AN ANALYSIS OF INELASTIC SCATTERING TO THE 2^+ STATE IN ^{54}Fe AT $1\cdot409$ MeV

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Abstract

The differential cross section, spin-flip probability amplitude, and asymmetry for the inelastic scattering of $19\cdot6$ MeV protons to the 2^+ state in ^{54}Fe at $1\cdot409$ MeV are analysed. The antisymmetrized distorted wave approximation is used to evaluate the scattering amplitudes in which the reaction mechanism is represented by a generalized core polarization model. Good fits are obtained to the differential cross section and spin-flip probability data. The exchange core polarization contributions are shown to be essential to obtain such fits. However, the asymmetry data cannot be fitted simultaneously, and this failure demonstrates the limitations of the current collective model representation of the core polarization process.

I. INTRODUCTION

In recent years there have been many experiments performed in which spin-dependent data (polarization, asymmetry, and spin-flip probability amplitude) associated with the inelastic scattering of nucleons from nuclei have been measured (see the survey by Satchler 1971). Asymmetries have been measured for a large number of reactions since the availability of polarized nucleon beams of reasonable current, but less data are available for the polarizations of emergent particles because the experiments require a second (analysing) scattering. As this analysing scattering is also required to measure spin-flip probabilities, a similar lack of data would be expected. However, for the special case of inelastic nucleon scattering leading to a 2^+ final state of the nucleus, which then de-excites by the emission of an E2 γ-ray, a simpler correlation experiment can determine the spin-flip probability amplitude. Specifically, the spin-flip probability amplitude can be related to the differential cross section for scattered nucleons which are in coincidence with an E2 γ-ray that emerges perpendicular to the scattering plane (Bohr 1959).

Obviously, the experiments that measure spin-dependent data are more difficult to perform than those in which only the differential cross sections for inelastic scattering are determined, and consequently most measurements to date have been for reactions with relatively large cross sections, namely the inelastic scattering of nucleons to the so-called "collective" states of nuclei. The structure of the differential cross sections of such reactions usually can be predicted very well from distorted wave approximation calculations based upon the collective (generalized optical) model theory of inelastic scattering (Satchler 1966, 1971; Sherif and Blair 1970). The magnitudes of the differential cross sections then determine the value of the deformation parameter of the generalized optical model potential. However, the

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values of the deformation parameters extracted by this procedure do not always agree with those obtained from analyses of other experiments (Stovall and Hintz 1964). Further, the collective model analyses of spin-dependent data from inelastic nucleon scattering are not too successful since the structural variations in results found for different isotopes and for a given nucleus with increasing projectile energy cannot be predicted (Reif and Hohn 1969; Satchler 1971).

The data for the inelastic scattering of 19·6 MeV protons to the first excited 2^+ state (1·409 MeV) in ^{54}Fe when compared with the predictions of the collective (generalized optical) model illustrate the inadequacies of the standard analysis. The differential cross section data can be fitted quite well by the collective model calculations but both the spin-flip probability and asymmetry data cannot (Hendrie *et al.* 1969). It has been suggested by McDaniel and Amos (1972) that the discrepancies between the data and the collective model analyses are due to the omission in the calculations of particle exchange processes. A "microscopic" theory of inelastic nucleon scattering (Amos *et al.* 1967; Geramb and Amos 1971) based upon an antisymmetrized distorted wave approximation automatically includes such exchange processes, the importance of which have been demonstrated in a number of applications (Love and Satchler 1970; Geramb and Amos 1971; Geramb 1972; Geramb *et al.* 1973).

For the strong collective transitions, such as the excitation of the 2^+ state in ^{54}Fe at 1·409 MeV, the dominant reaction process in the microscopic theory is that of core polarization (Love and Satchler 1967). This reaction process permits energy and momentum transfers to take place between the projectile and any individual bound nucleon in the target and to be mediated by the virtual excitation of the target as a whole. Direct and exchange core polarization components can be identified in the theory (Love and Satchler 1971): the direct core polarization components in the matrix elements are defined as those components in which the incident and emergent projectile have the same complete set of coordinates, while the exchange core polarization components are defined as those components of the matrix elements in which the emergent particle is a nucleon that was initially bound in the target.

