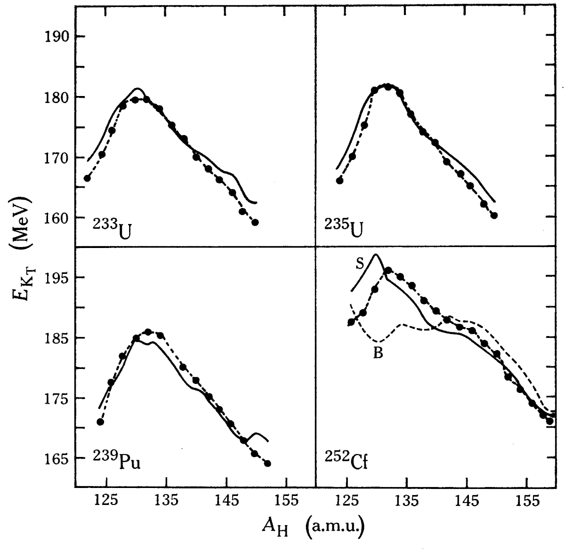

Fig. 4.—Dependences of the total kinetic energy E_{K_T} on the mass number A_H of the heavy fragment from the fission of the indicated nuclei. The continuous curves give the calculated results while the solid circles are experimental data from Schmitt *et al.* (1966), Neiler *et al.* (1966), and Pleasonton (1968). The calculated curves for ^{252}Cf are based on prompt neutron numbers from:

B, Bowman *et al.* (1963),

S, Schmitt *et al.* (1968).

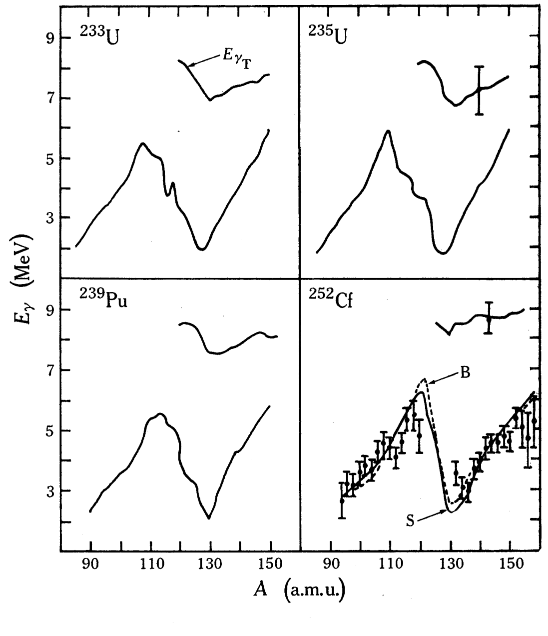

Fig. 5.—Dependences of γ-ray energy on fragment mass for the fission of the indicated nuclei. The upper curves for each nucleus show the calculated total γ-ray energy E_{γ_T} as a function of the heavy fragment mass number A_H and the lower curves are the calculated prompt γ-ray energy E_γ as a function of the mass number A of either fragment. For ^{252}Cf, the solid circles are the experimental data of Johansson (1964) while the calculated curves are based on prompt neutron numbers from:

B, Bowman *et al.* (1963),

S, Schmitt *et al.* (1968).

converted them to an absolute scale in units of MeV by using, as reference, our calculated value of 8·7 MeV for the total prompt γ-ray energy release by the most probable fragment pair: $A_H = 143$ and $A_L = 109$. The average total prompt γ-ray energy release measured by Bowman and Thompson (1958) for ^{252}Cf is 9·0 MeV, which is in agreement with our calculated value. In the case of the prompt γ-ray energy of the individual fragment, the agreement between the calculated and experimental values should be noted along with the fact that Johansson's (1964) experimental data take into account about 75% of the total prompt γ-rays.

(*c*) *Choice of a_s and a_c*

The calculated values of α_2 depend to a great extent on the values chosen for a_s and a_c in equation (7). Since several sets of values for the mass-equation coefficients are available in the literature, the following condition has been imposed in making a choice

$$E_T(\Delta m) \approx E_T(\exp), \tag{13}$$

where

$$E_T(\Delta m) = B(A_H, Z_H) + B(A_L, Z_L) + B_\nu - B(A_c, Z_c), \tag{14a}$$

$$E_T(\exp) = [E_{K_T} + E_{\gamma_T} + E_{\nu_H} + E_{\nu_L}]_{\exp}. \tag{14b}$$

The quantity $E_T(\Delta m)$ represents the total energy release for a split of the parent compound nucleus of mass and charge numbers A_c and Z_c, and B_ν is the thermal-neutron binding energy. In the case of the spontaneous fission of ^{252}Cf, equation (14a) is to be modified by putting $A = 252$ in place of A_c and omitting B_ν.

