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Parity-nonconserving Effects in n–p Capture at Thermal Energies*

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Abstract

The circular polarization and asymmetry, about the incident neutron spin direction, of the photon released in the reaction $n+p \rightarrow d+\gamma$ at thermal neutron energies are consequences of parity violation in the n–p interaction. A formalism is developed which expresses the polarization and asymmetry in terms of the half-off-shell transition matrix, and calculations are performed for several models of the weak parity-nonconserving N–N interaction. The results indicate that the polarization is sensitive to the choice of both the strong N–N interaction and weak N–N interaction while the asymmetry is sensitive mainly to the latter. The observed polarization is of the same sign and rather larger than that calculated with the conventional weak N–N potential. Difficulties in the present method indicate that this discrepancy cannot be regarded as significant.

1. Introduction

Studies of the manifestations of one-meson-exchange parity-nonconserving (PNC) internucleon interactions in finite nuclei have been pursued in recent years with the hope of distinguishing between several proposed models of the weak interaction. These models have been reviewed by Henley (1969), McKellar (1970), Fischbach and Tadić (1973) and Gari (1973). However, since the weak N–N interaction is of short range, its effects are sensitive to the nature of the two-nucleon wavefunction at small separations (i.e. to nucleon correlations). Consequently the nuclear physics aspects of the problem have to be disentangled before any model of the weak interaction can yield results which offer a useful comparison with experiment.

The most straightforward physical situation from the nuclear physics standpoint is the two-nucleon problem, in which the correlations are understood qualitatively and are described by the wavefunction, or equivalently by the off-shell behaviour of the transition matrix. Consequently the two-nucleon system offers the most promise of a complete theoretical treatment of PNC effects. Several such treatments have been attempted in the past by Blin-Stoyle and Feshbach (1961), Danilov (1965, 1971), Tadić (1968) and Hadjimichael and Fischbach (1971), but there is little consistency between their predictions. Moreover, all predictions of the circular polarization P_γ of the 2·23 MeV photon are at least an order of magnitude smaller than the recent experimental result of Lobashov *et al.* (1972), and all but one are of the opposite sign. These theoretical results for the conventional or Cabibbo (1963) model of the weak interaction in the factorization approximation are summarized in Table 1, together with the experimental result of Lobashov *et al.*

* Preliminary reports of this work have been given at the International Conference on Nuclear Physics, Munich, 1973 (Lassey and McKellar 1973), at the Symposium on Correlations in Nuclei, Balatonfüred, Hungary, 1973 (McKellar 1974), and by Lassey and McKellar (1974).

A study of the low energy n–p system also has the advantage that the isospin structure of the weak interaction is revealed: only the $\Delta T = 0$ (isoscalar) component contributes to the circular polarization, while only the $\Delta T = 1$ (isovector) component contributes to the asymmetry of the angular distribution of the photon about the spin of the incident neutron (Danilov 1965). An experimental measurement of the latter has not been made, but there is reason to believe that it is feasible if the asymmetry is $\sim 10^{-6}$ or greater in magnitude (R. Wilson, personal communication).

In the present work we calculate the circular polarization P_γ and the asymmetry α for several weak-interaction models currently in vogue, and for a variety of strong interactions. The weak nucleon–nucleon interaction is computed in the π and ρ exchange model, the parity-violating NNπ and NNρ couplings being determined by current algebra and factorizations respectively. Each of these approximations has been questioned recently (McKellar and Pick 1972, 1973; Körner 1973), but we employ them because we are primarily concerned here with the nuclear physics of the problem, and the use of standard weak PNC potentials provides an immediate comparison of our results with the earlier ones.

Table 1. Comparison of previous results for circular polarization

The theoretical values are for the Cabibbo weak interaction

Strong interaction	$P_\gamma \times 10^7$
Hulthen with hard core, for S-waves (Tadić 1968)	$+2{\cdot}2$
Yale (Hadjimichael and Fischbach 1971)	$+0{\cdot}031$
Livermore phase shifts and dispersion theory (Danilov 1971)	$-0{\cdot}18$
Experimental result (Lobashov *et al.* 1972)	$-(13{\cdot}0 \pm 4{\cdot}5)$

The strong interaction enters the present calculation through the half-off-shell transition matrices for n–p scattering. Two approximations are employed in the formalism: the first is that the M1 and E1 operators depend mainly upon the asymptotic nature of the incident and deuteron wavefunctions, and the second is that the D-state component of the deuteron can be neglected in calculating the electromagnetic matrix elements. With these approximations Danilov (1965) was able to express P_γ and α in terms of the PNC n–p scattering amplitude. This method, which is employed here, was also used by Tadić (1968) and in Danilov's (1971) dispersion calculation but was not employed by Hadjimichael and Fischbach (1971).

In Section 2 we review the weak-interaction models and PNC potentials used in the calculation, and in Section 3 we display the formalism of our method. The strong interactions are considered in Section 4. The final results are reported and discussed in Section 5, and some conclusions are drawn in Section 6.

2. Weak-interaction Models

The various models of the weak N–N interaction resulting from one pion and one vector meson exchange have been described in detail elsewhere (e.g. in the reviews cited in the first paragraph of the Introduction). The interaction appropriate to a $T_3 = 0$ system is taken to be

$$V_{\mathrm{PNC}} = V_{\mathrm{PNC}}(\Delta T = 0) + V_{\mathrm{PNC}}(\Delta T = 1), \tag{1}$$

where

$$V_{\mathrm{PNC}}(\Delta T=1) = AV_\pi[\boldsymbol{p}, r^{-1}\exp(-\mu_\pi r)]_- \cdot(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})T_{12}^{(-)} + V_\rho[\boldsymbol{p}, r^{-1}\exp(-\mu_\rho r)]_+ \cdot\{\tfrac{1}{4}\sqrt{3}\,C'(\boldsymbol{\sigma}^{(1)}\tau_0^{(1)}-\boldsymbol{\sigma}^{(2)}\tau_0^{(2)}) + \tfrac{1}{2}\xi C(\boldsymbol{\sigma}^{(1)}\tau_0^{(2)}-\boldsymbol{\sigma}^{(2)}\tau_0^{(1)})\}, \tag{2}$$

and

$$V_{\mathrm{PNC}}(\Delta T=0) = V_\rho[\boldsymbol{p}, r^{-1}\exp(-\mu_\rho r)]_+ \cdot(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\{T_{12}^{(+)}+\tfrac{1}{4}B\tau_0^{(1)}\tau_0^{(2)}+\tfrac{1}{2}\sqrt{3}\,\xi D\} + V_\rho[\boldsymbol{p}, r^{-1}\exp(-\mu_\rho r)]_- \cdot(\mathrm{i}\,\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)}) \times\{(1+\mu_V)(T_{12}^{(+)}+\tfrac{1}{4}B\tau_0^{(1)}\tau_0^{(2)})+\tfrac{1}{2}\sqrt{3}\,\xi D\}. \tag{3}$$

Table 2. Model-dependent parameters of PNC weak N–N interaction

The model-dependent parameters of equations (2) and (3) are listed in terms of the Cabibbo angle θ_c, which is taken to be given by $\sin\theta_c = 0{\cdot}21$; $\sin^2\theta_c$ has been neglected in comparison with unity