The direct core polarization matrix elements have a one-to-one correspondence with the matrix elements deduced from the collective (generalized optical) model theory of inelastic scattering, but the strengths of the direct core polarization matrix elements are not adjustable and must be specified by an analysis of the γ-ray transition rate for the de-excitation of the residual nuclear state in the inelastic scattering reaction (Love and Satchler 1967). Of course, model parameterization permits a wide range of values of the direct core polarization strengths. However, the strengths of the exchange core polarization components are not as well defined. The most useful representation for these latter components results if the reaction process is associated with the virtual excitation of giant multipoles in the target (Love and Satchler 1971; Geramb *et al.* 1973), in which case the strengths of the core polarization exchange amplitudes are determined by the energy shapes and strengths of the giant multipoles. With this interpretation of the core polarization components and, using a hydrodynamical model (Faessler and Greiner 1962) to define the giant multipole properties, excellent fits have been obtained to the differential cross sections in a number of reactions (Geramb *et al.* 1973).

In view of the above successes, and to elaborate upon the implications of an earlier study (McDaniel and Amos 1972), the differential cross section, spin-flip probability amplitude, and asymmetry for the inelastic scattering of 19·6 MeV protons leading to the 2^+ state in ^{54}Fe at 1·409 MeV excitation have been simultaneously analysed here using an antisymmetrized distorted wave approximation in a microscopic theory of the reaction in which it is assumed that the reaction mechanism is of the core polarization type. The details of the calculations are specified in the next section of this paper and the results are presented in Section III. The significance of the results is discussed in Section IV.

II. Calculations

The four measurable quantities associated with the inelastic scattering of protons from nuclei, namely the differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$, the polarization $P(\theta)$, the asymmetry $A(\theta)$, and the spin-flip probability amplitude $S(\theta)$, are defined by the equations

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sigma_{++}+\sigma_{+-}+\sigma_{-+}+\sigma_{--}\,, \qquad P(\theta) = \frac{\sigma_{++}-\sigma_{+-}+\sigma_{-+}-\sigma_{--}}{\mathrm{d}\sigma/\mathrm{d}\Omega}\,, \qquad \text{(1a, b)}$$

$$A(\theta) = \frac{\sigma_{++}+\sigma_{+-}-\sigma_{-+}-\sigma_{--}}{\mathrm{d}\sigma/\mathrm{d}\Omega}\,, \qquad S(\theta) = \frac{\sigma_{+-}+\sigma_{-+}}{\mathrm{d}\sigma/\mathrm{d}\Omega}\,. \qquad \text{(1c, d)}$$

Here θ is the scattering angle and $\sigma_{\nu\nu'}$ is the probability in the scattering that the incident and detected protons have spin projection quantum numbers ν and ν' respectively (only the signs of the projection quantum numbers are shown in equations (1)). The quantization axis is chosen to be perpendicular to the scattering plane and parallel to the angular momentum transfer vector. With this standard convention, the polarization equates to the left–right difference in the numbers of emergent protons following a second (analysing) scattering, while the asymmetry is determined by the left–right difference in the scattering of polarized protons.

In a direct reaction theory of inelastic scattering the partial cross sections $\sigma_{\nu\nu'}$ are proportional to transition matrix elements T defined by

$$T = T(\nu,\nu',M,M',\theta) = A\langle X_{\nu'}^{(-)}(0)|\langle\psi_{J'M'T'}(1\,...\,A)\,|\,t(0\,1)\,|\,\Psi_{JMT}^{\nu}(0\,1\,...\,A)\rangle\,, \qquad (2)$$

where the primes denote final state quantum numbers. The nucleus from which the protons are scattered has an atomic mass number A and states specified by the quantum numbers J, M, and T, while the spatial spin and isotopic spin coordinates of each nucleon are represented collectively by the numbers $0,1,2,\ldots$. It has been assumed that a local effective two-nucleon interaction represents the reaction mechanism and that this effective interaction is the same between the projectile and any of the bound nucleons.

