

Resonance Neutron Capture in $^{86,87}\text{Sr}$

G. C. Hicks,^A B. J. Allen,^B A. R. de L. Musgrove^B and R. L. Macklin^C

^A Department of Physics, James Cook University of North Queensland, Townsville, Qld 4811.

^B AAEC, Lucas Heights Research Laboratories, Private Mail Bag, Sutherland, N.S.W. 2232.

^C Oak Ridge National Laboratory, Oak Ridge, TN 37830, U.S.A.

Abstract

The neutron capture cross sections of $^{86,87}\text{Sr}$ have been measured with high energy resolution from 3 to 200 keV at the 40 m station of the Oak Ridge Electron Linear Accelerator. Individual resonances were analysed to 37 keV for ^{86}Sr and to 14 keV for ^{87}Sr , and average resonance parameters were deduced on the basis of assumed divisions between s- and p-wave resonances. The average radiative widths obtained on this basis are consistent with a capture mechanism which is predominantly statistical.

Introduction

Investigations of the resonance neutron capture mechanism for nuclei near the $N = 50$ closed shell (Toohey and Jackson 1974; Boldeman *et al.* 1975, 1976a, 1976b) have indicated the presence of significant non-statistical effects. For p-wave capture in ^{88}Sr it was found that valence processes dominate the capture mechanism. Important valence effects were also observed in the resonance neutron capture cross sections of $^{90,92,94}\text{Zr}$. In addition to valence effects, in the case of ^{90}Zr , further large non-statistical transitions to the low lying states in ^{91}Zr were indicated. The present work on $^{86,87}\text{Sr}$ extends the investigations in this mass region to the situation in which the capturing nuclei have neutron hole configurations.

The resonance capture cross sections of $^{86,87}\text{Sr}$ have been measured up to several hundred keV. From the analysis, parameters were obtained for 71 resonances in ^{86}Sr between 3.2 and 37.2 keV and for 93 resonances in ^{87}Sr between 3.0 and 14.2 keV. At higher energies, it is difficult to obtain reliable resonance parameters because of overlapping resonances. This work complements and extends the information on these resonances obtained from the total cross section measurements by Adamchuk *et al.* (1965) and Rahn (1971; personal communication cited in Mughabghab and Garber 1973). No spin assignments had been made on the basis of the total cross section measurements (in the energy region common to the present work), and therefore the extraction of average resonance parameters is not without some ambiguity.

Experimental Method and Data Analysis

As part of a collaborative project between the ORNL and AAEC, the $^{86,87}\text{Sr}(n, \gamma)$ reactions were measured using the 40 m flight path facility at the Oak Ridge Electron Linear Accelerator (ORELA) and the analyses were carried out at the AAEC Research

Table 1. ^{86}Sr resonance parameters

E_n (keV)	Present analysis				Rahn (1971)		Adamchuk <i>et al.</i> (1965)	
	l^A	κ^B (eV)	Γ_n (eV)	Γ_γ (eV)	E_n (keV)	$g\Gamma_n$ (eV)	E_n (keV)	Γ_n (eV)
3.258	(0)	0.581	3.4 ± 0.3	0.70	3.247	3.6 ± 0.4	3.280	4.7 ± 0.5
	(1; $g = 2$)	0.730	2.4 ± 0.3	0.45				
4.511		0.243			4.496	0.45 ± 0.08		
5.148		0.115			5.130	0.29 ± 0.05		
5.433		0.019 (1)						
5.498		0.102						
5.572		0.020						
5.837		0.080						
6.462		0.222			6.440	0.68 ± 0.13		
7.965		0.162			7.944	0.48 ± 0.09		
8.302		0.166			8.276	0.53 ± 0.11		
9.385		0.214			9.356	0.44 ± 0.09		
10.029		0.208						
10.195	(0)	0.196	13 ± 3	0.20	10.161	8.5 ± 1.5	10.400	12 ± 3
10.382		0.208						
10.556		0.046 (3)						
10.902		0.230	4.0 ± 1.5	0.15	10.869	1.7 ± 0.3		
11.139		0.093						
11.510	(0)	0.16 (3)	50 ± 15	0.17	11.475	14.5 ± 2.5	11.700	76 ± 18
11.961		0.215			11.925	0.16 ± 0.02		
12.815		0.119						
13.950		0.200			13.897	0.53 ± 0.08		
14.410		0.088						
14.540		0.115						
15.020	(0)	0.277	13 ± 4	0.28	14.980	9.2 ± 1.7		
15.065		0.023						
15.450	(0)	0.143 (13)	40 ± 10	0.14	15.399	9.1 ± 1.7		
16.340		0.301						
16.835		0.303						
17.488		0.109						
18.225		0.280 (25)						
18.440		0.034 (16)						
18.445	(0)	0.08 (8)	200 ± 100	0.08	18.374	37 ± 6	18.800	300 ± 60
19.058		0.378						
19.392		0.126 (8)						
20.02		0.041 (5)						
20.50		0.298						
21.25		0.172 (11)						
21.43		0.112 (9)			21.359	2.5 ± 0.4		
Not seen					22.031	0.11 ± 0.03		
22.46		0.027 (5)			22.394	0.18 ± 0.04		
22.79		0.196						
22.85	(0)	0.270	30 ± 15	0.29	22.778	1.6 ± 0.3		
23.10		0.117 (8)						
23.18		0.324			23.087	1.0 ± 0.15		
23.96		0.136 (9)						
24.73		0.261						
25.10		0.143 (11)						
25.61		0.077 (8)						
26.91		0.121 (13)						
27.31		0.237 (15)						
28.20		0.287 (17)						
28.38	(0)	0.37 (8)	100 ± 30	0.39				