Model	A	B	C	C'	D
1. Cabibbo (1963)	$-\sqrt{\frac{1}{2}}\tan\theta_c$	0	0	0	0
2A. Segré (1968), γ_5 invariant	0	0	0	0	$\frac{4}{3}$
2B. Segré (1968), γ_5 noninvariant	-4	0	0	$-\sqrt{\frac{4}{3}}$	$\frac{4}{3}$
3. Lee (1968)	$-\sqrt{\frac{1}{8}}\cot\theta_c$	1	0	$-\sqrt{3}$	0
4. d'Espagnat (1963)	$\sqrt{\frac{1}{2}}\cot\theta_c$	2	$-\sqrt{\frac{4}{3}}$	$-\sqrt{\frac{4}{3}}$	$\frac{2}{3}$
5. Oakes (1968)	$-\sqrt{8}\,\mathrm{cosec}\,2\theta_c$	-2	$2\sqrt{3}$	$2\sqrt{3}$	-6
6. Brunet (1967)	$-\frac{1}{3}\sqrt{2}\tan\theta_c$	2	$\sqrt{3}\sin^2\theta_c$	$\sqrt{3}\sin^2\theta_c$	$\frac{3}{2}\sin^4\theta_c$
7. Lee and Yang (1960)	$\sqrt{2}\tan\theta_c$	2	0	0	0
8. Michel (1967)	$\frac{3}{10}\sqrt{2}\cot\theta_c$	2	$\sqrt{\frac{4}{3}}\tan^2\theta_c$	$\sqrt{\frac{4}{3}}\tan^2\theta_c$	2
9A. Tomozawa (1970) octet	0	1	0	0	1
9B. Tomozawa (1970), nonet	$-\frac{1}{2}\tan\theta_c$	1	0	0	1

In equations (2) and (3) the terms $[\boldsymbol{p}, f(r)]_\pm$ denote $\boldsymbol{p}f(r) \pm f(r)\boldsymbol{p}$, in which $\boldsymbol{p} = -\mathrm{i}\nabla$ is the relative momentum operator; the superscripts (1) and (2) indicate the particle spinor or isospinor on which the operator acts;

$$T_{12}^{(\pm)} = (\tau_+^{(1)}\tau_-^{(2)} \pm \tau_-^{(1)}\tau_+^{(2)}),$$

with $\tau_\pm = \frac{1}{2}(\tau_x \pm \mathrm{i}\tau_y)$; and μ_V is the isovector anomalous nucleon magnetic moment. The strengths V_π and V_ρ are taken from Hadjimichael and Fischbach (1971) to have the numerical values

$$V_\pi = 8{\cdot}54\times10^{-6}\ \ \mathrm{MeV\,fm^2}, \qquad V_\rho = -1{\cdot}993\times10^{-5}\cos^2\theta_c\ \ \mathrm{MeV\,fm^2}.$$

The SU(3) parameter ξ is taken to be $(6\sqrt{3})^{-1}$ and the model-dependent parameters A, B, C, C' and D are set out in Table 2 in terms of the Cabibbo angle θ_c, which we take throughout to be given by $\sin\theta_c = 0{\cdot}21$. As was emphasized in the Introduction, these standard values of the parameters are used to permit easier comparison with earlier calculations. It has been suggested that they do not follow unambiguously from the weak Hamiltonian, but we will not discuss non-standard choices here.

It is convenient to rewrite equation (1) by adopting the notation

$$V_{\mathrm{PNC}} = \sum_{i=1}^{9} V_0^i [\boldsymbol{p}, r^{-1}\exp(-\mu_i r)]_{\pm} \cdot \boldsymbol{v}_i^{(\pm)}, \tag{4}$$

in which the summation spans the nine terms in equations (2) and (3), $\boldsymbol{v}_i^{(\pm)}$ contains the appropriate hermitian or antihermitian spin–isospin operator and V_0^i contains the model-dependent strength factors. The terms $i = 1,2,3$ span the isovector contribution and $i = 4,\ldots,9$ the isoscalar contribution. Matrix elements of $\boldsymbol{v}_i^{(\pm)}$ can be written for the mth spherical component as

$$\langle T',0;S',v'|v_{i,m}^{(\pm)}|T,0;S,v\rangle = (-)^S\hat{S}\langle S1vm|S'v'\rangle w_i^{(\pm)}(S,T), \tag{5}$$

where $\hat{S} = (2S+1)^{\frac{1}{2}}$ and $w_i^{(\pm)}(S,T)$ depends upon S' and T' implicitly, by $T' = T$ and $S' = 1-S$ for the isoscalar components or $T' = 1-T$ and $S' = S$ for the isovector components, and has the property

$$w_i^{(\pm)}(S,T) = \pm w_i^{(\pm)}(S',T'). \tag{6}$$

The operator $\boldsymbol{v}_i^{(+)}$ ($\boldsymbol{v}_i^{(-)}$) is hermitian (antihermitian).

3. Formalism

(*a*) *Danilov Formulation*

Danilov (1965) gave the scattering amplitude for low energy n–p scattering in the presence of PNC forces as

$$f(\boldsymbol{k}',\boldsymbol{k}) = -a_t P_t - a_s P_s + f_{\mathrm{PNC}}(\boldsymbol{k}',\boldsymbol{k}), \tag{7a}$$

where the contribution from PNC effects can be written as

$$\begin{aligned} f_{\mathrm{PNC}}(\boldsymbol{k}',\boldsymbol{k}) = {} & -Ca_t(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_p)\cdot(\boldsymbol{k}'+\boldsymbol{k}) \\ & +(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p)\cdot\{\lambda_t a_t(\boldsymbol{k}'P_t+\boldsymbol{k}P_s)+\lambda_s a_s(\boldsymbol{k}'P_s+\boldsymbol{k}P_t)\}. \end{aligned} \tag{7b}$$

In these equations a_s and a_t are the singlet and triplet scattering lengths, C characterizes the isovector weak interaction in the $J = 1$ state, and λ_s, λ_t are the isoscalar weak interactions in the $J = 0, 1$ states. The equations (7) are accurate up to terms linear in momentum. Danilov then wrote down the asymptotic forms of the capture wavefunction and of the $^3S_1$, $^3P_1$ and $^1P_1$ components of the deuteron (by analytic continuation to negative energies). By presuming that only the long range part of the wavefunction was important in evaluating the E1 and M1 matrix elements, Danilov used the asymptotic wavefunctions to obtain the following analytic expressions for the photon circular polarization P_γ and the asymmetry α with respect to the incident neutron spin direction:

$$P_\gamma = \frac{4\{(1-\frac{2}{3}a_s\beta)\lambda_t\mu_N + \frac{1}{3}a_s\beta\lambda_s\mu_N\}}{(\mu_p-\mu_n)(1-a_s\beta)} = 0{\cdot}61\,\lambda_t\mu_N - 0{\cdot}24\,\lambda_s\mu_N, \tag{8a}$$

$$\alpha = \frac{8C\mu_N}{3(\mu_p-\mu_n)(1-a_s\beta)} = 0{\cdot}087\,C\mu_N. \tag{8b}$$

Here μ_p and μ_n are the observed nucleon magnetic moments, $M = c\mu_N/\hbar$ is the nucleon mass and the deuteron binding energy is $\hbar^2\beta^2/M$. An outline of the derivation of equations (8), with the emphasis on the approximations employed, is given in Appendix 1.

We adopt here a procedure similar to that of Tadić (1968): the n–p scattering amplitude is calculated in the low energy limit and an identification of C and λ_s, λ_t is made from the Danilov (1965) expressions (7). Finally P_γ and α are calculated trivially from equations (8). Details of the identification of C and λ_s, λ_t may be found in Appendix 2.