The distorted wave approximation (Satchler 1966) to the scattering matrix element can be defined by using

$$\Psi_{JMT}^{\nu}(0\,1\,...\,A) = |X_{\nu}^{(+)}(0)\rangle\,|\psi_{JMT}(1\,...\,A)\rangle \qquad (3)$$

in equation (2), where the states $|X_{\nu}\rangle$ are to be determined by an optical model calculation, the results of which should "best fit" the appropriate elastic scattering data (Austern 1963). The collective (generalized optical) model description of inelastic scattering follows from this formalism by identifying the form factor

$$F(0) = A\langle\psi_{J'M'T'}(1\ldots A)\,|\,t(0\,1)\,|\,\psi_{JMT}(1\ldots A)\rangle\,, \tag{4}$$

with a deformation of the optical model potential (Satchler 1966, 1971; Sherif and Blair 1970). In this description of inelastic scattering, details of the structure of the target states are neglected and the theory is based upon a many-particle state (equation (3)) which is not antisymmetric with respect to the interchange of any target nucleon with the projectile.

In the microscopic theory of Amos *et al.* (1967) and Geramb and Amos (1971) an antisymmetrized distorted wave approximation can be made by using

$$\Psi^{\nu}_{JMT}(0\,1\ldots A) = \{X^{(+)}_{\nu}(0)\,\psi_{JMT}(1\ldots A) - X^{(+)}_{\nu}(1)\,\psi_{JMT}(0\,2\ldots A)\}\,. \tag{5}$$

Then, with the cofactor expansion

$$\psi_{JMT}(1\ldots A) = A^{-\frac{1}{2}}\sum_{jm\alpha}\phi_{jm\alpha}(1)\,a_{jm\alpha}\,\psi_{JMT}(1\ldots A) \tag{6}$$

of the target states, the scattering amplitude (equation (2)) can be re-expressed as

$$\begin{aligned} T = \sum_{jj'mm'\alpha\alpha' IN\tau\rho} & S(jj'JJ'TT';I\tau)\langle T\tau P\rho\,|\,T'P'\rangle \\ & \times\langle JIMN\,|\,J'M'\rangle(-)^{j-m}(-)^{\frac{1}{2}-\alpha}\langle jj'\,m-m'\,|\,I-N\rangle \\ & \times\langle\tfrac{1}{2}\tfrac{1}{2}\,\alpha-\alpha'\,|\,\tau-\rho\rangle\,[(2J'+1)(2T'+1)]^{-\frac{1}{2}} \\ & \times\langle X^{(-)}_{\nu'}(0)\,|\,\langle\phi_{j'm'\alpha'}(1)\,|\,t(0\,1)\,|\,\{X^{(+)}_{\nu}(0)\,\phi_{jm\alpha}(1) - X^{(+)}_{\nu}(1)\,\phi_{jm\alpha}(0)\}\rangle\,. \end{aligned} \tag{7}$$

The present notation is that used by Amos *et al.* (1967) and Geramb and Amos (1971), with the trivial exception that primes are used here instead of their suffixes 1 and 2 or i and f. Equation (7) shows that the scattering amplitude is a multiple sum of two-body matrix elements, each of which is weighted by a single-particle density factor S that contains all the many-nucleon properties of the target states. To evaluate the scattering amplitude, multipole expansions of the distorted wavefunctions (the partial wave series) and the effective interaction must be made. As all the details of these expansions and the resultant form of the scattering amplitude have been given by Amos *et al.* (1967) and Geramb and Amos (1971), only the quantities relevant to later discussions are shown explicitly here.

