Aust. J. Phys., 1976, **29**, 97–105

An Evolutionary Model for Quasars

Natalia V. Zotov and W. Davidson

Department of Mathematics, University of Otago, Box 56, Dunedin, New Zealand.

Abstract

A simple, though not necessarily unique, model for the cosmic population dynamics of quasars is presented. This model incorporates a double evolutionary trend: (i) a strong luminosity evolution, the presence of which we have discussed previously (Zotov and Davidson 1970, 1973) and (ii) an evolution of total coordinate density. This double trend is the same as that already found to be applicable to the total radio source population, if account is taken only of radio characteristics (Davidson *et al.* 1971). The predictions of the scheme are compared with five sets of data, covering a wide range of frequencies and flux density limits, and they are found to give good agreement in all cases.

1. Introduction

As is now customary, we shall use the term 'quasar' for objects of starlike appearance and large redshift (assumed to be cosmological). For quasars selected on both optical and radio criteria we use the designation QSS.

In previous papers (Zotov and Davidson 1970, 1973) we have shown that the mean optical luminosity of *observed* quasars increases steadily with redshift, regardless of the value of the deceleration parameter q_0 of the cosmological model. In our 1973 paper we also demonstrated that schemes relying solely on density evolution of quasars, with the assumption that their luminosity function remains constant while their total density varies with epoch, appear to be incompatible with the count data now available. In the present paper, we propose a scheme which incorporates both a luminosity evolution consistent with that evidenced by observable quasars, and a density evolution. It is to be expected in advance that the mean luminosity evolution of the total quasar population will be less than that found by Zotov and Davidson (1970) for the powerful quasars actually observed. This same remark is relevant to the radio luminosity evolution of the QSS also. The evolution in both cases will be governed by an adjustable parameter, and will be compatible with the assumption that the total quasar population between the redshifts z and $z+dz$ tends to coincide with the *observable* population when $z \to 0$. We also postulate an evolution of total coordinate density, similarly governed by an adjustable parameter. This we may expect in advance to be less than that of Schmidt (1972), who neglected the luminosity evolution of quasars. Finally we assume that between z and $z+dz$ the distributions of radio and optical luminosity for the total quasar population are normal about their respective means at redshift z, and in practice lower cutoffs at the 3σ level have been adopted for the purposes of calculation. This assumption appears to be consistent with an analysis presented to the colloquium held in 1974 at Northwestern University (Madeleine Barnothy, personal communication). This analysis dealt with data from the 3C and 4C catalogues, the Parkes surveys at 1410 and 2700 MHz, a survey at 365 MHz by Wills *et al.* (1973), and a list of B2 quasars. Because of the strong luminosity evolution present, the successful matching of the data on the basis of this normal-distribution assumption is a strong indication that intrinsically very weak quasars do not exist at high redshifts. This is precisely the same phenomenon found previously for models fitting the total radio source population, having regard only to their radio characteristics (Davidson *et al.* 1971).

With the simple schematic apparatus outlined above we are in a position to calculate the numbers of quasars expected above particular optical and radio limits. In these calculations the total local ($z \approx 0$) density of quasars will be one of the parameters to be determined from a comparison of predictions and observations.

2. Details of Evolutionary Scheme

The average optical luminosity of observed quasars as a function of the cosmic time lapse τ was found previously (Zotov and Davidson 1970) to be described by

$$|\bar{M}|_{\text{obs}} = 20 + 1\cdot086\,\beta\tau^{2/3}. \tag{1}$$

Here M denotes the absolute magnitude related to luminosity L by the usual formula

$$M/M_0 = 2\cdot5\log_{10}(L_0/L). \tag{2}$$

The parameter β was found to have a best-fitting value of $1\cdot76\times10^{-6}$ essentially for all q_0 in the range $0 \leqslant q_0 \leqslant 0\cdot5$ (for τ measured in years). These findings have not been altered by the considerable number of identifications made since that analysis. The plots of the later data of 229 quasars given by Zotov and Davidson (1973) confirm this result.