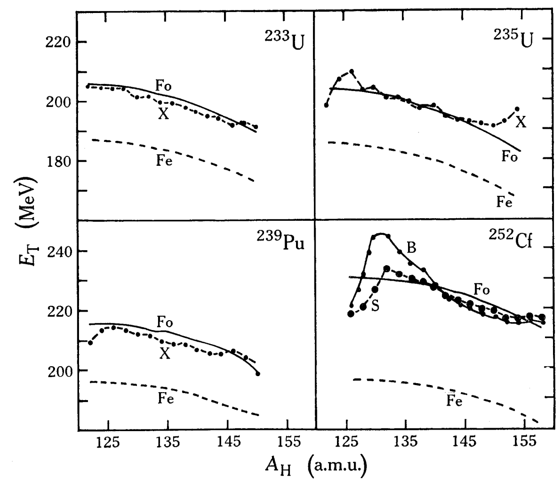

Fig. 6.—Dependences of the total energy release E_T on the mass number A_H of the heavy fragment from the fission of the indicated nuclei. Symbols used are:
Fo, $E_T(\Delta m)$ with Fowler's coefficients;
Fe, $E_T(\Delta m)$ with Fermi's coefficients;
X, $E_T(\exp)$.
For ^{252}Cf, $E_T(\exp)$ is based on prompt neutron numbers from:
B, Bowman *et al.* (1963);
S, Schmitt *et al.* (1968).

The semi-empirical mass formula with the numerical coefficients of Fermi, Fowler, and Green (Green 1955), was used in calculating the binding energies in equation (14a) for the different complementary fragment-pairs. In calculating the corresponding values of $E_T(\exp)$ from equation (14b), the values of $E_{K_T}(\exp)$ were taken from Schmitt *et al.* (1966), Neiler *et al.* (1966), and Pleasonton (1968). Since the values of $E_{\gamma_T}(\exp)$ are not well known for different fragment pairs, we used 9·0 and

7·5 MeV for all fragment pairs in the cases of ^{252}Cf and ^{235}U respectively (Bowman and Thompson 1958; Rau 1963), and an assumed value of 7·5 MeV for all fragment pairs in the cases of ^{233}U and ^{239}Pu. The values of E_{ν_H} and E_{ν_L} were calculated from the experimental prompt-neutron numbers as outlined in subsection (*a*) above.

Figure 6 shows the values of $E_T(\Delta m)$ and $E_T(\exp)$ plotted against the heavy-fragment mass number. As the value of $E_T(\Delta m)$ obtained with Green's (1955) values for the mass-equation coefficients was practically identical with that obtained with Fowler's values (Green 1955), the former has been omitted. It is clear from Figure 6 that Fowler's values (Fo curves) for the mass-equation coefficients lead to a fair estimate of the total energy release and hence they were used in equations (7) and (8) for the calculation of the deformation parameters in subsection (*a*) above.

(*d*) *Derivative* $(\partial\nu/\partial E_{K_T})_r$

The rate of change $(\partial\nu/\partial E_{K_T})_r$ of the total number of prompt neutrons with total kinetic energy, for a fixed mass ratio $r = A_H/A_L$, is a model-dependent quantity whose calculated value may be compared with the available experimental result. The calculated value of $-0\cdot142$ MeV^{-1}, which is derived in the Appendix along with the value $-0\cdot00036$ MeV^{-1} for $E_{\gamma_T}^{-1}(\partial E_\gamma/\partial E_{K_T})_r$, is in excellent agreement with Stein and Whetstone's (1958) experimental value of $-0\cdot141\pm0\cdot002$ MeV^{-1}.

