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Ratio of Sheath Thickness to Debye Length for a Slightly Ionized Continuum Plasma

John A. Hamilton

Division of Mineral Chemistry, CSIRO, P.O. Box 124, Port Melbourne, Vic. 3207.

Abstract

The penetration of plasma sheaths for spherical probes in a slightly ionized continuum plasma has been computed for values of ε (the ratio of ion to electron temperature) of 0·01 and 1·0 with ρ_p (the ratio of probe radius to plasma Debye length) set at 5, 10, 20 and 30. Values of the potential drops at the sheath boundaries are presented.

Introduction

Electrostatic probes have become indispensable diagnostic tools for determining plasma properties. In the simplest form, a probe biased at a certain potential is inserted into a gaseous discharge and a plot is made of the probe current versus probe voltage, from which local parameters of the plasma can be calculated. This is in contrast to many other diagnostic methods which give values averaged over a large volume, e.g. microwave techniques (Huddlestone and Leonard 1965). Single, double and triple probe methods have been used, with particular advantages claimed for each.

Currently, much interest lies in the high pressure region (>1 torr; i.e. >133 Pa) where collisions cannot be neglected (Ul'yanov 1970, 1978). Collisional theory applies to those cases where the mean free path of the particles is much smaller than the sheath thickness. Particles entering the probe sheath are influenced by the probe potential, and it is thought that the collisions they experience produce random walks up to within one collision path from the probe (Cozens and von Engel 1965). A different type of collection occurs in low pressure plasmas; here only the high energy particles capable of overcoming the retarding potential are collected. High pressure theories can be classified as either general, continuum or continuum plus free fall. All three cases are complicated because there is no simple equation of motion for the particles at high pressure.

Although probe measurements are simple experimentally, the underlying theory is complex because of the many factors to be considered. Progress in plasma theory has been impeded by the lack of computational techniques for solving nonlinear differential equations. Detailed analyses have been restricted to approximations and limiting cases only, leaving a great amount of extrapolation necessary for experimentalists working in the field of plasmas. Recently, we have demonstrated by use of iterative techniques (Dorman and Hamilton 1977*a*) that an analogue computer (Hamilton 1975) is ideal for examining boundary value problems, particularly those describing plasma behaviour. The computer used in our studies permitted an investigation of Cohen's (1963) continuum theory. Dorman (1973) demonstrated how the continuum equations could be solved numerically using Runge's method. About the same time, Yastrebov (1972) solved the continuum equations in a manner similar to Dorman. Both authors commenced integration at the probe surface and varied the initial conditions of the equations until the curve of the sheath solution joined the curve of the quasi-neutral solution smoothly. Such methods were tedious and few cases were solved, thus restricting further investigation of the sheath region. Barad and Cohen (1974) extended Cohen's (1963) earlier work by numerically solving some cases of spherical probes in a moderately ionized plasma. This theory further complicates the continuum equations because transport coefficients vary with position in the plasma. The limiting case of large Debye length was examined and solved by Chang and Laframboise (1976). An extension was made of Cohen's theoretical probe current–potential characteristics (limited to a ratio of probe radius to plasma Debye length of $\rho_p > 50$) to lower values of ρ_p (Dorman and Hamilton 1977*b*). The results led to an examination (Dorman and Hamilton 1976) of existing theories, particularly the use suggested by Huddlestone and Leonard (1965) of an ion shielding factor in determining physical parameters such as charged particle concentration, kinetic energy and electron temperature.