(*b*) *Calculation of Danilov Parameters*

By writing the components of the weak T-matrix (that is, T-matrix elements connecting states $|l,S,J\rangle$ to states $|l\pm1,S',J\rangle$) in a Born approximation to first order in the PNC potential (but to all orders in the parity-conserving potential), and making the appropriate identification of the Danilov (1965) parameters C and λ_s, λ_t, we obtain (see Appendix 2)

$$C = \frac{\sqrt{\frac{3}{2}}\pi M}{4\hbar^2 a_t} \sum_{i=1}^{3} V_0^i w_i^{(\pm)}(1,0)$$
$$\times \{(-2/\pi\mu_i^2) + t_1^{(\pm)}(\mu_i) + x_{11}^{(\pm)}(\mu_i) + u_{11}(\mu_i) + w_{11}^{(\pm)}(\mu_i) + y_{11}^{(\pm)}(\mu_i)\}, \tag{9a}$$

$$\lambda_{2S+1} = -\frac{\pi M}{4\hbar^2 a_{2S+1}} \sum_{i=4}^{9} V_0^i w_i^{(\pm)}(S,1-S)$$
$$\times \{(-2/\pi\mu_i^2) + t_S^{(\pm)}(\mu_i) + x_{1-S,S}^{(\pm)}(\mu_i) + u_{1-S,S}(\mu_i) + w_{1-S,S}^{(\pm)}(\mu_i) + y_{1-S,S}^{(\pm)}(\mu_i)\}. \tag{9b}$$

In these expressions we have employed the notation of equations (5) and (6); $c\mu_i/\hbar$ is the π or ρ meson mass as appropriate, and $(2S+1) = 1, 3$ is an alternative label to s, t. Equations (9) are expressed in such a way as to separate the roles of S-wave and P-wave correlations and those of S–D coupling. The term $(-2/\pi\mu_i^2)$ is the 'plane wave contribution', i.e. effects of uncorrelated waves, and the remaining contributions are defined by

$$t_S^{(\pm)}(\mu) = \frac{2M}{\pi\hbar^2}\int_0^\infty \left(\frac{1}{p^2+\mu^2} \pm \frac{2p^2}{3(p^2+\mu^2)^2}\right) R_{0S,0S}^S(p)\,\mathrm{d}p, \tag{10a}$$

$$x_{S'S}^{(\pm)}(\mu) = \pm\frac{2M}{\pi\hbar^2}\left(\sqrt{\tfrac{1}{2}}\,\delta_{S',S} - \sqrt{2}\,\delta_{|S'-S|,1}\right)\delta_{S,1}\int_0^\infty \frac{2p^2}{3(p^2+\mu^2)^2} R_{21,01}^1(p)\,\mathrm{d}p, \tag{10b}$$

$$u_{S'S}(\mu) = \frac{2M}{\pi\hbar^2}\int_0^\infty \frac{p}{p^2+\mu^2} R_{1S',1S'}^S(p)\,\mathrm{d}p, \tag{10c}$$

$$w_{S'S}^{(\pm)}(\mu) = -\frac{1}{\pi}\left(\frac{M}{\hbar^2}\right)^2 \int_0^\infty\int_0^\infty \left(\frac{Q_0(z)}{p} \pm \frac{Q_1(z)}{q}\right) R_{1S',1S'}^S(q)\, R_{0S,0S}^S(p)\,\mathrm{d}p\,\mathrm{d}q, \tag{10d}$$

$$y_{S'S}^{(\pm)}(\mu) = -\frac{1}{\pi}\left(\frac{M}{\hbar^2}\right)^2\left(\sqrt{\tfrac{1}{2}}\,\delta_{S',S} - \sqrt{2}\,\delta_{|S'-S|,1}\right)\delta_{S,1}$$

$$\times \int_0^\infty \int_0^\infty \left(\frac{Q_2(z)}{p} \pm \frac{Q_1(z)}{q}\right) R^S_{1S',1S'}(q)\, R^1_{21,01}(p)\, \mathrm{d}p\, \mathrm{d}q\,. \tag{10e}$$

In these expressions, z is an abbreviation for $(p^2+q^2+\mu^2)/2pq$ and $Q_l(z)$ is an irregular Legendre function. The functions R are defined in terms of the half-off-shell T-matrix elements $T^J_{l'S',lS}(p,k;k^2)$ by

$$R^J_{l'S',lS}(p) = \lim_{k\to 0}\{k^{-l}\, T^J_{l'S',lS}(p,k;k^2)\}\,. \tag{11}$$

It is immaterial whether T is the incoming or outgoing solution because at zero energy the singularity in the kernel of the Lippmann–Schwinger equation disappears. The $(\pm)$ index on the functions (10) corresponds to the commutator (antihermitian) or anticommutator (hermitian) nature of $[p, r^{-1}\exp(-\mu r)]_\pm$, the appropriate weak-interaction term.

As is evident from equations (10), the functions t, x and u describe the effects of S-wave, coupled $^3S_1$–$^3D_1$ wave and P-wave correlations respectively while the functions w and y describe transitions from the 'defect' part of the capture wavefunction to the 'defect' part of the deuteron wavefunction. Separating their roles in this manner enables us to test in a systematic way the effect upon the polarization and asymmetry of 'switching on' the various correlations. We find that P waves do play an important role, so that their neglect by Tadić (1968) is a serious one.

4. Strong Interaction

The strong n–p interaction enters our formalism only through the half-off-shell transition matrix at zero energy, as defined by equation (11). It is well known that the transition matrix is most readily obtained from a given N–N potential if the latter is in separable form. Because of this fact there have been attempts to fit phenomenological rank-N separable potentials to phase shift data by Tabakin (1964), Mongan (1969*a*), Sirohi and Srivastava (1972, 1973) and others. More recently, attempts to expand a reliable local potential in terms of a rank-N separable potential have been made. These include the unitary pole approximation (UPA) (Levinger *et al.* 1969; Harms and Laroze 1971; Bhatt *et al.* 1972) and the unitary pole expansion (UPE) of Harms (1970). Both of these approximations rely upon a strongly resonant behaviour in the appropriate partial wave for their success. The UPE is an expansion based upon the UPA as the first term. The UPA and UPE have been studied in great detail recently by the above authors and by Harms and Newton (1970), Jackson and Lande (1972), Siebert *et al.* (1972), Afnan and Read (1973) and Srivastava and Sirohi (1973). We have employed the UPA to the soft-core potential of Reid (1968) for both the $^1S_0$ and $^3S_1$–$^3D_1$ channels. The $^1S_0$ case appears to be very reliable in that the UPA results of either Harms and Laroze (1971) or Jackson and Lande (1972) produce very good agreement with the two-term UPE of Jackson and Lande (1972), but there is some doubt about the adequacy of a UPA to the Reid triplet potential (A. D. Jackson, personal communication). For example, the UPA used by Bhatt *et al.* (1972) and Siebert *et al.* (1972) produces values for the coupling constant,

tan ε_1, and $^3D_1$ phase shift which are in poor agreement with those predicted directly by the Reid potential. Afnan and Read (1973; personal communication) are able to improve upon the latter and they also point out that Bhatt *et al.* and Siebert *et al.* define their coupling constant with the wrong sign, an observation supported by Peiper (1974). From the work of Jackson (personal communication) and Jackson and Lande (1974) it appears that, in contrast to the $^1S_0$ case, the UPE to Reid's $^3S_1$–$^3D_1$ soft-core potential is only a slowly converging series, so that inaccuracy with a rank-1 or rank-2 UPE is inevitable. The $^3S_1$ potential of Malfliet and Tjon (1969), however, was found to lead to a rapidly converging UPE series by Jackson and Lande (1972, 1974), so that a UPA or two-term UPE appears to be useful. Unfortunately, no coupling to the $^3D_1$ state is included in this potential.

Because of the absence of resonant behaviour in P waves it is unlikely that a UPE would have rapid convergence in these partial waves. However, a direct solution of the Lippmann–Schwinger equation at zero energy (where the integral equation is of the Fredholm form) is quite feasible for those Reid P-wave potentials for which $J \leqslant 1$. The same can also be said of the S-wave channels, but we have chosen instead to utilize the less cumbersome UPE.