IV. Discussion

No satisfactory method for the calculation of either the angular momentum or the magnitude of the prompt γ-ray energy for an individual fission fragment exists in the literature (Maier-Leibnitz *et al.* 1965*a*). Hoffman (1964) suggested that a non-axial orientation of the fission fragments at the scission point results in a torque that produces the angular momentum. However, without a knowledge of the initial orientation and degrees of distortion of the complementary fragments at the scission point, calculation of the angular momentum of a given fission fragment is not possible. Statistical-model calculations, made by Leachman and Kazek (1957) and Terrell (1959) on the simple assumption that prompt γ-ray emission occurs only after prompt neutron emission becomes energetically impossible, lead to rather low values for the average total γ-ray energy. Furthermore, such calculations fail to predict the distinctly saw-tooth type of mass dependence of the prompt γ-ray energy which Johansson's (1964) experimental results show. Thomas and Grover (1967) have provided calculations on the assumption that γ-ray emission can compete with neutron emission at excitation energies nearly equal to the sum of the neutron separation energy and the yrast level for the spin of the excited level. Although their results on the total γ-ray energy and the fragment angular momentum are in fair agreement with the corresponding experimental values, the main drawback of this method is that it requires, as input data, a knowledge of the average angular momentum as well as the distribution of the initial excitation energies.

Recently, Rasmussen *et al.* (1969) have shown that zero-order bending-mode vibrations of a distorted fragment at the scission point may lead to values of the angular momentum observed experimentally. The somewhat arbitrary subdivision of the total scission-point potential energy in the present work, represented by equations (4)–(5), may be explained in terms of this model. For small-angle bendings, our formulation regarding the potential energy at the scission point coincides with that of Rasmussen *et al.* if $\alpha_3 = 0$. The energy that goes into the bending-mode vibration is, in this case, equal to the appropriate monopole–quadrupole interaction energy. The generation of angular momentum through bending-mode vibrations may be considered as introducing a sort of "memory" into a fission fragment in the sense that most of this angular momentum has to be removed, after prompt neutron emission, along with the prompt γ-rays. The post-scission de-excitation process for a single fragment may be viewed as starting with a fragment possessing an initial excitation energy of $E_D + E_Q$ (where E_D is the shape deformation energy and E_Q is the monopole–quadrupole interaction energy) and a total angular momentum I. A number ν of prompt neutrons is then evaporated, thereby removing an amount of energy *equivalent* to E_D and leaving the residual fragment with an excitation energy of E_Q. Further de-excitation then proceeds through γ-ray emission.

The predominantly quadrupolar nature, the magnitude of the half-life, and the mean energy of about 0·8 MeV of the prompt γ-rays led Johansson and Kleinheinz (1965) to suggest, as an extension of Mollenauer's (1962) conclusion, that the prompt γ-rays may be the result of de-excitation of the fragments through vibrational cascades. This view seems to be favoured by the experimentally measured anisotropies of the prompt γ-rays as a function of the fragment mass (Armbruster *et al.* 1969). The statistical model predicts that the anisotropy should be dependent on the initial fragment spin and hence on the fragment mass number, but the experimental results (Armbruster *et al.* 1969) do not show any such pronounced mass dependence. With our previously calculated values of the individual prompt γ-ray energies, the assumption that the final stage of de-excitation of a fragment proceeds through vibrational cascades allows the number of quadrupolar γ-ray quanta emitted by a given fragment to be calculated as follows.

For quadrupolar vibrations of a collective nature, the energy per quantum is given by (Bohr and Mottelson 1953)

$$\hbar\omega_2 = \hbar(C_2/B_2)^{\frac{1}{2}}. \tag{15}$$

If the nucleus behaves simply as a charged liquid drop, the values of C_2 and B_2, as obtained from hydrodynamic considerations, are

$$(C_2)_{\text{hydr}} = (a_s/\pi)A^{2/3} - (0\cdot3/\pi)Z^2e^2R_0^{-1}, \tag{16a}$$

$$(B_2)_{\text{hydr}} = (3/8\pi)AMR_0^2, \tag{16b}$$

where M is the nucleonic mass. The experimental values of C_2 and B_2, however, show considerable departure from those predicted by equations (16). Away from the closed shells the values of B_2 are found to be about 10 times the values of $(B_2)_{\text{hydr}}$ (Alder *et al.* 1956). For a very approximate calculation of the quantum energy $\hbar\omega_2$ of a fragment with the "most probable" charge number Z for its mass number A, we have used $C_2 = (C_2)_{\text{hydr}}$ and $B_2 = 10 \times (B_2)_{\text{hydr}}$ in equation (15). Fowler's value (Green 1955) for a_s has been used in equation (16a) and $R_0 = r_0 A^{1/3}$ has been calculated with $r_0 = 1\cdot16$ fm. The number N_γ of prompt γ-ray quanta emitted by a fragment of mass number A is given by