Boyd (1951) calculated the potential distribution around a spherical probe, assuming the sheath radius to be known. From this, he was able to compute the current of positive ions to the probe. The potential around the probe was separated into four regions: (i) the sheath, close to the probe, where the ion concentration n_+ is much greater than the electron concentration n_-; (ii) the abnormal extra-sheath region; (iii) the normal mobility region, where $n_+ = n_-$; (iv) the undisturbed region. Only electronegative probes were considered, so that the electron current J_- could be ignored. Although Boyd's treatment of the regions has been criticized by Cohen (1963), because of the difficulty in matching the boundaries, it was shown by Dorman and Hamilton (1976) that the use of Sena's (1946) equation for the ion velocity in the abnormal extra-sheath region is valid over a certain region. It was the present author's opinion that ions do move to the probe by an ion exchange mechanism, but that the resulting changes due to mobility dependence on the continuum equations would make computations extremely difficult. This paper demonstrates the calculation of sheath thicknesses for different probe potentials y_p and the parameter ρ_p, for various values of the ratio ε of ion temperature T_+ to electron temperature T_-.

Basic Equations

For a slightly ionized (<1%) gaseous plasma at a pressure $p \gtrsim 1$ torr, the differential equations describing particle concentration and potential are, as stated by Cohen (1963),

$$\varepsilon\, dn_+/d\zeta - n_+\, dy/d\zeta = -\varepsilon J_+ , \tag{1}$$

$$dn_-/d\zeta + n_-\, dy/d\zeta = -J_- , \tag{2}$$

$$(\zeta^4/\rho_p^2) d^2y/d\zeta^2 = n_+ - n_- , \tag{3}$$

with the boundary conditions

$$\zeta = 0, \qquad y = 0, \qquad n_+ = n_- = 1 ; \tag{4a}$$

$$\zeta = 1, \qquad y = y_p, \qquad n_+ = n_- = 0 . \tag{4b}$$

Here $n_{\pm}$ is the normalized positive ion or negative electron concentration; $\zeta = r_p/r$ is the ratio of probe radius r_p to radial distance r from the probe centre; $J_{\pm}$ is the normalized ion or electron current received by the probe; y is the normalized potential, with $y = y_p$ at the probe surface; and $\rho_p = r_p/\lambda_D$ and $\varepsilon = T_+/T_-$, as defined above (λ_D being the Debye length for electrons).

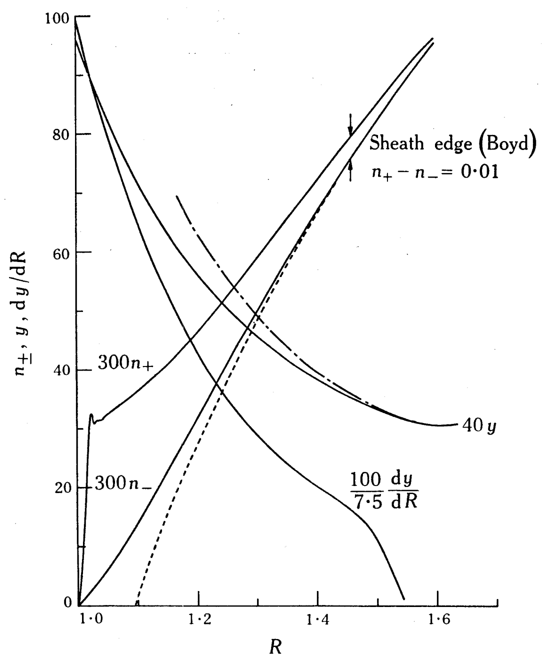

Fig. 1. Scaled (to 100 V) curves of $n_{\pm}$, y and dy/dR versus R, obtained on an analogue computer. The quasi-neutral solutions are shown by the dashed curve for n and by the chain curve for y. When $R = 1{\cdot}46$ we have $n_+ - n_- = 0{\cdot}01$ (Boyd's (1951) criterion).

It was convenient to change the independent variable ζ to $R = \zeta^{-1}$ for use on the computer; thus equations (1)–(3) become

$$dn_+/dR = \varepsilon^{-1}(\varepsilon J_+/R^2 + n_+\, dy/dR), \tag{5}$$

$$dn_-/dR = J_-/R^2 - n_-\, dy/dR, \tag{6}$$

$$d^2y/dR^2 = \rho_p^2(n_+ - n_-) - 2R^{-1}\, dy/dR, \tag{7}$$

with the boundary conditions

$$R = \infty, \qquad y = 0, \qquad n_+ = n_- = 1; \tag{8a}$$

$$R = 1, \qquad y = y_p, \qquad n_+ = n_- = 0. \tag{8b}$$

Graphs of potential and of ion and electron concentrations were presented by Dorman and Hamilton (1977*a*).