As a check upon the dependence of our results on the strong n–p interaction, we have also employed the phenomenological rank-2 separable interaction of Mongan (1969*a*) (his case II) and of Sirohi and Srivastava (1972, 1973).

5. Results and Discussion

(*a*) *Method of Calculation*

As already noted, the strong interaction enters the calculation through the half-off-shell transition matrix at zero energy, as defined by equation (11), and this is most easily calculated if the strong nucleon–nucleon potential is in separable form. Consequently, we have used local soft-core S-wave potentials, in the form of the UPA to the Reid (1968) potentials as given by Harms and Laroze (1971) and Bhatt *et al.* (1972) or the UPE to the Reid and Malfliet–Tjon (1969) potentials as given by Jackson and Lande (1972). In the former, the form factors are expanded in linear combinations of Yamaguchi form factors and the $R(q)$ can thus be expressed analytically in terms of the expansion coefficients. Jackson and Lande quote their form factors numerically over a grid in momentum space.

The Reid P-wave potentials can be converted into the transition matrix by directly solving the Lippmann–Schwinger equation at zero energy, dividing out a factor of k so that the solution is $R^J_{1S,1S}(q)$ directly. Obtaining the momentum space potential $V(p,q)$ in analytical form represents the biggest hurdle to be overcome in solving the Lippmann–Schwinger equation. The solution is then obtained by writing the integral as a sum over the momentum grid

$$q_i = \gamma \tan\{\tfrac{1}{4}\pi(1+x_i)\}, \tag{12}$$

where

$$W_i = \tfrac{1}{4}\pi w_i(\gamma + q_i^2/\gamma)$$

are the appropriate weights for integration. The x_i and w_i are the roots and weights appropriate to Gaussian integration of order N; with $N = 80$ and $\gamma = 3{\cdot}0\,\mathrm{fm}^{-1}$, this grid is identical with that of Jackson and Lande (1972). We find that the solutions

Table 3. Calculated contributions to equations (9) for four strong interactions

The listed values are the contributions from the terms (10) to the parameters C (with $S'=S=J=1$; $\mu=\mu_\pi$), λ_s ($S=J=0, S'=1; \mu=\mu_\rho$) and λ_t ($S=J=1, S'=0; \mu=\mu_\rho$) of equations (9), calculated using the strong interaction models R, RMT, M and SS described in the text

Contributing term	± index	Contributions (fm²) to: C	λ_s	λ_t	Contributions (fm²) to: C	λ_s	λ_t
		(*a*) *Model R*			(*b*) *Model RMT*		
$-2/\pi\mu^2$		−1·27	−0·0423	−0·0423	−1·27	−0·0423	−0·0423
$t_S^{(\pm)}(\mu)$	+		−0·2064	0·0716		−0·1719	0·1129
	−	2·30	−0·3981	0·1043	2·35	−0·3797	0·1320
$x_{S'S}^{(\pm)}(\mu)$	+			0·0360			
	−	0·28		−0·0360			
$u_{S'S}(\mu)$		0·16	0·0233	0·0295	0·16	0·0233	0·0295
$w_{S'S}^{(\pm)}(\mu)$	+		−0·0287	−0·0377		−0·0583	−0·0704
	−	−0·40	0·1341	−0·0655	−0·44	0·1181	−0·0892
$y_{S'S}^{(\pm)}(\mu)$	+			−0·0296			
	−	−0·07		0·0212			
Sum	+		−0·2542	0·0275		−0·2492	0·0297
	−	1·00	−0·2830	0·0112	0·80	−0·2806	0·0300
		(*c*) *Model M*			(*d*) *Model SS*		
$-2/\pi\mu^2$		−1·27	−0·0423	−0·0423	−1·27	−0·0423	−0·0423
$t_S^{(\pm)}(\mu)$	+		−0·3319	0·1567		−0·6593	0·2340
	−	2·44	−0·5622	0·1649	2·45	−0·6582	0·2032
$x_{S'S}^{(\pm)}(\mu)$	+			−0·0035			0·0083
	−	0·05		0·0035	0·07		−0·0083
$u_{S'S}(\mu)$		0·21	0·0786	0·0602	0·13	−0·0831	0·0084
$w_{S'S}^{(\pm)}(\mu)$	+		−1·5500	−0·1702		−1·2982	−0·0475
	−	−0·76	−0·0953	−0·2196	−0·45	−1·2994	−0·0413
$y_{S'S}^{(\pm)}(\mu)$	+			0·0109			−0·0018
	−	0·00		−0·0059	−0·01		0·0016
Sum	+		−1·8456	0·0118		−2·0829	0·1591
	−	0·67	0·6212	−0·0392	0·92	−2·0830	0·1213

of the Lippmann–Schwinger equation are indistinguishable for $N = 40$ and 80 (both with $\gamma = 3{\cdot}0\,\mathrm{fm}^{-1}$).

We have also employed the rank-2 separable potentials of Mongan (1969*a*), case II, and Sirohi and Srivastava (1972, 1973), for which the transition matrix can be written down analytically. Both of these potentials have been derived phenomenologically as fits to the Livermore phase shifts of McGregor *et al.* (1969), similar to those used by Reid (1968) in fitting his local potentials.

In all cases, the integrals in equations (10) have been performed numerically using the grid (12) with $N = 80$ and $\gamma = 3{\cdot}0\,\mathrm{fm}^{-1}$; results for $N = 40$ are indistinguishable. The following numerical values were assumed:

$$\left.\begin{aligned} \mu_\pi = 0{\cdot}708\,\mathrm{fm}^{-1}, \quad \mu_\rho = 3{\cdot}88\,\mathrm{fm}^{-1}, \quad \hbar^2/M = 41{\cdot}46\,\mathrm{MeV\,fm}^2, \\ a_s = -23{\cdot}679\,\mathrm{fm}, \quad a_t = 5{\cdot}397\,\mathrm{fm}. \end{aligned}\right\} \tag{13}$$

Table 4. Parameter values for different strong and weak interactions

The tabulated results are the parameter values corresponding to (*a*) the Cabibbo weak interaction and the four strong interactions described in the text, and (*b*) the RMT strong interaction and the weak-interaction models as designated in Table 2

(*a*) *Cabibbo model results*

Strong interaction	Parameter (10^{-7} units)				
	$C\mu_N$	$\lambda_s\mu_N$	$\lambda_t\mu_N$	P_γ	α
R	−0·431	2·301	0·170	−0·448	−0·038
RMT	−0·347	2·277	0·709	−0·113	−0·030
M	−0·297	6·917	−1·247	−2·419	−0·026
SS	−0·397	17·241	2·622	−2·533	−0·035

(*b*) *RMT model results*

Weak interaction	Parameter (10^{-7} units)				
	$C\mu_N$	$\lambda_s\mu_N$	$\lambda_t\mu_N$	P_γ	α
1	−0·347	2·277	0·709	−0·113	−0·030
2A	0·0	2·363	0·709	−0·134	0·0
2B	−9·248	2·363	0·709	−0·134	−0·806
3	3·573	1·708	0·886	−0·131	0·311
4	7·403	1·181	1·063	0·366	0·645
5	−15·390	3·031	0·355	−0·510	−1·342
6	−0·224	1·139	1·063	0·376	−0·020
7	0·693	1·139	1·063	0·376	0·060
8	4·512	1·267	1·063	0·345	0·393
9A	0·0	1·772	0·886	0·116	0·0
9B	−0·245	1·772	0·886	0·116	−0·021

(*b*) *Numerical Results*

We have employed four sets of strong interactions, denoted by R, RMT, M and SS. The first of these employs the tabulations by Harms and Laroze (1971) and Bhatt *et al.* (1972) of the UPA to the Reid (1968) soft-core S-wave potentials, together with the local soft-core P-wave potentials of Reid. The $^1S_0$ UPA transition matrix has been renormalized to the scattering length of (13); this is necessary because the Reid $^1S_0$ potential is determined from p–p scattering data and consequently the UPA of Harms and Laroze predicts $a_s = -17{\cdot}18$ fm. The RMT interaction incorporates the Jackson and Lande (1972) two-term UPE to the Reid $^1S_0$ and Malfliet–Tjon (1969) uncoupled $^3S_1$ interactions, together with the Reid local P-wave interactions. Both of the UPE's have been renormalized to the scattering lengths of (13). The M and SS potentials are the rank-2 separable potentials of Mongan (1969*a*), case II, and Sirohi and Srivastava (1972, 1973) respectively.