$$N_\gamma = E_\gamma(\hbar\omega_2)^{-1}. \tag{17}$$

Using the previously calculated values of the prompt γ-ray energy E_γ in equation (17), N_γ has been calculated for the fission fragments from ^{235}U and the values are shown in Figure 7. A comparison with experimental values measured by Maier-Leibnitz *et al.* (1965*b*) is also given. The experimental values were transformed from the relative to the absolute scale in two different ways: (*a*) by assuming that the average number of quanta emitted by the light-fragment group is the same as that emitted by the heavy-fragment group and (*b*) by assuming that the ratio of the average number of quanta from the heavy-fragment group to that from the light-fragment group is 1·68, as shown by our calculated results. In both cases the total number of quanta averaged over all fragments has been taken to be equal to our calculated value of 8·3, which is in good agreement with the measurements of Maienschein *et al.* (1958) and Rau (1963). Since there is no *a priori* reason for favouring assumption (*a*), Figure 7(*b*), which corresponds to assumption (*b*), possibly shows good agreement between the calculated and the experimental results.

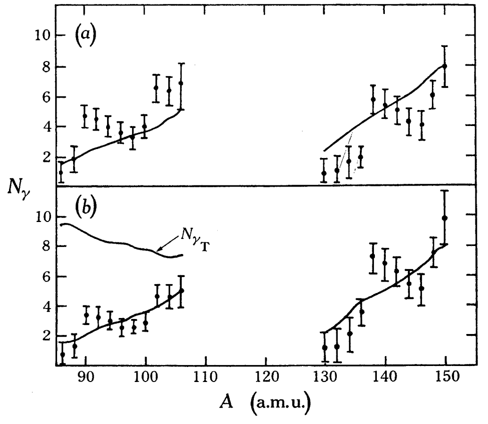

Fig. 7.—Dependences of the number of prompt γ-ray quanta N_γ on the mass number A of either fragment from the fission of ^{235}U. The continuous curves give the calculated results while the solid circles are experimental data from Maier-Leibnitz *et al.* (1965*b*). Parts (*a*) and (*b*) are based on the two different assumptions described in the text.

V. Conclusions

The present calculations are very approximate in nature because of the various simplifying assumptions we have made and particularly because of the use of the simple liquid-drop model without any corrections for shell effects. The agreement between the calculated and the experimental quantities therefore is meaningful only in so far as it serves as an exploratory step towards testing the validity of our hypotheses. The general trend of the calculated results seems to support our main hypothesis that the Coulomb excitation energy of a fragment at the scission point may be identified with its subsequent prompt γ-ray decay energy. However, more rigorous calculations, based on realistic assumptions regarding the scission-point shapes of the fragments and their deformation energies, are needed to establish the present model on a firmly quantitative basis.

VI. Acknowledgments

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VII. References

Alder, K., Huus, T., Bohr, A., Mottelson, B. R., and Winther, A. (1956).—*Rev. mod. Phys.* **28,** 432.

Apalin, V. F., Gritsyuk, Y. N., Kutikov, I. E., Lebedev, V. I., and Mikaelian, L. A. (1965).—*Nucl. Phys.* **71,** 553.

Armbruster, P., Hossfeld, F., Labus, H., and Reichelt, K. (1969).—Proc. I.A.E.A. Symp. on Phys. Chem. of Fission, Vienna, p. 545.

Bohr, A., and Mottelson, B. R. (1953).—*Mat.-fys. Meddr.* **27,** No. 16.

Bowman, H. R., Milton, J. C. D., Thompson, S. G., and Swiatecki, W. J. (1963).—*Phys. Rev.* **129,** 2133.

Bowman, H. R., and Thompson, S. G. (1958).—Proc. 2nd Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, Vol. 15, Paper P/652.

Ferguson, J. M., and Read, P. M. (1966).—*Phys. Rev.* **150,** 1018.

Green, A. E. S. (1955).—"Nuclear Physics." p. 287. (McGraw-Hill: New York.)

Hoffman, M. M. (1964).—*Phys. Rev.* B **133,** 714.