We note that equation (1) gives $|\bar{M}|_{\text{obs}} = 22$ at $z \approx 0\cdot1$, and we shall take this value to be the mean value of the $|M|$ for the total population of quasars at $z \approx 0\cdot1$. If quasars are a different type of object and brighter on average than normal galaxies, as evidenced so far, then this assumption would seem to be appropriate. For the total population of quasars we therefore postulate that the median absolute magnitude (in the sense of a complete normal distribution) follows one of the family of curves defined by

$$|\bar{M}| = 22 + k_1(\beta\tau^{2/3} - 2), \tag{3}$$

where k_1 is an adjustable parameter such that $0 \leqslant k_1 \leqslant 1$. Curves of this family all have $|\bar{M}| = 22$ at $z \approx 0\cdot1$ for all likely values of q_0. The spread of absolute magnitudes for the observed quasars has a standard deviation of approximately unity at all redshifts. If at any redshift the mean optical luminosity for the total quasar population is less than that for the observed population ($k_1 < 1$) we may expect the standard deviation for the whole population to be less than unity, since the observed population would represent the high-powered limb of the assumed normal distribution.

Similar considerations apply to the radio power of quasars. We find that for *observed* quasars the mean power $\bar{P}_{\rm obs}$ at 500 MHz satisfies

$$\{\log_{10}(\bar{P}/\bar{P}_0)\}_{\rm obs} \approx 4\tau/T_0 . \tag{4}$$

Here $(\bar{P}_0)_{\rm obs} \approx 2\times 10^{25}\ \mathrm{W\,sr^{-1}\,Hz^{-1}}$ is the local value of $\bar{P}_{\rm obs}$, and T_0 is the Hubble time measured in years, taken in this paper to be $1\cdot 3\times 10^{10}$ years. The above value for $(\bar{P}_0)_{\rm obs}$ will be taken to be the median radio power $\bar{P}_0$ for all local quasars. Let $\bar{P}$ be the median radio power at general z. Then we adopt as possible median luminosity curves for the total population of QSS the set

$$\log_{10}(\bar{P}/\bar{P}_0) = k_2\,\tau/T_0 , \tag{5}$$

where k_2 is a parameter taken in the range $0 \leqslant k_2 \leqslant 4$. Once again it is to be expected that the standard deviation of the total population will be less than that of observed QSS, for which at 500 MHz we have $\sigma(\log_{10} P^{(500)}) \approx 0\cdot 4$.

Since luminosity evolution combined with density evolution is postulated here, the density evolution for the total population required to account for observed numbers will be described by a function $\alpha(z)$ which must lie between $\alpha(z) \equiv 1$, corresponding to the conservation of numbers in coordinate volume, and $\alpha(z) \equiv (1+z)^6$ as proposed by Schmidt (1968, 1972). Accordingly our density evolution will be taken in the simple form

$$\alpha(z) = (1+z)^{\lambda} , \tag{6}$$

where λ is a parameter lying in the range $0 \leqslant \lambda \leqslant 6$.

In our view, the optical and radio luminosities of quasars are independent *at a given redshift*. For the quasars under present consideration, this suggestion was investigated by assigning quasars at each frequency into small redshift ranges of z to $z+\mathrm{d}z$, where $\mathrm{d}z = 0\cdot 2$, and calculating the optical and radio powers for the case $q_0 = 0\cdot 1$. No significant correlation between radio and optical power at a given redshift was found at any frequency. On the other hand, according to our scheme of luminosity evolution at both radio and optical frequencies, there clearly will be a correlation between *mean* radio and optical powers as z increases. This may explain the evidence for correlation presented by Schmidt (1970), Petrosian (1973) and Fanti *et al.* (1973).