The contributions listed in equations (9) to C and λ_s, λ_t are displayed in Tables 3*a*–3*d* for the strong interactions R, RMT, M and SS respectively. The differences in the results due to the $^1S_0$ potential, between R and RMT, are entirely attributable to the improvement of a rank-2 approximation over the rank-1 approximation to Reid's (1968) potential. The most marked differences between the results are noticeable in the P-wave contributions (u and w) to λ_s. This largely reflects the differences in the $^3P_0$ and $^1P_1$ potentials off the energy shell. In particular the numerical value

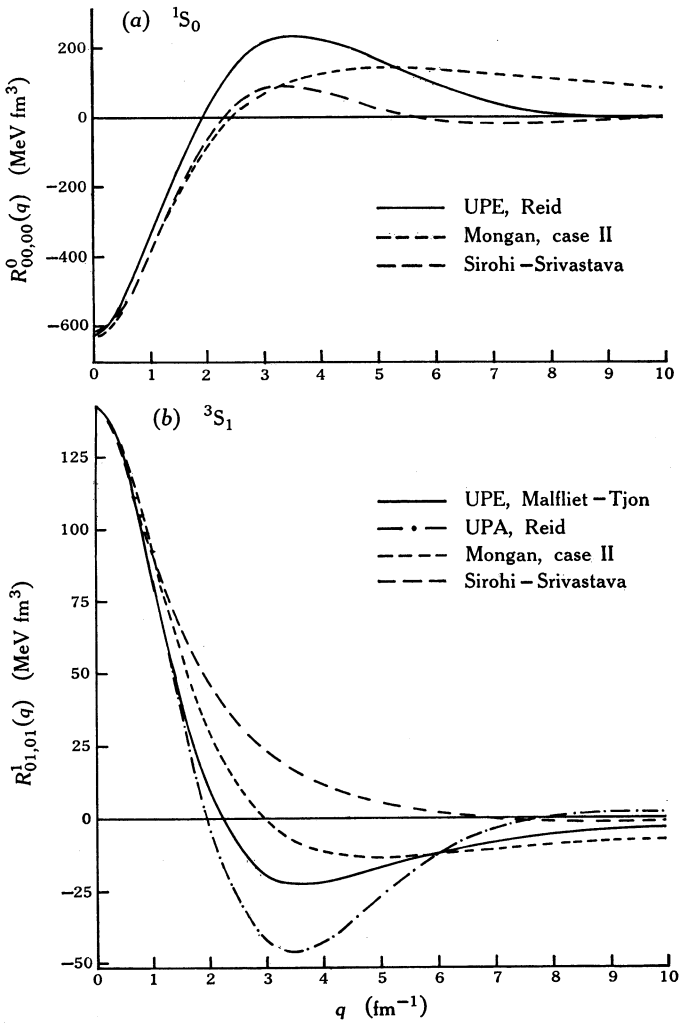


Fig. 1. Zero-energy half-off-shell transition matrices as defined by equation (11) for S waves:

(*a*) $R^0_{00,00}(q)$ for the $^1S_0$ wave, using the two-term UPE to the Reid (1968) soft-core potential as given by Jackson and Lande (1972) (the UPA of Jackson and Lande or Harms and Laroze (1971) is indistinguishable on this scale) and the potentials of Mongan (1969*a*), case II, and Sirohi and Srivastava (1972);

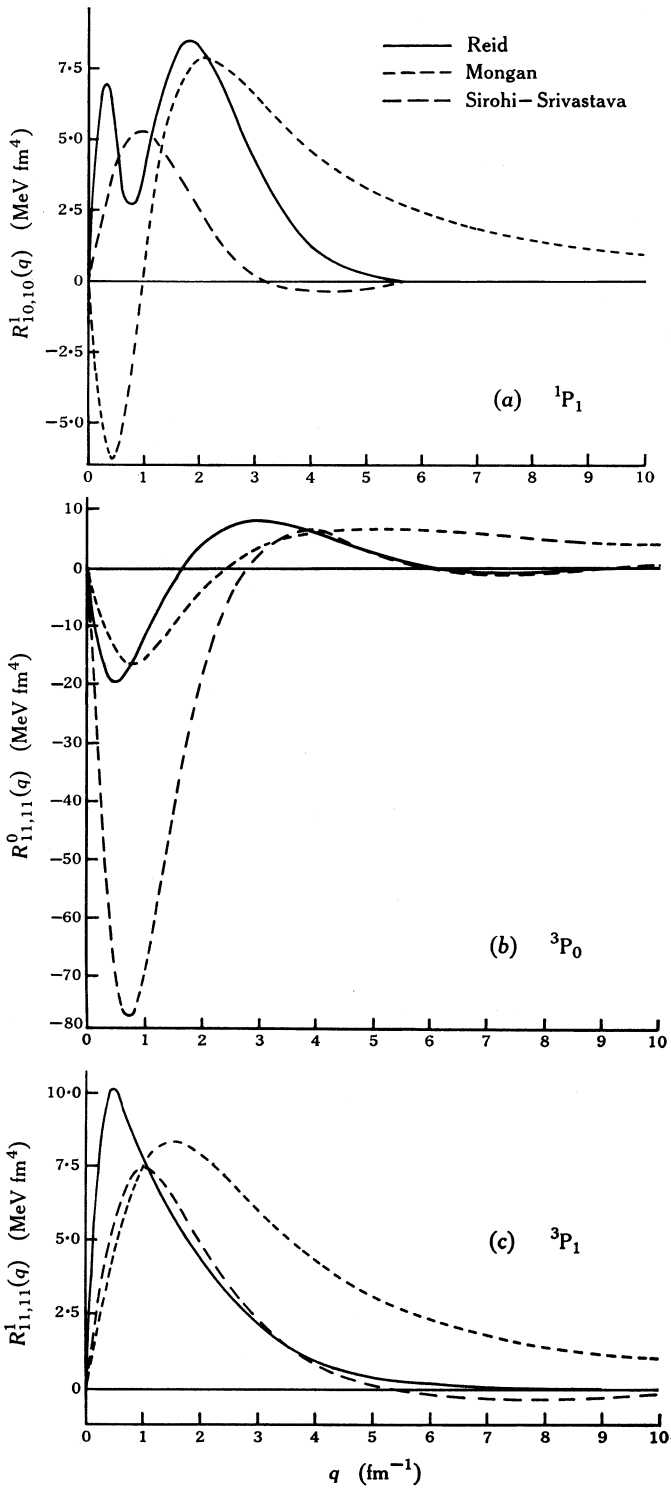
(*b*) $R^1_{01,01}(q)$ for the $^3S_1$ wave, using the two-term UPE to the Malfliet and Tjon (1969) potential as given by Jackson and Lande (1972), the UPA to the Reid (1968) soft-core potential as given by Bhatt *et al.* (1972) and the potentials of Mongan (1969*a*) and Sirohi and Srivastava (1973).