Johansson, S. A. E. (1964).—*Nucl. Phys.* **60,** 378.

Johansson, S. A. E., and Kleinheinz, P. (1965).—*In* "Alpha, Beta and Gamma-ray Spectroscopy". (Ed. K. Siegbahn.) p. 805. (North-Holland: Amsterdam.)

Leachman, R. B., and Kazek, C. S. (1957).—*Phys. Rev.* **105,** 1511.

Maienschein, F. C., Peele, R. W., Zobel, W., and Love, T. A. (1958).—Proc. 2nd Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, Vol. 15, Paper P/670.

Maier-Leibnitz, H., Armbruster, P., and Specht, H. J. (1965*a*).—Proc. I.A.E.A. Symp. on Phys. Chem. of Fission, Salzburg, Vol. 2, p. 113.

Maier-Leibnitz, H., Schmitt, H. W., and Armbruster, P. (1965*b*).—Proc. I.A.E.A. Symp. on Phys. Chem. of Fission, Salzburg, Vol. 2, p. 143.

Milton, J. C. D., and Fraser, J. S. (1962).—*Can. J. Phys.* **40,** 1626.

Mollenauer, J. F. (1962).—*Phys. Rev.* **127,** 867.

Mukherji, S. (1969).—*Nucl. Phys.* A **129,** 297.

Neiler, J. H., Walter, F. J., and Schmitt, H. W. (1966).—*Phys. Rev.* **149,** 894.

Pleasonton, F. (1968).—*Phys. Rev.* **174,** 1500.

Rasmussen, J. O., Norenberg, W., and Mang, H. J. (1969).—*Nucl. Phys.* A **136,** 465.

Rau, F. E. W. (1963).—*Ann. Phys.* **10,** 252.

Schmitt, H. W., Lide, R. W., and Pleasonton, F. (1968).—*Nucl. Instrum. Meth.* **63,** 237.

Schmitt, H. W., Neiler, J. H., and Walter, F. J. (1966).—*Phys. Rev.* **141,** 1146.

Stein, W. E., and Whetstone, S. L. (1958).—*Phys. Rev.* **110,** 476.

Stephens, F. S., Diamond, R. M., and Perlman, I. (1959).—*Phys. Rev. Lett,* **3,** 435.

Swiatecki, W. J. (1956).—*Phys. Rev.* **104,** 993.

Swiatecki, W. J. (1958).—Proc. 2nd Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, Vol. 15, Paper P/651.

Terrell, J. (1959).—*Phys. Rev.* **113,** 527.

Terrell, J. (1965).—Proc. I.A.E.A. Symp. on Phys. Chem. of Fission, Salzburg, Vol. 2, p. 3.

Thomas, T. D., and Grover, J. R. (1967).—*Phys. Rev.* **159,** 980.

Vandenbosch, R. (1963).—*Nucl. Phys.* **46,** 129.

Appendix

Calculation of $(\partial v/\partial E_{\mathrm{KT}})_r$

From the expansion for E_v in the form of equation (7), we obtain

$$(\partial E_v/\partial \alpha_2)_r = \{0\cdot 4000\,\alpha_2(2-\rho)-3\times 0\cdot 0381\,\alpha_2^2(1+\rho)-4\,\alpha_2^3(0\cdot 3771-0\cdot 2082\,\rho)$$
$$+5\,\alpha_2^4(0\cdot 1011+0\cdot 0465\,\rho)+6\,\alpha_2^5(0\cdot 3091-0\cdot 1827\,\rho)\}E_s^0\ldots, \qquad \text{(A1)}$$

where $E_s^0 = a_s A^{2/3}$ is the surface energy and

$$\rho = E_c^0/E_s^0 = (a_c/a_s)Z^2/A\,.$$

Choosing the complementary fragment pairs of mass numbers 143 and 109 in the case of the fission of ^{252}Cf, the rate of change of the total number of prompt neutrons for the fragment pair with the total kinetic energy is given by

$$\left(\frac{\partial E_{\nu\mathrm{H}}}{\partial \alpha_{2\mathrm{H}}}\right)_r + \left(\frac{\partial E_{\nu\mathrm{L}}}{\partial \alpha_{2\mathrm{L}}}\right)_r = \left(\frac{\partial E_{\nu}}{\partial \alpha_{2}}\right)_r . \tag{A2}$$

