Short Communications

Aust. J. Phys., 1975, **28**, 353–5

Non-existence of Axially Symmetric Massive Complex Scalar Fields

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Abstract

It is established that time-dependent axially symmetric complex scalar fields (charged scalar fields) do not sustain the mass parameter m within the context of the Einstein–Rosen metric.

The study of scalar meson fields in general relativity has drawn the attention of many workers (e.g. Bergmann and Leipnik 1957; Buchdahl 1959; Bramhachary 1960; Stephenson 1962; Penney 1968, 1969; Gautreau 1969; Misra and Pandey 1971; Rao *et al.* 1972; Roy and Rao 1972). The particular scalar fields which find more applications in high energy physics are the complex fields (charged scalar fields). Das (1963) has obtained exact solutions of the combined Einstein–Maxwell and Klein–Gordon equations for a complex scalar field in the case of spherical symmetry. In the present note we show that the mass parameter of a complex scalar field vanishes for the axially symmetric Einstein–Rosen metric. An analogous result in the case of real massive scalar fields has been derived by Roy and Rao (1972), and that result is recovered when the present scalar field is specialized to be real.

We consider the axially symmetric Einstein–Rosen metric

$$ds^2 = e^{2\alpha-2\beta}(dt^2-d\rho^2)-\rho^2 e^{-2\beta} d\phi^2 - e^{2\beta} dz^2, \tag{1}$$

where α and β are functions of ρ and t only and ρ, ϕ, z and t correspond respectively to x^1, x^2, x^3 and x^4. The combined Einstein–Maxwell and Klein–Gordon equations for complex scalar fields $\varphi = \varphi(\rho,t)$ are

$$K \equiv (D^i D_i + m^2)\varphi = 0, \tag{2}$$

$$\bar{K} \equiv (\bar{D}^i \bar{D}_i + m^2)\bar{\varphi} = 0, \tag{3}$$

$$M^i \equiv \nabla_j F^{ij} + (4\pi)^{\frac{1}{2}} i\varepsilon(\varphi\,.\,\bar{D}^i\bar{\varphi} - \bar{\varphi}\,.\,D^i\varphi) = 0, \tag{4}$$

$$E_{ij} \equiv R_{ij} - \tfrac{1}{2}g_{ij}R + 8\pi\{(\bar{D}_i\bar{\varphi}\,.\,D_j\varphi + \bar{D}_j\bar{\varphi}\,.\,D_i\varphi) - g_{ij}(\bar{D}^a\bar{\varphi}\,.\,D_a\varphi - m^2\varphi\bar{\varphi}) - F_{ik}F^k_j + \tfrac{1}{4}g_{ij}F_{ab}F^{ab}\}. \tag{5}$$

The operator D_i and its conjugate $\bar{D}_i$ are defined by

$$D_i = \nabla_i + (4\pi)^{\frac{1}{2}} i\varepsilon A_i, \qquad \bar{D}_i = \nabla_i - (4\pi)^{\frac{1}{2}} i\varepsilon A_i, \tag{6}$$

with $D_i\varphi$ denoting the corresponding operation on φ and an accompanying dot denoting ordinary multiplication. In the above equations, ∇_i indicates covariant differentiation of a function with respect to x^i and the symbol i when it is not an index has its usual value $\sqrt{(-1)}$. The mass and charge parameters of the field are denoted respectively by m and ε, the latter parameter being related to the fine-structure constant which is in turn related to the charge of an electron. R_{ij} and R have their usual meanings. The electromagnetic field tensor F_{ij} is given by

$$F_{ij} = A_{i,j} - A_{j,i},$$

A_i being a four-potential satisfying the Lorentz gauge condition $\nabla_i A^i = 0$. Here and below a subscript comma is used to denote a partial derivative with respect to the following index.

In a charged scalar field a particle is described in terms of a complex wavefunction

$$\varphi(x) = 2^{-\frac{1}{2}}(\varphi_1(x) + \mathrm{i}\varphi_2(x)). \tag{7}$$

If the arbitrary masses m_1 and m_2 associated with the fields φ_1 and φ_2 are identified with a single mass parameter m, we have only one equation of the complex field:

$$\varphi = 2^{-\frac{1}{2}}(\varphi_1 + \mathrm{i}\varphi_2), \qquad \bar{\varphi} = 2^{-\frac{1}{2}}(\varphi_1 - \mathrm{i}\varphi_2),$$

where φ, $\bar{\varphi}$ satisfy the Klein–Gordon equations (2) and (3) so that

$$\varphi\bar{\varphi} = \tfrac{1}{2}(\varphi_1^2 + \varphi_2^2), \tag{8}$$

with φ_1 and φ_2 real. The quantity $\varphi\bar{\varphi}$ may represent the matter density of the field and is always positive (Das and Coffman 1967).

The non-vanishing components of the mixed Einstein tensor of the metric (1) are given by

$$G_1^1 = e^{2\beta-2\alpha}\{\rho^{-1}\alpha_{,1} - \beta_{,1}^2 - \beta_{,4}^2\}, \tag{9a}$$

$$G_2^2 = e^{2\beta-2\alpha}\{\alpha_{,11} - \alpha_{,44} + \beta_{,1}^2 - \beta_{,4}^2\}, \tag{9b}$$

$$G_3^3 = -e^{2\beta-2\alpha}\{2(\beta_{,11} - \beta_{,44} + \rho^{-1}\beta_{,1}) + (\alpha_{,44} - \alpha_{,11} + \beta_{,4}^2 - \beta_{,1}^2)\}, \tag{9c}$$

$$G_4^4 = -G_1^1 = -e^{2\beta-2\alpha}\{\rho^{-1}\alpha_{,1} - \beta_{,1}^2 - \beta_{,4}^2\}, \tag{9d}$$

$$G_1^4 = G_4^1 = -e^{2\beta-2\alpha}\{2\beta_{,1}\beta_{,4} - \rho^{-1}\alpha_{,4}\}. \tag{9e}$$

From these equations we observe that

$$G_1^1 + G_4^4 = 0,$$

which implies

$$g^{11}G_{11} + g^{44}G_{44} = 0. \tag{10}$$

After a straightforward but tedious calculation using equations (5), (6) and (10), we get

$$g^{11}G_{11} + g^{44}G_{44} \equiv -8\pi\{-8g^{22}\pi\varepsilon^2 A_2^2\varphi\bar{\varphi} - 8g^{33}\pi\varepsilon^2 A_3^2\varphi\bar{\varphi} + 2m^2\varphi\bar{\varphi} - g^{11}g^{44}(F_{14})^2 + g^{22}g^{33}(F_{23})^2\} = 0. \tag{11}$$

Since we have that g^{22} and g^{33} are negative and g^{44} and $\varphi\bar{\varphi}$ are positive, the terms within the braces in equation (11) are all positive. This implies

$$A_2 = A_3 = 0, \qquad F_{23} = F_{14} = 0 \qquad \text{and} \qquad m = 0. \tag{12}$$

Hence we conclude that a mass parameter of a complex scalar field cannot exist for the axially symmetric Einstein–Rosen metric.

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Manuscript received 9 December 1974