Fig. 2 (*opposite*). Zero-energy half-off-shell transition matrices as defined by equation (11) for P waves, using the potentials of Reid (1968), Mongan (1969*a*) and Sirohi and Srivastava (1973):

(*a*) $R^1_{10,10}(q)$ for the $^1P_1$ wave;

(*b*) $R^0_{11,11}(q)$ for the $^3P_0$ wave;

(*c*) $R^1_{11,11}(q)$ for the $^3P_1$ wave.


Reid
Mongan
Sirohi–Srivastava
$R^1_{10,10}(q)$ (MeV fm^4)
(a) $^1P_1$
$R^0_{11,11}(q)$ (MeV fm^4)
(b) $^3P_0$
$R^1_{11,11}(q)$ (MeV fm^4)
(c) $^3P_1$
q (fm^{-1})

of $w_{10}(\mu_\rho)$ is very different in the M and SS potentials from that in the R and RMT potentials, and in the former cases it dominates λ_s. This term arises from the weak interaction between $^1S_0$ and $^3P_0$ and, since we are near the 'singlet deuteron' anti-bound resonance energy, the importance of the interaction between the $^1S_0$ and $^3P_0$ states is enhanced, a point also noted by Blin-Stoyle and Feshbach (1961). We shall further consider this in the following subsection.

Table 4*a* records values of C, λ_s, λ_t, P_γ and α corresponding to each of the strong interactions, and appropriate to the Cabibbo (1963) weak-interaction model, while Table 4*b* lists these parameters for each of the weak-interaction models represented in Table 2, using RMT as a representative strong interaction.

(*c*) *Discussion*

From Table 3 it is evident that both the circular polarization P_γ and asymmetry α involve a delicate cancellation of contributions which renders them sensitive to the choice of the strong interaction describing the short-range correlations. Hence a discussion of the merits of the four strong interaction models is in order.

All of the interactions chosen have been designed to fit the recent Livermore phase-shift data and consequently have very similar on-shell behaviour. The differences in their predictions in this work therefore reflect their different off-shell behaviours. The off-shell behaviour is illustrated in Figs 1 and 2 where the zero-energy half-off-shell transition matrices of equation (11), which we hereinafter refer to as the half-shell functions,* are displayed. It is immediately apparent that the Mongan (1969*a*) half-shell function decays to zero with increasing momentum much more slowly than for the other potentials, as was also noted by Mongan (1969*b*). This feature can be expected to have its largest influence on those results ($\lambda_s, \lambda_t, P_\gamma$) which depend upon the short-range ρ-meson exchange weak force, for which the form factor dies off more slowly with increasing momentum.

As expected, the interactions R and RMT lead to similar results, particularly for λ_s where the only difference is due to the different separable representations of the Reid $^1S_0$ potential. The nonzero $^3S_1$–$^3D_1$ coupling in R accounts for most of the numerical discrepancies in C and λ_t. However, since the UPA of Bhatt *et al.* (1972) to the Reid $^3S_1$–$^3D_1$ potential gives a poor representation of the coupling constant, the effects of the coupled transition matrix elements in R should not be taken too seriously. We note that both SS and M provide effects due to $^3S_1$–$^3D_1$ coupling which are very much smaller, and generally negligible. (However, Peiper (1974) has recently questioned the accuracy of the coupled interaction of both the Mongan and Sirohi–Srivastava potentials.) Moreover, since we have neglected the D-state of the deuteron in computing P_γ and α from the scattering amplitude, there is some consistency in also neglecting the contributions $x(\mu)$ and $y(\mu)$ to C and λ_t.

A comparison of the values in Table 3 makes it clear that P-state effects play an important role in determining λ_s, λ_t and C (through $u(\mu)$ and $w(\mu)$) and it is their role which depends most sensitively upon the potential. An inspection of Fig. 2 confirms that there are wide variations in the off-shell behaviour of the P-wave potentials. Of these, the least important is the $^3P_1$ potential (Fig. 2*c*) because the longer range

* For S waves, $R(q)$ is to within a factor the half-shell function as used by Kowalski (1965) and Noyes (1965*a*). For P waves of zero energy the Kowalski–Noyes half-shell function is not defined because of the vanishing denominator; if the appropriate power of k is removed before putting $k = 0$ then $R(q)$ is again the half-shell function to within a factor.

one-pion-exchange (OPE) weak force is sensitive to only $\sim 1\ \mathrm{fm}^{-1}$ off the energy shell. The off-shell behaviour for both the $^3P_0$ and $^1P_1$ potentials shows considerable sensitivity to the choice of potential, and the importance of these is shown by the values of $w_{1-S,S}(\mu_\rho)$, in particular, in Table 3. Since the capture state energy is very close to the energy of the 'antibound' $J = 0$ resonance of the deuteron (this is reflected in the large magnitude of a_s), the $^3P_0$ potential plays an important role and must not be neglected. In fact its consequences are quite capable of dominating λ_s, as they do in the M and SS potentials. The $^1P_1$ potential has a strong but less spectacular influence on λ_t.

Table 5. Effective-range coefficients for low energy P waves

Potential[A]	Coefficient	$^1P_1$	$^3P_0$	$^3P_1$
Livermore phase shifts	f (fm^{-3})	−0·53	0·35	−0·52
	g (fm^{-1})	−8·9	2·1	−5·5
Reid soft core	f (fm^{-3})	−0·661	0·365	−0·652
Mongan, case II	f (fm^{-3})	+1·13	0·714	−2·64
Sirohi–Srivastava	f (fm^{-3})	−2·72	0·147	−2·00

[A] References are McGregor *et al.* (1969), Reid (1968), Mongan (1969*a*) and Sirohi and Srivastava (1973) respectively.

It is instructive to understand why the half-shell functions for P waves differ so markedly when they are supposed to be constrained to fit the same phase shifts. Consider the 'effective-range' expansion for P waves

$$k^3 \cot \delta(k) = f + gk^2 + \ldots . \tag{14}$$

At small energies and momenta, the on-shell T-matrices therefore behave like

$$T^{(+)}(k,k;k^2) = -(2\hbar^2/\pi M)k^2(f+gk^2+\ldots - ik^3)^{-1}$$

and our half-shell functions have the behaviour

$$R(q) = -(2\hbar^2/\pi M)q\{f^{-1} + O(q)\}, \tag{15}$$

where the second term in the series (15) does not depend only upon f and g. Thus, the slope of the linear relationship between $R(q)$ and q is the only constraint imposed by the on-shell data upon the low momentum behaviour of $R(q)$. Just how stringent this constraint is depends upon the extent to which the leading term in the expansion (14) dominates the expansion at small k ($k < 1\ \mathrm{fm}^{-1}$). An analysis of the Livermore phase shifts (McGregor *et al.* 1969) indicates that f fails to dominate the series, even for $E_{\mathrm{lab}} = 1$ MeV ($k \sim 0{\cdot}1\ \mathrm{fm}^{-1}$), but that a cutoff at the quadratic term provides a reasonable description of all P-wave phase shifts up to at least $E_{\mathrm{lab}} = 3$ MeV. The values of f and g resulting from this analysis are displayed in Table 5. The values of f which are predicted by the P-wave potentials of Reid (1968), Mongan (1969*a*) and Sirohi and Srivastava (1973) are also contained in this table. It is immediately clear that a fit to P-wave phase shifts over an energy range $< 10^2$ MeV does not necessarily provide an accurate fit to f. This fact reflects the unimportance of f in the expansion (14), which in turn is responsible for the failure of a phase-shift fit to significantly constrain the off-shell behaviour. This feature is to be contrasted

with the S-wave behaviour in which the scattering length dominates the low energy phase shifts (up to ~ 10 MeV), which is why phenomenological S-wave potentials are required to reproduce the scattering lengths very closely. This point has been noted by Srivastava (1973), who demonstrated that S-wave phase-shift data up to momentum p primarily determines the off-shell behaviour of $T(q,p;p^2)$ and $T(p,q;p^2)$ for $q < p$. While this explains why our half-shell functions for S-waves are largely potential-independent, we wish to emphasize that Srivastava's argument does not hold for P waves.*

Despite the strong influence of the potential upon both λ_s and λ_t, much of this influence is not carried through to P_γ, as can be seen from Table 4*a*. In fact the contributions of λ_s and λ_t to P_γ (equation (8a)) largely cancel one another to leave a value of P_γ which is not appreciably different from zero. This feature makes a more careful calculation of P_γ essential; in particular the Danilov formulae (8) must be improved upon. A discussion of some of the approximations in the Danilov formalism is given in Appendix 1.