Within the redshift range z to $z+\mathrm{d}z$, the probability of observing a quasar as a QSS above a given radio flux density limit S_l is the proportion having $P > P_l$, where $P_l(z)$ is the minimum power of sources having $S \geqslant S_l$ at redshift z. Likewise the probability of observing a quasar above a given optical limit m_l is the proportion having $|M| > |M_l|$, where $|M_l(z)|$ defines a lower limit of $|M|$ corresponding to m_l at redshift z. Let $\mathrm{d}N_{\rm tot}$ be the total number of quasars in this range of redshift. Then

$$\mathrm{d}N_{\rm tot} = n_0\,\alpha(z)\,\mathrm{d}V , \tag{7}$$

where n_0 is the local density of quasars, $\alpha(z)$ describes the variation of number density in comoving coordinate volume and $\mathrm{d}V$ is an element of such volume. Since we are assuming that the radio and optical luminosities of a quasar are independent, the probability of a quasar having both $P > P_l$ and $|M| > M_l$ is the product of the probabilities. Thus let $\xi = \log_{10}(P/\bar{P})$ and $\zeta = |M|$, ξ_l and ζ_l being the values corresponding to $P = P_l(z)$ and $M = M_l(z)$ respectively, and let $\eta(\xi)$ and $\eta(\zeta)$ be the normal distribution functions describing the radio and optical luminosity functions of quasars respectively. Then the number of QSS observable in volume $\mathrm{d}V$ is

$$\mathrm{d}N = \frac{\int_{\xi_l}^{\xi_2} \eta(\xi)\,\mathrm{d}\xi}{\int_{\xi_1}^{\xi_2} \eta(\xi)\,\mathrm{d}\xi} \times \frac{\int_{\zeta_l}^{\zeta_2} \eta(\zeta)\,\mathrm{d}\zeta}{\int_{\zeta_1}^{\zeta_2} \eta(\zeta)\,\mathrm{d}\zeta} \times n_0\,\alpha(z)\,\mathrm{d}V. \tag{8}$$

Here ζ_1 and ζ_2 are effective lower and upper limits of ζ for the total quasar population, and they are defined respectively by $\zeta_1 = \bar{\zeta} - 3\sigma_\zeta$ and $\zeta_2 = \bar{\zeta} + 3\sigma_\zeta$, where $\bar{\zeta} = |\bar{M}|$ and σ_ζ is the standard deviation of absolute magnitudes (taken to be adjustable). For the total quasar population, ξ_1, ξ_2 and σ_ξ are defined similarly as the corresponding radio quantities. Hence the number of quasars above given radio and optical limits, with redshift up to some determined value Z, will be

$$N = \int_{z=0}^{z=Z} \mathrm{d}N, \tag{9}$$

where $\mathrm{d}N$ is given by equation (8). When the six parameters n_0, λ, k_1, k_2, σ_ζ and σ_ξ are specified, the above expression (8) and hence the integral (9) are determined.

3. Data

The QSS data employed represent the most complete sets of data available over a wide range of frequencies. These data, labelled according to the abbreviations adopted in Table 1, are:

3CR-Q, fifty-five QSS from the 3CR survey having $S_{178} \geqslant 9$ Jy, excluding the obscured regions within 10° of the galactic plane. Apparent magnitudes are listed by Burbidge (1967), except for the more recent identifications for which the comprehensive catalogue by Veron and Veron (1974) was consulted. The region covers 46% of the sky, and identifications are likely to be complete for QSS having $m_v \leqslant 19$.

PKS1400-Q, QSS identified from the Parkes observations of radio sources at 1410 MHz, having declinations between 0° and −20°. Identifications of 64 QSS having $S_{1410} \geqslant 0{\cdot}8$ Jy have been proposed by Bolton and Ekers (1966*a*, 1966*b*, 1967). If the region within 10° of the galactic equator is discounted, the area covered by the survey is 1·874 sr.

4C-Q, QSS from the 4C catalogue, contained in an area of 0·35 sr surveyed by Lynds and Wills (1972) and having $S_{178} \geqslant 2$ Jy. Lynds and Wills found 31 QSS in this category, and considered their identifications complete for sources having $m_v \lesssim 19{\cdot}4$.

PKS2700*A*-Q, QSS having $S_{2700} \geqslant 0{\cdot}35$ Jy, identified from the Parkes 2700 MHz survey which covered an area of 0·77 sr in the declination range $|\delta| \leqslant 4°$. The observations and QSS identifications, made originally by Bolton and Wall (1970), were reported by Wall *et al.* (1971).