Calculation of Sheath Thickness

Fig. 1 shows the computer curves of $n_{\pm}$, y and $\mathrm{d}y/\mathrm{d}R$ versus R for $\varepsilon = 0{\cdot}01$, $y_p = 2{\cdot}42$ and $\rho_p = 12{\cdot}7$.

To determine the sheath thickness λ_s it is necessary to extrapolate slightly, for each computer curve (approximately 100 altogether), the curves for n_+ and n_- along the quasi-neutral line (Dorman and Hamilton 1977*a*). The criterion for the position of the sheath edge is an arbitrary one, but Boyd's (1951) criterion $n_+ - n_- = 0{\cdot}01$ is adopted here. From each computer plot the distance R and hence ζ is determined, where $n_+ - n_- = 0{\cdot}01$, this being the sheath thickness. The Debye length λ_D is calculated from the ratio of probe radius r_p to ρ_p (four different values of ρ_p were used). With knowledge of the sheath thickness it is also possible to determine the

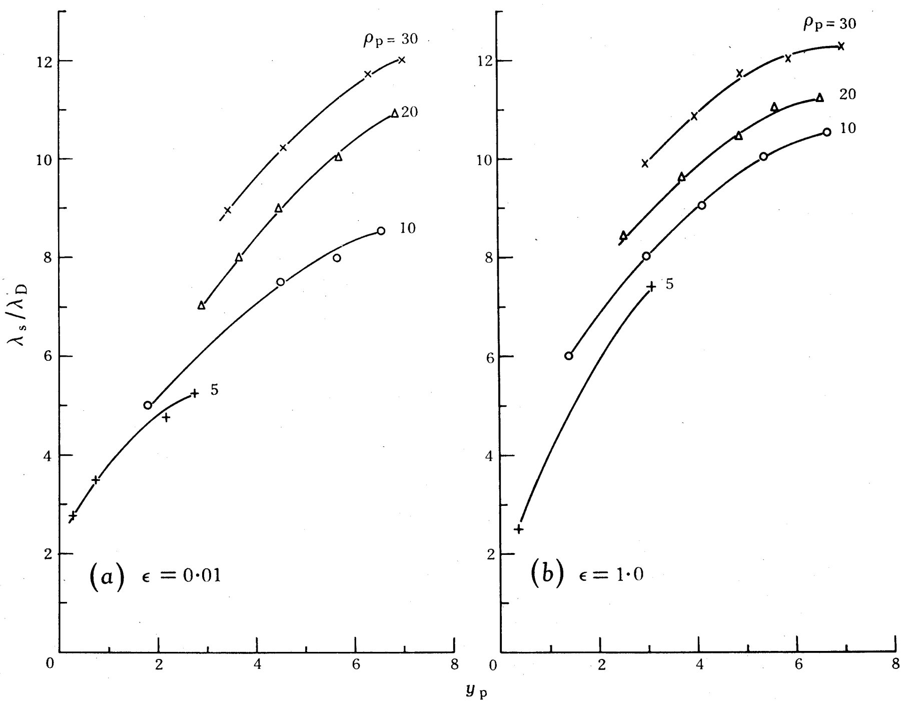

Fig. 2. Ratio of the sheath thickness λ_s to Debye length λ_D versus the normalized potential y_p at the probe surface for four values of ρ_p and $\varepsilon = 0{\cdot}01$ and $1{\cdot}0$.

potential drop across the sheath, since the initial value of y_p is known for each computer plot. The potential is read from the computer plot and subtracted from the initial value, enabling the value to be expressed as a percentage.