The transition interaction is usually split into two parts. The first part, which gives the "valence" contributions to the scattering amplitude, is an effective interaction between the projectile and that bound nucleon initially described by the single-particle state $\phi_{jm\alpha}$. This effective interaction is explicitly the two-nucleon t-matrix in nuclear matter (McCarthy 1968). Because of calculational problems, gross simplifying approximations must be made; from other studies of inelastic scattering (Love and Satchler 1970), the long range part of the central Hamada–Johnston two-nucleon potential is an adequate approximation for this t-matrix.

The second part of the transition interaction gives the "core polarization" contributions to the scattering amplitude. In this core polarization process the active bound state nucleon in the scattering amplitude (7) receives energy and momenta indirectly from the projectile and all such transfers are mediated by a virtual excitation of the nucleus as a whole. A collective model is normally used to represent these virtual excitations (Love and Satchler 1967).

Both components of the effective interaction can be expanded using the same multipole series, namely

$$t(0\,1) = \sum_{LMji} t_{LM}^{ij}(r_0, r_1)\, Y_{LM}(\Omega_0)\, Y_{LM}^*(\Omega_1)\, \boldsymbol{S}_i(0)\,.\,\boldsymbol{S}_i(1)\, \boldsymbol{T}_j(0)\,.\,\boldsymbol{T}_j(1)\,, \tag{8}$$

where i and j take the values 1 and 2 for which the operators are

$$\boldsymbol{S}_1 = \boldsymbol{T}_1 = \mathbf{1}\,, \qquad \boldsymbol{S}_2 = \boldsymbol{\sigma}\,, \qquad \boldsymbol{T}_2 = \boldsymbol{\tau}\,, \tag{9}$$

with $\mathbf{1}$ being the unit matrix. The valence and core polarization components of the effective interaction are associated with radial multipole terms (Love and Satchler 1967),

$$t_{LM}^{ij}(r_0, r_1) = v_L^{ij}(r_0, r_1) + y_L^{ij}(\varepsilon)\, k_L(r_0)\, k_L(r_1)\,. \tag{10}$$

The second term in equation (10) is the core polarization radial multipole for which the strength factors† $y_L^{ij}(\varepsilon)$ are energy dependent. The energy at which the strength factor is to be found in the direct matrix elements is the modulus of the reaction Q value, while in the exchange matrix elements the core polarization strength factors are to be evaluated at an energy equal to the single-particle energy change, namely

$$\varepsilon = E_{\mathrm{p}} - E_{\mathrm{bd}}\,, \tag{11}$$

where E_{p} is the kinetic energy of the projectile and E_{bd} is the binding energy of the bound nucleon in the final state (Love and Satchler 1971; Geramb 1972). The radial form factors in the core polarization component of the effective interaction are usually represented by the generalized optical model (Satchler 1966) and thus have the form

$$k_L(r_0) = R\,\partial\{V_{0m}(r_0)\}/\partial r_0\,, \tag{12}$$

where $V_{0m}(r_0)$ is the optical model potential of radius R that best fits the elastic scattering data (for simplicity, the spin–orbit potentials are assumed to be spherical). Similarly, the bound state potential is assumed to be deformed and the deformation defines the radial form factor $k_L(r_1)$.