PKS2700*B*-Q, QSS having $S_{2700} \geqslant 0{\cdot}1$ Jy, identified from the Parkes 2700 MHz survey of six selected areas covering 0·0753 sr, were described by Wall *et al.* (1971), following the original observations by Bolton and Wall (1970).

The numbers (extrapolated to whole sky values) of QSS in the above five sets of data having $15 \cdot 5 \leqslant m \leqslant 22 \cdot 5$ are listed in Table 1 together with (in italics) the predicted numbers which are described in Section 4 below. The QSS have been assigned to intervals of apparent magnitude by distributing those with $m = 18 \cdot 5$ exactly, say, evenly between the 18 and 19 magnitude groups. (Zotov and Davidson (1973) adopted the alternative procedure of considering intervals such as $17 \cdot 5 < m \leqslant 18 \cdot 5$, $18 \cdot 5 < m \leqslant 19 \cdot 5, \ldots$.)

Table 1. Comparison of observed and predicted whole-sky numbers of QSS

The predicted numbers are printed in italics

Data	$S_l^{(500)}$	$m =$ 16	17	18	19	20	21	22	Total
3CR-Q	4·0	14	22	50	22	6	4	2	120
		17	*44*	*58*	*37*	*11*	2	*0*	*169*
PKS1400-Q	1·7	17	70	205	84	50	3	0	429
		25	*115*	*216*	*171*	*57*	*7*	*0*	*591*
4C-Q	1·0	72	287	467	251	0	0	0	1077
		74	*297*	*476*	*346*	*103*	*18*	*1*	*1315*
PKS2700*A*-Q	0·64	25	212	473	424	122	0	0	1256
		45	*229*	*510*	*502*	*216*	*40*	*3*	*1545*
PKS2700*B*-Q	0·18	83	501	668	2587	1001	0	0	4840
		84	*583*	*1823*	*2590*	*1498*	*345*	*30*	*6953*

Each survey from which data have been taken has a lower radio flux density limit that can be converted to a limit $S_l^{(500)}$ at 500 MHz, upon assuming a particular value for the spectral index x ($S_\nu \propto \nu^{-x}$). The data sets are listed in Table 1 in order of descending 500 MHz limits, calculated for $x = 0 \cdot 75$. This serves merely to show the trends as a function of flux density at a constant frequency. The source totals apply to the frequencies of the surveys; they would not necessarily be the totals observed at the 500 MHz limits of flux density.

The PKS1400-Q group of QSS is included in the Parkes catalogue, for which the preliminary survey was made at 408 MHz. Katgert (1974) has criticized the use of this particular sample in our earlier paper (Zotov and Davidson 1973) on the grounds that the Parkes catalogue is complete only for $S_{408} \geqslant 3 \cdot 5$ Jy, whereas a complete sample at 1410 MHz having $S_{1410} \geqslant 0 \cdot 8$ Jy should include sources having $S_{408} \gtrsim 2$ Jy for a mean spectral index of 0·75. In this case we find it hard to understand why all sources chosen for the Parkes catalogue at 408 MHz were required to have $S_{1410} \geqslant 0 \cdot 4$ Jy, and therefore $S_{408} \gtrsim 1$ Jy (Shimmins *et al.* 1966). However this may be, we do not believe that it affects the case presented in our earlier paper, since the alternative sample chosen by Katgert (1974), namely one having $S_{408} \geqslant 3 \cdot 5$ Jy, still reveals essentially the same shortcomings of Schmidt's models which we pointed out in the samples having $S_{2700} \geqslant 0 \cdot 35$ and 0·1 Jy.

It is evident that Katgert (1974) accepted our calculations of the predictions at 2700 MHz according to Schmidt's models. The discrepancies, such as they are, arise in the presentation of the observed counts. Dr Katgert shares the QSS having $m = 18 \cdot 5$, say, equally between the 18 and 19 magnitude groups. This procedure produces the differences between her results and ours (Zotov and Davidson 1973) and in particular has the effect of nearly doubling the $m = 20$ count. (To remove this source of discrepancy we have adopted her procedure in the present paper.) But it will be seen that her 2700 MHz calculations reinforce our criticisms of Schmidt's models, i.e. consistently, there are statistically significant shortfalls in the predicted numbers at intermediate m values as well as statistically significant excesses at the highest m values. In fact at $m = 20$ Schmidt's predictions, both for $S_{2700} \geqslant 0 \cdot 35$ and $0 \cdot 1$ Jy, exceed the observed counts by a factor of three or four. Therefore either the Parkes identifications are drastically incomplete at $m \approx 20$ or Schmidt's evolutionary schemes based on pure density evolution with a progressive cutoff of the highest powered sources as z increases are incorrect.