Results and Discussion

Fig. 2 shows graphs of λ_s/λ_D versus y_p for $\rho_p = 5$, 10, 20 and 30 with $\varepsilon = 0{\cdot}01$ and $1{\cdot}0$. The percentage potential drop across the sheaths for these cases is plotted in Fig. 3.

(a) λ_s/λ_D versus y_p

Boyd (1951) defined a sheath edge as being that region near a probe where $n_+ - n_- = 0{\cdot}01$. Because of the difficulty in solving equations (1)–(3) above, other authors (e.g. Waymouth 1964) have assumed that the ratio of sheath thickness λ_s to Debye length λ_D was typically 5–10. By extrapolating the computer curves it is now

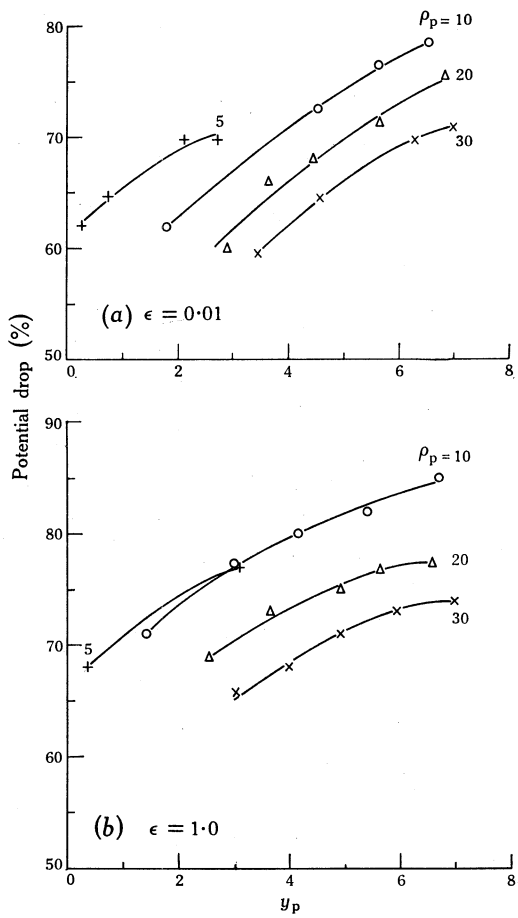

Fig. 3. Percentage potential drop across the sheath versus y_p for the cases shown in Fig. 2.

possible for us to be more precise. As can be seen from Fig. 2, there is little difference for various values of ε; for $\varepsilon = 1{\cdot}0$ the ratio λ_s/λ_D is about one more unit for a given y_p than for $\varepsilon = 0{\cdot}01$. The results quoted by Waymouth (1964) were for a free-fall sheath based on the Child–Langmuir law (equation 12.24 in the text by Brown 1966), so it can be said that values of λ_s/λ_D for both the collisional sheath and the collisionless sheath are comparable.

It is informative to calculate the minimum number of collisions that occur within a sheath. Using the typical values for λ_- (the mean free path of electrons) and λ_s from

Dorman and Hamilton (1976) of 0·01 cm and 0·5 cm respectively, we see that at least 50 collisions occur. The actual number would be greater because the charged particles do not move radially into the probe, especially at the sheath edge.

The random walk model proposed by Cozens and von Engel (1965) can now be examined more closely with the aid of the computer curves. If particles exhibited purely random walk then equations (1)–(3) would not apply, and instead the particle concentration from the centre of the probe would approximately follow an inverse square law. Cozens and von Engel also claim that the energy distribution remains position independent, except for the last electron path. However, this would imply that there is no field penetration, and from our computer curves (Dorman and Hamilton 1977*a*) this is known to be incorrect. Also, the variation in sheath thickness with potential cannot be explained by their theory. Nevertheless, random walks probably do occur farther out from the probe where potential gradients are small.