In the direct matrix elements (Geramb and Amos 1971) the selection rules limit contributions from the multipoles of the effective interactions to those with quantum numbers determined by the orbital angular momentum transfer in the reaction. The corresponding polarization strength factors then have a direct relationship to the effective charges and g factors required by the nuclear spectroscopy to predict the observed transition rate of the γ-ray de-excitation of the final state of the

† These strength factors incorporate the volume integral weights used by Love and Satchler (1971).

nucleus (Love and Satchler 1967, 1971). Hence the effective charges associated with the spectroscopy of the states of the target nucleus define the strength factors $y_L^{ij}(\mathrm{Q})$ required in the direct core polarization components of inelastic scattering. The exchange core polarization matrix elements (Geramb and Amos 1971) permit contributions from all multipoles of the effective interaction. To define the strength factors $y_L^{ij}(\varepsilon)$ in this case, it is convenient to associate each multipole in the matrix elements with the virtual excitation of a giant resonance in the target nucleus (Geramb *et al.* 1973). Unfortunately there are few data available from which detailed properties of these giant multipole resonances can be extracted (the giant dipole being perhaps an exception), but a hydrodynamic two-fluid model (Faessler and Greiner 1962) can be used to specify their properties for use in the exchange core polarization components in inelastic scattering.

Specifically, we now consider the excitation of the 2^+ state at $1\cdot409$ MeV in ^{54}Fe by the inelastic scattering of $19\cdot6$ MeV protons. The distorted waves (initial and final channels) are calculated from an optical model, the potential parameters of which give a best fit to the experimental elastic scattering data (Hendrie *et al.* 1969). The ground and first excited 2^+ states in ^{54}Fe are represented by the dominant terms of a standard shell model calculation (Lips and McEllistrŏm 1970), namely

$$|^{54}\mathrm{Fe, g.s.}\rangle = |(1\mathrm{f}_{7/2})^{-2};0^+\rangle \tag{13a}$$

$$|^{54}\mathrm{Fe}, 1\cdot409\rangle = (0\cdot8)^{\frac{1}{2}}|(1\mathrm{f}_{7/2})^{-2};2^+\rangle + (0\cdot2)^{\frac{1}{2}}|(1\mathrm{f}_{7/2})^{-1}(2\mathrm{p}_{3/2})^{-1};2^+\rangle, \tag{13b}$$

where the single-particle proton states are represented by harmonic oscillator wave-functions calculated for an oscillator energy of 11 MeV.

The valence interaction is not used in these calculations since previous analyses of the inelastic scattering of $19\cdot6$ MeV protons from ^{54}Fe (McDaniel and Amos 1972) have shown that only a small fraction of the transition strength results from the long-range part of the central Hamada–Johnston potential. At most therefore the valence interaction components can provide a “fine tuning” of the predictions obtained using the core polarization reaction mechanism.

The direct core polarization component of the scattering amplitude is determined solely by the quadrupole term in the usual collective model expansion. A quadrupole strength $y_2(\mathrm{Q})$ of $2\cdot85\times10^{-3}$ MeV^{-1} is used (McDaniel and Amos 1972). This value corresponds to the effective charge of $2\cdot5\,e$ which is required by the nuclear spectroscopy to predict a $B(\mathrm{E2})$ value in agreement with the observed data on the γ-decay of the 2^+ state in ^{54}Fe.

The exchange core polarization component of the scattering amplitude is restricted to receive a contribution only from an isovector giant dipole resonance of the target. For ^{54}Fe, hydrodynamic model calculations predict that the giant dipole, quadrupole, and octupole resonances have energy maxima at 15, 18, and 22 MeV respectively.* However, the dipole approximation should be good since γ-induced reactions have shown that the giant dipole exists at about 15 MeV excitation in ^{54}Fe and is particularly strong. The isovector nature of the resonance results

* Specifically, these energies result by using the hydrodynamic model scaling of $A^{\frac{1}{3}}$ upon the resonance shapes of Geramb *et al.* (1973).

by restricting contributions in the effective interaction of equation (8) to those in which $i = 1$ and $j = 2$. The dipole strength factor $y_1(\mathrm{E})$ is taken as $0 \cdot 0251\ \mathrm{MeV}^{-1}$, a value based solely upon the fit to the magnitude of the differential cross section.