Schmidt's scheme of density evolution without luminosity evolution means that in every column of his Table 6 (Schmidt 1972) he is dealing with the same luminosity function that he started with, except that, sticking to density evolution, he has to progressively cut off the high powered end as m increases; otherwise he would exceed the number of such bright sources observable. This means that by the time he gets to $m = 22$ his total population is made up of very weak sources not much brighter than galaxies. If such models are correct then they feature just the opposite evolutionary trend established for the radio population. Support for the criticisms by Zotov and Davidson (1973) of these models is now to be found in the recently reported work of Browne (1974). In a detailed search for identifications of quasars in the Parkes 2700 MHz catalogue for $S_{2700} \geqslant 0 \cdot 35$ Jy, Browne found that the number of identifications of quasars fainter than $m = 19$ fell significantly short of the numbers predicted by Schmidt. It is interesting also that a deep plate search to $m = 23 \cdot 5$ in the 3CR Catalogue by Longair and Gunn (1975) has produced no more quasars than are shown in our 3CR-Q set of Table 1.

When we consider for each set in our Table 1 the number of QSS per range of apparent magnitude, we can see that the peak shifts gradually, as we move down the table, from $m = 18$ to $m = 19$, with almost equal numbers at $m = 18$ and 19 for the PKS2700A-Q set. Again, proceeding down Table 1, we see that the total number in each set increases rapidly as the calculated lower flux density limit $S_l^{(500)}$ decreases, the total number in the PKS2700B-Q set being approximately 40 times the number in the 3CR-Q set. (The frequency of the observations increases at the same time, except that the PKS1400-Q and 4C-Q sets are out of order in this respect.)

The tapering off of numbers observed at $m = 19$ and 20 for low frequencies and at $m = 20$ for high frequencies is too substantial to be attributed to the incompleteness of QSS identifications. If we can reproduce this effect in a theoretical scheme we avoid postulating a very rapid increase in the total numbers, particularly of QSS near or below the limits of observation. The completeness of QSS identifications in the 2700 MHz surveys has been discussed in detail by Shimmins *et al.* (1968) and Bolton (1969). Shimmins *et al.* argued that, since a majority of identified QSS have spectral index $x < 0 \cdot 8$ (78 out of 150 radio sources identified), while the great majority of the 190 unidentified sources have $x > 0 \cdot 8$, these are probably radio galaxies. Bolton argued further that, because of the falling off of QSS identifications for $m \gtrsim 19$, in strong contrast to the situation with regard to radio galaxies, and because of the polarization properties of unidentified sources, it is very unlikely that a high proportion of these could be QSS. We may conclude from this that the Parkes 2700 MHz QSS identifications have a high degree of completeness, and that it is therefore implied that a theoretical scheme predicting numbers of QSS must not overpredict these numbers by a large factor at $m \approx 19$ and 20.

We follow the procedure proposed by Schmidt (1972) and adopted by Zotov and Davidson (1973) of dividing the theoretical distribution of QSS into three ranges of spectral index x, with means as observed at low frequencies and assumed to apply strictly to a reference frequency of 500 MHz. The assumed distribution of spectral index x for QSS at 500 MHz is:

Range	$x \leqslant 0\cdot50$	$0\cdot50 < x \leqslant 0\cdot88$	$0\cdot88 < x \leqslant 1\cdot20$
Mean x	0·30	0·75	1·00
Fraction of QSS	0·3	0·4	0·3

4. Comparison of Theory and Observation

Despite the fact that our evolutionary scheme depends on the six parameters listed at the end of Section 2, it was found that matching all the features of the observations described in Section 3 left little choice in the value of these parameters. Previous investigations (Zotov and Davidson 1970) suggested that the value of the parameter q_0 describing the deceleration of the expansion of the universe is small, so the computations have been performed for $q_0 = 0\cdot01$. A larger value of q_0 would imply that all evolution of luminosity and density is less than that found with the small value of q_0. This will give a feeling for what is involved, as an upper limit to evolutionary trends.