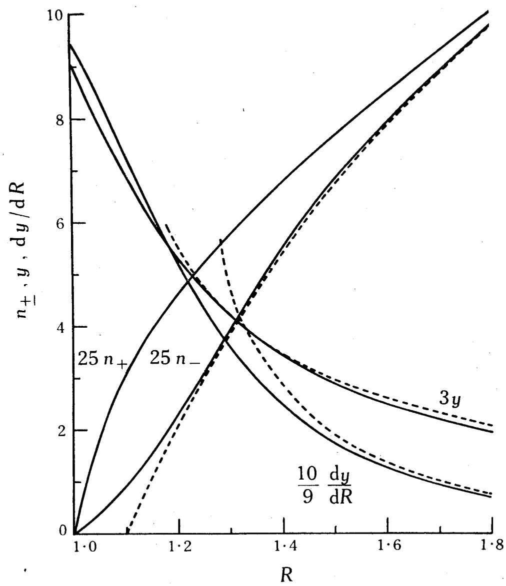

Fig. 4. Scaled (to 10 V) curves of $n_\pm$, y and $\mathrm{d}y/\mathrm{d}R$ versus R, for $\varepsilon = 1\cdot 0$, $y_p = 3$ and $\rho_p = 5$. Note the absence of a clearly defined diffusion region near the probe surface ($R = 1\cdot 0$); cf. Fig. 1.

(*b*) *Potential Drop across Sheath*

Fig. 3 shows the potential drop across the different plasmas (probe to sheath). For the same y_p, values of the potential drop were 5% higher for $\varepsilon = 1\cdot 0$ than for $\varepsilon = 0\cdot 01$. Since in practice double probes are usually at least 1 cm apart, and from the computer curves the potential at 1 cm distance was found to be less than 10% of the maximum value, we can conclude that interference between probes is negligible. Waymouth (1964) claimed that the dimension of the region within which the quasi-neutral plasma is disturbed is of the order of $20\,r_p$. However, this value is rather conservative, since even at $10\,r_p$ the potential will have fallen to about 1% of its initial value.

A basic assumption in the use of the continuum theory is that the diffusion coefficient and the mobility of the particles remain constant. As seen in Fig. 5 of Dorman and Hamilton (1977*a*), the potential gradients at the sheath edge and probe surface are 20 and 40 $\mathrm{V\,cm^{-1}}$ respectively. Hornbeck (1951) has shown that the ionic mobility is approximately inversely proportional to the square root of the field strength at high fields. For 40 $\mathrm{V\,cm^{-1}}$ and a pressure of 1 torr, i.e. the worst case in the experiment performed by Dorman and Hamilton (1976), it can be seen from Fig. 1 of Frost (1957) that the positive ion mobility varies only 10% between the undisturbed plasma and a point close to the probe surface. Higher pressure would result in a smaller mobility variation. The corresponding changes in the diffusion coefficient given by the Einstein relation are similar.

(*c*) *Diffusion Widths*

As shown by the computer curves (Fig. 1), the gradient of the positive ion curve (n_+) is high across the diffusion region, whose width, to a first approximation, is of the order of one mean free path. Su and Lam (1963) determined the diffusion width to be of the order of $\varepsilon(1+\varepsilon)^{\frac{1}{2}}\lambda_{\mathrm{D}}$; for $\varepsilon \sim 0{\cdot}01$ and $\rho_{\mathrm{p}} = 10$, the diffusion thickness is $0{\cdot}001\,r_{\mathrm{p}}$ from their definition. In the present study, Fig. 5 of Dorman and Hamilton (1977*a*) was repeated with the solution time reduced by a factor of 10 in order to measure it more accurately. From the plot, the diffusion width was found to be $0{\cdot}0075\,r_{\mathrm{p}}$, thus approximately agreeing with the Su and Lam definition.

From Fig. 4 it can be seen that the diffusion region, when $\varepsilon \sim 1{\cdot}0$ with y_{p} small, is not clearly defined because for these cases the high energy ions have a thermal energy comparable with the diffusion energy. However, cold ions gain little energy between collisions, and hence the potential drop across the last collision path is large.

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