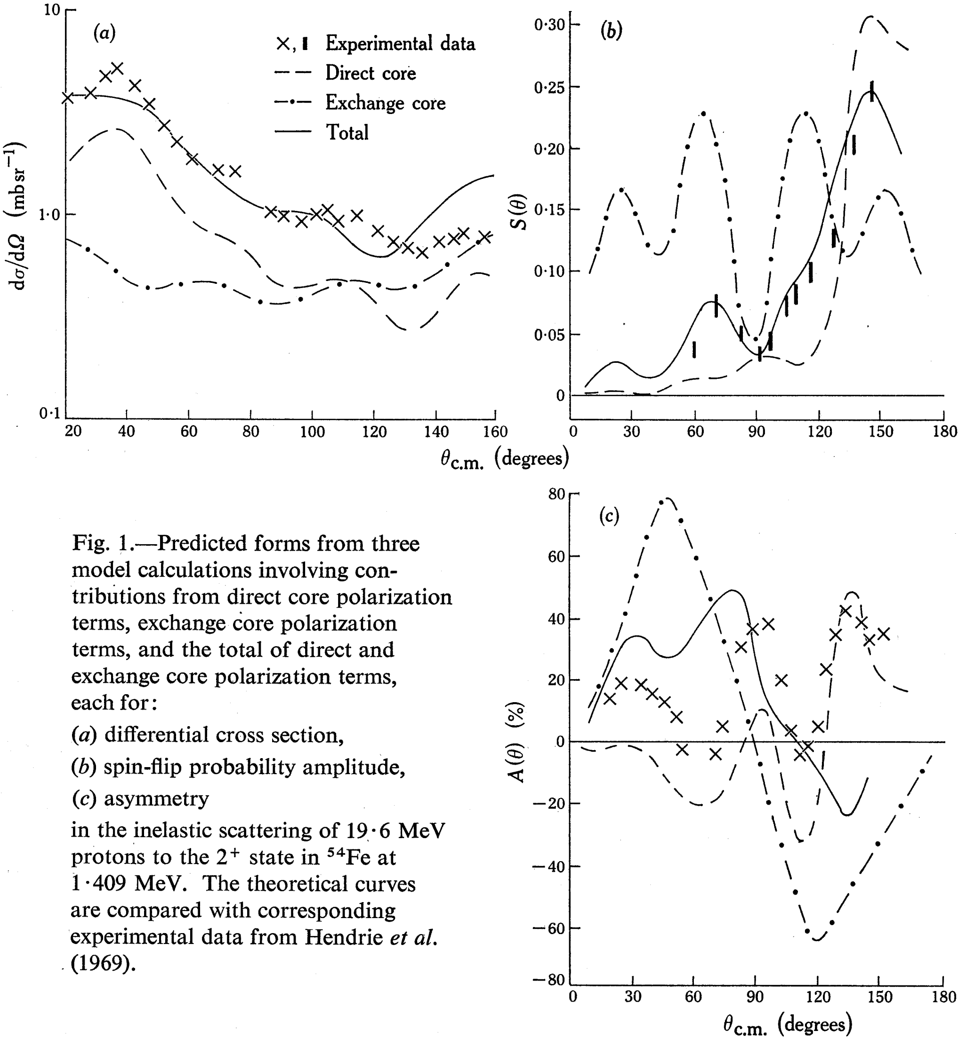

Fig. 1.—Predicted forms from three model calculations involving contributions from direct core polarization terms, exchange core polarization terms, and the total of direct and exchange core polarization terms, each for:

(*a*) differential cross section,

(*b*) spin-flip probability amplitude,

(*c*) asymmetry

in the inelastic scattering of 19·6 MeV protons to the 2^+ state in ^{54}Fe at 1·409 MeV. The theoretical curves are compared with corresponding experimental data from Hendrie *et al.* (1969).