Assuming the average neutron binding energy to be 5 MeV and the centre-of-mass neutron kinetic energy to be $1\cdot 2$ MeV, the observed rate of change of neutron numbers with α_2 is given by

$$\left(\frac{\partial \nu}{\partial \alpha_2}\right)_r = \frac{1}{6\cdot 2}\left\{\left(\frac{\partial E_{\nu\mathrm{H}}}{\partial \alpha_{2\mathrm{H}}}\right)_r + \left(\frac{\partial E_{\nu\mathrm{L}}}{\partial \alpha_{2\mathrm{L}}}\right)_r\right\}. \tag{A3}$$

Furthermore, from equation (4a), we obtain

$$\left(\frac{\partial E_{\mathrm{K_T}}}{\partial \alpha_2}\right)_r \approx -\frac{Z_\mathrm{H} Z_\mathrm{L} e^2}{r_0\,(A_\mathrm{H}^{1/3} + A_\mathrm{L}^{1/3})(1+\bar{\alpha}_2)}\,, \tag{A4}$$

where, for simplicity we have used $\bar{\alpha}_2 = \frac{1}{2}(\alpha_{2_\mathrm{H}} + \alpha_{2_\mathrm{L}})$. It therefore follows that

$$\left(\frac{\partial \nu}{\partial E_{\mathrm{K_T}}}\right)_r = \left(\frac{\partial \nu}{\partial \bar{\alpha}_2}\right)_r \bigg/ \left(\frac{\partial E_{\mathrm{K_T}}}{\partial \bar{\alpha}_2}\right)_r . \tag{A5}$$

It can be seen from Figure 2 that, for the relevant fragment pairs, $\alpha_{2_\mathrm{H}} = 0\cdot 35$ and $\alpha_{2_\mathrm{L}} = 0\cdot 4$. This gives $\bar{\alpha}_2 = 0\cdot 375$, from which we obtain the calculated value of

$$(\partial \nu/\partial E_{\mathrm{K_T}})_r = -0\cdot 142 \quad \mathrm{MeV}^{-1}.$$

Calculation of $E_{\gamma_\mathrm{T}}^{-1}(\partial E_\gamma/\partial E_{\mathrm{K_T}})_r$

Considering the simplest case of symmetric fission with $Z_\mathrm{H} = Z_\mathrm{L} = Z$, $\alpha_{2_\mathrm{H}} = \alpha_{2_\mathrm{L}} = \alpha_2$, and $r_0\,A_\mathrm{H}^{1/3} = r_0\,A_\mathrm{L}^{1/3} = R$, we obtain from equation (4a)

$$(\partial E_{\mathrm{K_T}}/\partial \alpha_2)_r = -Z^2 e^2/D(1+\alpha_2). \tag{A6}$$

Furthermore, from equations (4b) and (11a),

$$E_\gamma \approx (0\cdot 6\,Z^2 R^2 e^2/D^3 \lambda^2)(\alpha_2 + 0\cdot 25\,\alpha_2^2), \tag{A7}$$

whence

$$\left(\frac{\partial E_\gamma}{\partial \alpha_2}\right)_r \approx \{0\cdot 6\,Z^2 R^2 e^2/D^3 \lambda^2 (1+\alpha_2)\}(1 - 1\cdot 5\,\alpha_2 - 0\cdot 25\,\alpha_2^2). \tag{A8}$$

From equations (A6) and (A8) and assuming $E_{\gamma_\mathrm{T}} = 7\cdot 5$ MeV, we obtain

$$E_{\gamma_\mathrm{T}}^{-1}(\partial E_\gamma/\partial E_{\mathrm{K_T}})_r \approx -(0\cdot 15/7\cdot 5)(1 - 1\cdot 5\,\alpha_2 - 0\cdot 25\,\alpha_2^2)/(1+\alpha_2)^2 . \tag{A9}$$

Equation (A9) predicts that, for $\alpha_2 > 0\cdot 6$, E_γ approximately increases with $E_{\mathrm{K_T}}$. In the symmetric region we have $\alpha_2 = 0\cdot 4$ and the calculated value of

$$E_{\gamma_\mathrm{T}}^{-1}(\partial E_\gamma/\partial E_{\mathrm{K_T}})_r \approx -0\cdot 00036 \quad \mathrm{MeV}^{-1}.$$