The following values of the parameters were found to give good matching of predicted and observed numbers over the whole range of radio frequencies and flux density limits considered, with some surplus of predicted numbers at $m = 19$ and 20, which allows scope for the unidentified objects at these apparent magnitudes: $n_0 \approx 2\times10^{-8}$ Mpc^{-3}; $\lambda = 5$, that is, density evolution is described by $(1+z)^5$; $k_1 = 0\cdot84$, $k_2 = 1\cdot9$, that is, $|\bar{M}| = 22 + 0\cdot84(\beta\tau^{2/3} - 2)$ and $\log_{10}(\bar{P}/\bar{P}_0) = 1\cdot9(\tau/T_0)$; and $\sigma_\zeta = 0\cdot95$, $\sigma_\xi = 0\cdot3$. The values of k_1 and k_2 are less than those for observed quasars, as is expected.

A detailed comparison of numbers predicted from this theoretical scheme with numbers in each of the data sets described in Section 3 is given in Table 1. The value of n_0 used is 2×10^{-8} Mpc^{-3}. This is the value which gives best matching for all sets of data, without underpredicting any of the peak numbers.

There is a broad measure of agreement between theory and observation for the 4C-Q, PKS2700*A*-Q, and PKS2700*B*-Q sets for $m \lesssim 18$ or 19, apart from the major discrepancy at $m = 18$ for the last set. This discrepancy must be regarded as a peculiarity of the particular very small region of the sky supplying the original observations. For each of the three sets, a shortfall of observed sources becomes evident at $m \gtrsim 19$, which may reasonably be attributed to incompleteness of identification. The differences may be seen in better perspective if we estimate errors based on the number in each sample and the area of sky involved. Thus for the PKS2700*A*-Q and PKS2700*B*-Q sets, the observed numbers with the estimated sample errors are, for faint apparent magnitudes:

Apparent magnitude	PKS2700*A*-Q	PKS2700*B*-Q
19	424 ± 86	2587 ± 657
20	122 ± 46	1001 ± 409

The predicted numbers actually come within these limits at $m \approx 19$, and are not far outside at $m \approx 20$. Thus the observed tapering off of counts at faint apparent magnitudes is predicted by this theoretical scheme, and only a small proportion of further identifications is needed to give agreement.

For the PKS1400-Q set, agreement between theory and observation is good at $m \approx 18$ and 20, satisfactory at $m \approx 16$, and less satisfactory elsewhere. Similar characteristics hold for the 3CR-Q set. However, as we move through the sets toward higher radio flux density limits $S_l^{(500)}$, the anisotropies in the regions of small redshift are more likely to dominate. The theoretical scheme therefore gives a reasonable explanation of the observations over a wide range of radio frequencies.

5. Conclusions

The overall matching of predictions and observations is remarkably good for our simple model, which could only be expected to give a rough approximation to the actual situation. All the features of the observations commented on in Section 3 are reproduced in the predictions. The peak of numbers above each flux density limitis seen to shift in the manner discussed, and the predictions are generally just plausibly greater than the observed number of quasars for $m \approx 19$ and 20.

The values of the parameters k_1 and k_2 imply that the median optical luminosity of all quasars is not much below that of observed quasars even at large redshifts, i.e. that optical luminosity evolution is very strong but that the evolution of radio power is much less marked, being slightly less than half that of observed QSS. Moreover, since we find $\lambda = 5$, the density evolution in our scheme is very much less rapid than that required by Schmidt ($\lambda = 6$). This density evolution and the evolution of radio power and optical luminosity adduced on the basis of the observations are such that there would be very few quasars below the 3σ limits for both. For example, there would be only a relatively small number of quasars having radio power $P^{(500)} < 2{\cdot}2 \times 10^{25}\ \mathrm{W\,sr^{-1}\,Hz^{-1}}$ and absolute magnitude $|M| < 23{\cdot}07$ at $z = 1$ (for $m > 20{\cdot}8$), or $P^{(500)} < 5{\cdot}8 \times 10^{25}\ \mathrm{W\,sr^{-1}\,Hz^{-1}}$ and $|M| < 24{\cdot}57$ at $z = 2{\cdot}5$ (for $m > 22{\cdot}2$). With the proposed scheme of evolution, the observed counts are matched fairly closely, without the need to postulate large numbers of sources below the limits of observation.