III. Results

The experimental data for the differential cross section, spin-flip probability amplitude, and asymmetry from Hendrie *et al.* (1969) are each compared in Figure 1 with three predictions from the model calculations: direct core polarization results, obtained by omitting exchange contributions to the scattering amplitude; exchange core polarization results, obtained by omitting direct contributions; and total results from the complete calculations. The direct core polarization calculations can be related to calculations of inelastic scattering in which the usual collective (generalized optical) model approximation is used (Satchler 1966). The correspondence is complete when the deformation parameter is defined by

$$\beta_L = \sum_{jj'mm'\alpha\alpha' IN\tau\rho} S(jj'JJ'TT';I\tau)$$
$$\times\langle JIMN\,|\,J'M'\rangle\langle T\tau P\rho\,|\,T'P'\rangle(-)^{j-m}(-)^{\frac{1}{2}-\alpha}\langle jj'\,m-m'\,|\,I-N\rangle$$
$$\times\langle\tfrac{1}{2}\tfrac{1}{2}\alpha-\alpha'\,|\,\tau-\rho\rangle(2T'+1)^{-\frac{1}{2}}y_L(\mathrm{Q})\langle\phi_{j'm'\alpha'}\,|\,k_L(r_1)\,Y^*_{LM}(\Omega_1)\,|\,\phi_{jm\alpha}\rangle\,. \tag{14}$$

Hence, to the extent that the direct core polarization predictions are inadequate so also will be the analyses in which the collective model approximation for inelastic scattering is used.

The differential cross section analyses are shown in Figure 1(*a*). The structure and magnitude of the experimental data are reasonably reproduced by the total calculation except at large scattering angles. The direct core polarization results also reproduce the structure of the data reasonably well; however, as the predicted magnitude is small by a factor of two, the usual collective model analyses would predict deformation strengths too large by up to 40%. A better fit to the structure of the differential cross section data could be obtained with the models described above, but this is not essential for the present discussion.

The results for the spin-flip probability amplitude are compared with the experimental data in Figure 1(*b*). The direct core polarization calculations do not reproduce the observed structure (although a large experimental back angle peak is correctly predicted). The exchange core polarization calculations by themselves are of little interest save that strong peaks are predicted at 60° and 110° in the centre of mass system with a relatively small peak at 150°, and this structure is important since the data have peaks at these scattering angles. The total result, obtained by allowing the direct and exchange amplitudes to interfere, fits the observations very well. The fit at large scattering angles must be considered fortuitous, however, since the predictions of the spin-flip probability amplitudes are normalized by the predictions of the differential cross section (see equation (1d)). Nevertheless, the importance of exchange contributions and consequently the inadequacy of the usual collective model calculations is established.

The predictions and experimental data for the asymmetry are shown in Figure 1(*c*). The direct core polarization results reproduce the general structure of the data but with magnitudes that are wrong everywhere except at large scattering angles. This fit to the structure is deceptive, since in order to improve the predictions the model must be modified by a mechanism that considerably enhances the scattering probabilities of projectiles having spin projection $+\frac{1}{2}h$ compared with the scattering probabilities of those having spin projection $-\frac{1}{2}h$ (see equation (1c)). This problem occurs in collective model analyses of other inelastic scattering reactions (Reif and Hohn 1969; Sherif and Blair 1970), and therefore appears to be a general inadequacy of the usual collective model of inelastic scattering. The compaıative success of the core polarization calculations in fitting the differential cross section and spin-flip probability amplitude data in Figures 1(*a*) and 1(*b*) respectively makes the poor fit to the asymmetry data in Figure 1(*c*) disappointing at first sight. However, the exchange core polarization terms contribute significantly to the predictions and provide the necessary trend in the forward directions. The complete inadequacy at backward scattering angles can be discounted, to a large extent, since the differential cross section is not fitted at these angles. Further, unlike differential cross sections and spin-flip probability amplitude calculations, the asymmetry calculations reflect differences between partial cross sections (see equations (1)). Hence asymmetry data should be most sensitive to details of the reaction mechanism and to small contributions to the exchange core polarization amplitude from multipoles other than the dipole term.