From Table 4*b* it is clear that the asymmetry α exhibits much more sensitivity to the choice of the weak-interaction model than does P_γ (in fact the latter probably depends more upon the choice of the strong interaction). The sensitivity of α is partly because of the extreme sensitivity of the PNC, NNπ amplitude to the choice of the weak-interaction Hamiltonian, but also because the ρ meson term (present in all the isovector interactions except for the Cabibbo (1963), Segré (1968) γ_5-invariant, Lee–Yang (1960) and Tomazawa (1970) models) makes an important contribution, supplementing the OPE contribution.

In comparing our results with those of other investigators (Table 1), we can make the following observations. Our results for λ_s and λ_t agree closely with those of Tadić (1968) provided we neglect all P-wave contributions (as Tadić did); our values for C in the absence of P-wave correlations are comparable with Tadić's, but of opposite sign. (Tadić's OPE weak potential also has the opposite sign to ours.) Our results appear to be irreconcilable with those of Hadjimichael and Fischbach (1971), who used the same PNC potential for those models that we have in common, but whose results for P_γ are two orders of magnitude smaller than ours. To within an order of magnitude, the Hadjimichael–Fischbach result for P_γ is independent of neutron energy, so that their small result is unlikely to result from a delicate cancellation of contributions. Our values for C and λ_s, λ_t in the Cabibbo model also agree closely with Danilov's (1971) dispersion theory result. Since this approach differs significantly from that adopted here, we feel that it is the Danilov formulae (8) themselves which represent the most serious approximation.

6. Conclusions

As has been pointed out in recent reviews of this subject, previous calculations have encountered considerable difficulty in reconciling the measurement by Lobashov *et al.* (1972) of the photon circular polarization in $n+p \rightarrow d+\gamma$ with the observation by Hättig *et al.* (1970) of the parity-forbidden α decay $^{16}O(2^-;\ 8{\cdot}88\ \text{MeV}) \rightarrow {}^{12}C+\alpha$. Both of these observables depend only on the isoscalar PNC potential, yet calculations

* *Note added in proof*: Srivastava (1974) has recently extended his study of off-shell behaviour to include P waves, and has come to a similar conclusion about the constraints imposed by the on-shell behaviour.

using the standard potential obtained from the factorization approximation to the ρ exchange potential with the Cabibbo weak-interaction Hamiltonian were able to obtain agreement with the decay width for the above α decay, Γ_α, of Hättig *et al.* but not with the P_γ of Lobashov *et al.* The calculations of Γ_α were made by Gari and Kümmel (1969) and Henley *et al.* (1969). The present results for P_γ alter this picture, and suggest that much more work is required before one can assert that reasonable agreement cannot be obtained between both Γ_α and P_γ using the same PNC potential. This is because the cancellations observed in the present calculation show that the Danilov (1965) method of calculating P_γ must be improved before definitive conclusions can be drawn. The possibility of reconciling P_γ and Γ_α is heartening since any order of magnitude alteration in the potential strength needed to achieve agreement between the earlier calculation and experiment for P_γ would have destroyed the agreement for Γ_α. Table 4*a* shows that our calculations give a value for P_γ in the range $-0{\cdot}113\times10^{-7}$ to $-2{\cdot}53\times10^{-7}$, and we can summarize the present experimental and theoretical situation as follows.

	Experiment	Cabibbo theory calculation
Γ_α (eV)	$(1{\cdot}0\pm0{\cdot}2)\times10^{-10}$ (Hättig *et al.* 1970)	2×10^{-10} [Gari and Kümmel 1969; Henley *et al.* 1969]
P_γ	$-(1{\cdot}3\pm0{\cdot}45)\times10^{-6}$ (Lobashov *et al.* 1972)	$-(0{\cdot}13\pm0{\cdot}1)\times10^{-6}$ (Present work)

The isovector PNC potential contains much more information about the weak-interaction Hamiltonian, so that a measurement of α would in principle contribute greatly to our knowledge of this interaction. For example, it is in this term that one would expect to see the effects of the neutral currents introduced in the Weinberg (1967) model of the weak interaction (Salam 1969).

We conclude by re-emphasizing the approximations that have gone into our calculation: the use of the standard prescription for generating a PNC potential from the weak Hamiltonian, the treatment of the 2π exchange contribution to the PNC entirely in terms of ρ exchange, the neglect of possible effects of exchange currents (Gari and Huffman 1971; McKellar 1972; Henley 1973), and the computation of the electromagnetic transition amplitudes from the asymptotic parts of S and P wavefunctions following the Danilov (1965) method. We hope to relax some of these approximations in future work, and envisage that an abandonment of the Danilov prescription for calculating E1 and M1 matrix elements could have a profound influence upon P_γ, but probably not upon α.

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Appendix 1. Danilov Prescription

The main steps in the derivation of the Danilov (1965) formulae (8) are outlined here, with emphasis on where the important approximations enter. Assuming a scattering amplitude of the form (7), the incident wavefunctions $\phi_{JM}(r)$ for $J = 0, 1$ states have the asymptotic forms (with arbitrary normalization)

$$\phi_{00}(r) \sim (1-r^{-1}a_s)\mathscr{Y}_{00}^{00}(\hat{r}) + 2ir^{-2}\lambda_s a_s \mathscr{Y}_{11}^{00}(\hat{r}), \tag{A1a}$$

$$\phi_{1M}(r) \sim (1-r^{-1}a_t)\mathscr{Y}_{01}^{1M}(\hat{r}) - 2ir^{-2}a_t\{\sqrt{\tfrac{1}{3}}\lambda_t \mathscr{Y}_{10}^{1M}(\hat{r}) - \sqrt{\tfrac{2}{3}}C\mathscr{Y}_{11}^{1M}(\hat{r})\}, \tag{A1b}$$

in terms of the properly normalized eigenfunctions of total angular momentum

$$\mathscr{Y}_{lS}^{JM}(\hat{r}) = \sum_{vm} \langle lSmv \mid JM\rangle Y_l^m(\hat{r})\chi_s^v. \tag{A2}$$

The asymptotic S and P components of the deuteron are obtained by analytic continuation of the scattered wavefunction to negative energy $-\hbar^2\beta^2/M$, as

$$\phi_{d,M}(r) \sim N[\mathscr{Y}_{01}^{1M}(\hat{r}) + 2ir^{-1}(1+\beta r)\{\sqrt{\tfrac{1}{3}}\lambda_t \mathscr{Y}_{10}^{1M}(\hat{r}) - \sqrt{\tfrac{2}{3}}C\mathscr{Y}_{11}^{1M}(\hat{r})\}]r^{-1}\exp(-\beta r). \tag{A3}$$

The normalization constant N plays no role in the evaluation of P_γ and α.