It is not intended to suggest that this is the only model which could fit all the observational data, but the model shows clearly that it is possible to explain economically and simply all observations, including such features as the evolution of the optical and radio powers of observed quasars, the shifting of the peak of observed numbers from 18^m at low to 19^m at high radio frequencies, and the required increase in total numbers as the frequency of observations increases and the flux limit drops.

It should be possible before long to decide observationally between a scheme, such as this one which incorporates a natural tapering off of numbers at high apparent magnitudes, and others, such as Schmidt's (1972) which predicts very large numbers at these apparent magnitudes, numbers which seem to be greater than the observations at the higher frequencies by a factor of about three at $m \approx 20$. If identifications can be made complete with reasonable certainty to $m = 20$, this should be sufficient to decide the issue, and therefore to provide valuable qualitative insight into the problem of model fitting.

Acknowledgment

One of us (N.V.Z.) has had the opportunity recently of having a discussion with Dr Jeannette Katgert. We wish to thank her for helpful comments.

References

Bolton, J. G. (1969). *Astron. J.* **74**, 131.
Bolton, J. G., and Ekers, Jennifer (1966*a*). *Aust. J. Phys.* **19**, 559.
Bolton, J. G., and Ekers, Jennifer (1966*b*). *Aust. J. Phys.* **19**, 713.
Bolton, J. G., and Ekers, Jennifer (1967). *Aust. J. Phys.* **20**, 109.
Bolton, J. G., and Wall, J. V. (1970). *Aust. J. Phys.* **23**, 789.
Browne, I. W. A. (1974). *Observatory* **94**, 282.
Burbidge, E. M. (1967). *Annu. Rev. Astron. Astrophys.* **5**, 399.
Davidson, W., Davies, M., and Cox, B. G. (1971). *Aust. J. Phys.* **24**, 403.
Fanti, R., Formiggini, L., Lari, C., Padrielli, L., Katgert, J. K., and Katgert, P. (1973). *Astron. Astrophys.* **23**, 161.
Katgert, Jeannette (1974). *Observatory* **94**, 20.
Longair, M. S., and Gunn, J. E. (1975). *Mon. Not. R. Astron. Soc.* **170**, 121.
Lynds, C. R., and Wills, D. (1972). *Astrophys. J.* **172**, 531.
Petrosian, V. (1973). *Astrophys. J.* **183**, 359.
Schmidt, M. (1968). *Astrophys. J.* **151**, 393.
Schmidt, M. (1970). *Astrophys. J.* **162**, 371.
Schmidt, M. (1972). *Astrophys. J.* **176**, 273.
Shimmins, A. J., Bolton, J. G., and Wall, J. V. (1968). *Nature (London)* **217**, 818.
Shimmins, A. J., Day, G. A., Ekers, R. D., and Cole, D. J. (1966). *Aust. J. Phys.* **19**, 837.
Veron, M. P., and Veron, P. (1974). *Astron. Astrophys. Suppl. Ser.* **18**, 309.
Wall, J. V., Shimmins, A. J., and Merkelijn, Jeannette K. (1971). *Aust. J. Phys. Astrophys. Suppl.* No. 19.
Wills, B. J., Wills, D., and Douglas, J. N. (1973). *Astron. J.* **78**, 521.
Zotov, Natalia V., and Davidson, W. (1970). *Aust. J. Phys.* **23**, 351.
Zotov, Natalia V., and Davidson, W. (1973). *Mon. Not. R. Astron. Soc.* **162**, 127.

Manuscript received 16 December 1974, revised 8 September 1975