IV. Conclusions

Analyses of the differential cross section, spin-flip probability amplitude, and asymmetry data for the inelastic scattering of 19·6 MeV protons to the 2^+ state in ^{54}Fe have shown the importance of exchange core polarization contributions to the scattering amplitude. As a consequence, analyses of inelastic nucleon scattering can be inaccurate when the usual collective (generalized optical) model approximation is used.

The structure of the differential cross sections is fitted reasonably well with or without the exchange core polarization contributions. However, if the direct core polarization component strength is determined from an analysis of the γ-ray transition rate for the subsequent decay of the excited 2^+ state, a significant exchange core polarization contribution to the inelastic scattering process is necessary. The correspondence between the direct core polarization and the usual collective model calculations then implies that the latter model can seriously overestimate the deformation of the nucleus.

The spin-flip probability amplitude analyses have demonstrated the significance of the exchange core polarization components in the reaction mechanism. Structure that has previously defied explanation can be interpreted as being due to the virtual excitation of a giant dipole resonance in the reaction. Finally, the analysis of the asymmetry has shown that exchange core polarization contributions are essential to account for the experimental data. However, as the asymmetry reflects the differences between the partial cross sections, such data are very sensitive to details of the model calculations. The current models are not yet sufficiently sophisticated to adequately represent the reaction mechanism. Studies of the effects of multipole resonances other than an isovector dipole and of the importance of a deformed spin–orbit potential are in progress.

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VI. References

Amos, K., Madsen, V. A., and McCarthy, I. E. (1967).—*Nucl. Phys.* A **94**, 103.

Austern, N. (1963).—*In* "Selected Topics in Nuclear Theory". (Ed. F. Janouch.) (I.A.E.A.: Vienna.)

Bohr, A. (1959) —*Nucl. Phys.* **10**, 486.

Faessler, A., and Greiner, W. (1962).—*Z. Phys.* **168**, 425.

Geramb, H. V. (1972).—*Nucl. Phys.* A **183**, 582.

GERAMB, H. V., and AMOS, K. (1971).—*Nucl. Phys.* A **163**, 337.
GERAMB, H. V., SPRICKMANN, R., and STROBEL, G. (1973).—*Nucl. Phys.* A **199**, 545.
HENDRIE, D. L., GLASSHAUSER, C., MOSS, J. M., and THIRION, J. (1969).—*Phys. Rev.* **186**, 1188.
LIPS, K., and MCELLISTRÖM, M. T. (1970).—*Phys. Rev.* C **1**, 1009.
LOVE, W. G., and SATCHLER, G. R. (1967).—*Nucl. Phys.* A **101**, 424.
LOVE, W. G., and SATCHLER, G. R. (1970).—*Nucl. Phys.* A **159**, 1.
LOVE, W. G., and SATCHLER, G. R. (1971).—*Nucl. Phys.* A **172**, 449.
MCCARTHY, I. E. (1968).—"Introduction to Nuclear Theory." (Wiley: New York.)
MCDANIEL, F. D., and AMOS, K. (1972).—*Nucl. Phys.* A **180**, 497.
REIF, R., and HOHN, J. (1969).—*Nucl. Phys.* A **137**, 65.
SATCHLER, G. R. (1966).—*In* "Lectures in Theoretical Physics". Vol. VIIIC. (Eds. P. D. Kunz, D. A. Lind, and W. E. Brittin.) (Univ. Colorado Press.)
SATCHLER, G. R. (1971).—*In* "Polarization Phenomena in Nuclear Reactions". (Eds. H. H. Barschall and W. Haeberli.) (Univ. Wisconsin Press.)
SHERIF, H., and BLAIR, J. S. (1970).—*Nucl. Phys.* A **140**, 33.
STOVALL, T., and HINTZ, N. M. (1964).—*Phys. Rev.* **135**, B330.