The circular polarization P_γ and asymmetry α about the spin direction of the incident neutron result from interference between the 'irregular' E1 transitions (between S and P states) and the 'regular' M1 transition. Danilov estimates the matrix elements by presuming that the asymptotic wavefunctions (A1) and (A3) adequately represent the wavefunctions at all distances (the zero-range approximation). In this case the M1 matrix element describing capture from the $^1S_0$ state to the $^3S_1$ deuteron state is

$$\mathscr{M} = \int_0^\infty (1-r^{-1}a_s)r^{-1}\exp(-\beta r)r^2\,dr = \beta^{-2} - \beta^{-1}a_s. \tag{A4}$$

Similar results for the E1 matrix elements lead to the P_γ and α expressions (8). The approximations of this procedure are then apparent. One depends upon the accuracy of results such as (A4) and similar expressions for the E1 matrix elements, as well as the neglect of the deuteron (and capture) D states. Neglect of the deuteron D state is responsible for the neglect of the E1 transitions $^3P_{0,1} \rightarrow {}^3D_1$.

It was shown by Bethe and Longmire (1950), Austern and Rost (1959) and Noyes (1965*b*) that the result (A4) underestimates the M1 transition strength and that this effect alone would increase both P_γ and α by $\sim 27\%$. This value excludes corrections to the normalization constant N, which is $N_0 = (2\beta)^{\frac{1}{2}}$ for 'zero-range' wavefunctions and $N_g = P_s^{\frac{1}{2}}N_0(1-\beta r_0)^{-\frac{1}{2}}$ (where P_s is the deuteron S-state probability and r_0 the triplet scattering length) for more accurate deuteron $^3S_1$ wavefunctions (Bethe and Longmire). However, some compensation for this increase might be expected from finite-range effects in the E1 matrix elements. Nevertheless, an overall enhancement of P_γ and α does seem probable from such considerations.

In addition to finite-range effects (i.e. effects due to the presumption that the asymptotic wavefunctions adequately represent the wavefunctions at all nucleon separations), the most serious approximation of the Danilov formulation is the

neglect of the deuteron D-state component. (The capture state D-state component is very small since its asymptotic component falls off linearly with energy). Because our value of P_γ is not appreciably different from zero, due to the cancellation of terms in λ_s and λ_t, the role of E1 transitions to the deuteron D state may be appreciable. Preliminary estimations of this effect indicate that P_γ may be enhanced in magnitude and remain negative.

Appendix 2. Derivation of Danilov Parameters

We outline here the derivation of equations (9). The transition matrix has a partial-wave decomposition

$$\langle \boldsymbol{k}', S'v' \,|\, T(s) \,|\, \boldsymbol{k}, Sv\rangle = \sum_{l'lJ} T^J_{l'S',lS}(k', k; s) \sum_{m'mM} Y^{m'}_{l'}(\hat{\boldsymbol{k}}')\, Y^{m*}_{l}(\hat{\boldsymbol{k}}) \langle l'S'm'v' \,|\, JM\rangle \langle lSmv \,|\, JM\rangle \tag{A5}$$

(with no restriction that $|l-l'|$ be even) and is related to the scattering amplitude by

$$\langle S'v' \,|\, f(\boldsymbol{k}', \boldsymbol{k}) \,|\, Sv\rangle = -(2\pi^2 M/\hbar^2) \langle \boldsymbol{k}', S'v' \,|\, T(k^2) \,|\, \boldsymbol{k}, Sv\rangle \tag{A6}$$

for $|\boldsymbol{k}| = |\boldsymbol{k}'| = k$. In the low energy limit, retaining only terms linear in momentum in the expansion (A5) and comparing the result (A6) with the Danilov expressions (7) and (8), we make the identifications

$$a_t C = -(\pi M/4\hbar^2)\sqrt{\tfrac{3}{2}}\, R^1_{01,11}(0), \tag{A7a}$$

$$a_{2S+1}\lambda_{2S+1} = -(\pi M/4\hbar^2)(-)^S \hat{S}\, R^S_{0S',1S}(0), \tag{A7b}$$

in which $\hat{S} = (2S+1)^{\frac{1}{2}}$, $S' = 1-S$, and $R^S_{0S',1S}(k)$ is defined as in equation (11). This identification is assisted by writing the Danilov scattering amplitude as

$$\begin{aligned}\langle S'v' \,|\, f(\boldsymbol{k}', \boldsymbol{k}) \,|\, Sv\rangle &= 2k(\tfrac{4}{3}\pi)^{\frac{1}{2}}(-)^S \hat{S} \sum_m \langle S1vm \,|\, S'v'\rangle [C\sqrt{\tfrac{2}{3}}\, a_t \{Y^{m*}_1(\hat{\boldsymbol{k}}') + Y^{m*}_1(\hat{\boldsymbol{k}})\}\delta_{S',S,1} \\ &\quad - \{\lambda_{2S+1}\, a_{2S+1}\, Y^{m*}_1(\hat{\boldsymbol{k}}') + \lambda_{2S'+1}\, a_{2S'+1}\, Y^{m*}_1(\hat{\boldsymbol{k}})\}\delta_{|S'-S|,1}],\end{aligned} \tag{A8}$$

in which there is only one term in the sum, corresponding to $m = v'-v$.

The PNC T-matrix elements of equations (A7) are evaluated to first order in the PNC interaction by writing

$$\langle \boldsymbol{k}' \,|\, T^{(+)}_{\mathrm{PNC}}(s) \,|\, \boldsymbol{k}\rangle = \langle \psi^{(-)}_{\boldsymbol{k}'}(s) \,|\, V_{\mathrm{PNC}} \,|\, \psi^{(+)}_{\boldsymbol{k}}(s)\rangle, \tag{A9}$$

where

$$|\psi^{(\pm)}_{\boldsymbol{k}}(s)\rangle = [1 + G^{(\pm)}(s)\, T^{(\pm)}_{\mathrm{strong}}(s)]\,|\boldsymbol{k}\rangle,$$

in terms of the Green's function $G^{(\pm)}(s) = (s+\nabla^2 \pm \mathrm{i}\varepsilon)^{-1}$. A partial-wave decomposition of equation (A9) at zero energy leads to the result

$$R^S_{1S',0S}(k) = F^S_{1S',0S}(k) - \frac{M}{\hbar^2}\int_0^\infty F^S_{1S',0S}(q)\, T^S_{1S',1S'}(q, k; 0)\, \mathrm{d}q, \tag{A10}$$

where

$$F^S_{1S',0S}(q) = V^S_{1S',0S}(q,0) - \frac{M}{\hbar^2}\sum_L \left(\int_0^\infty V^S_{1S',LS}(q,p)\, T^S_{LS,0S}(p,0;0)\, \mathrm{d}p \right),$$

in which the summation is over $L = 0, 2$ for $S = 1$ or $L = 0$ for $S = 0$. The partial-wave components $V^J_{1S',LS}(q,p)$ of the weak potential are defined analogously to equation (A5), and in terms of the notation of equations (4) and (5) are given by

$$V^J_{l'S',lS}(q,p) = B^J_{l'S',lS} \sum_i V^i_0\, w_i^{(\pm)}(S,T)\{p^{-1}Q_l(z_i) \pm p^{-1}Q_{l'}(z_i)\}, \qquad \text{(A11)}$$

where

$$z_i = (p^2+q^2+\mu_i^2)/2pq$$

and the B's are purely geometrical factors, given by

$$B^J_{l'S',lS} = \pi^{-1}\hat{S}\hat{S}'(-)^{S'-S+J+l}\,\hat{l}\langle l100 \mid l'0\rangle \begin{Bmatrix} S & 1 & S' \\ l' & J & l \end{Bmatrix}. \qquad \text{(A12)}$$

The isospin quantum number T is 0 or 1 such that $(l+S+T)$ is odd. A substitution of equation (A10) into equations (A7) leads to the contributions (9) to C and λ_s, λ_t.

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