Table 1 (Continued)

E_n (keV)	κ^B (eV)	E_n (keV)	κ^B (eV)	E_n (keV)	κ^B (eV)
28.76	0.366	32.94	0.163 (15)	35.69	0.162 (15)
30.45	0.243 (17)	33.18	0.287 (20)	35.73	0.139 (13)
30.63	0.193 (14)	33.84	0.191 (19)	36.66	0.218 (22)
30.73	0.432	34.17	0.115 (11)	36.89	0.157 (21)
31.12	0.138 (13)	34.22	0.226 (16)	37.06	0.129 (19)
31.34	0.349	34.32	0.182 (15)	37.18	0.125 (15)
31.87	0.235 (17)	35.00	0.202 (20)		

^A The l values in parenthesis are probable assignments.

^B The error in the least significant figure for $\kappa = g\Gamma_n\Gamma_\gamma/\Gamma$ is given in parenthesis if $> 5\%$; for example $0.208(25) \equiv 0.208 \pm 0.025$.

Laboratories. Capture γ rays were detected by two non-hydrogenous C_6F_6 liquid scintillators (Macklin and Allen 1971; Allen *et al.* 1973), and events were weighted according to the observed γ -ray energy of the capture reaction. A 0.5 mm ^6Li glass scintillator, 0.5 m upstream from the capture sample, was operated as a neutron monitor (Macklin *et al.* 1971). The ^6Li neutron flux was obtained in a separate measurement at a later date and related to the $^{86,87}\text{Sr}$ data using a fission monitor situated in the neutron source cell. An uncertainty of $\pm 10\%$ was assigned to the normalization. The $^6\text{Li}(n, \gamma)$ cross section and efficiency perturbations caused by the glass constituents were previously parametrized by Macklin *et al.* (1975) and the absolute efficiency was determined by the saturated resonance method for the 4.9 eV resonance in gold. Enriched SrCO_3 targets were used (84.1% ^{86}Sr and 91.2% ^{87}Sr with respective thicknesses of 0.00641 and 0.00727 at. b $^{-1}$). The neutron energy resolution was approximately 0.2%.

After dead-time and background corrections, the capture cross sections were analysed using a modified version of the ORNL/RPI Monte Carlo code (Sullivan *et al.* 1969). Breit-Wigner single level theory was used to generate the cross sections. Initial guesses for the resonance neutron width Γ_n and radiative width Γ_γ were made to determine self-shielding and multiple scattering corrections. After the multiple scattering component was subtracted, an iterative fit to the capture area for each resonance ($2\pi^2\lambda^2 g\Gamma_n\Gamma_\gamma/\Gamma$) was accomplished by varying whichever of the initial values of Γ_γ and Γ_n was the smaller. A linear background beneath each resonance was assumed. As a fit can be used to estimate the neutron width when $\Gamma_n > 0.2\Gamma_R$, where Γ_R is the energy resolution, the neutron width was determined where possible. A prompt background correction was also made to account for the sensitivity of the detectors to resonance scattered neutrons. For $l > 0$ resonances this correction is negligible, but for s-wave resonances where $\Gamma_\gamma/\Gamma_n < 10^{-4}$ a substantial correction is required. This correction has been deduced from measurements of the potential scattering in C and ^{208}Pb , and resonance capture in many nuclides for energies from 7.6 to 440 keV. The Monte Carlo calculation is described in detail in Allen *et al.* (1977).

The γ -ray detectors used in this experiment have an efficiency proportional to the total γ energy emitted. Hence the γ -ray yield is proportional to the product of the number of neutrons captured and their binding (plus c.m.) energy. However, for ^{86}Sr the total energy of the capture reaction is reduced by 5% when the 0.388 MeV

Table 2. ^{87}Sr resonance parameters

E_n (keV)	I^A	Present analysis			Rahn (1971)		Adamchuk <i>et al.</i> (1965)	
		κ^B (eV)	Γ_n (eV)	Γ_γ^C (eV)	E_n (keV)	$2g\Gamma_n$ (eV)	E_n (keV)	$2g\Gamma_n$ (eV)
3.067		0.023			3.053	0.06 ± 0.01	3.030	0.7 ± 0.4
3.221		0.002 (0.2)						
3.299		0.009						
3.401		0.035						
3.443		0.003 (0.2)						
3.671	(0)	0.071	1.0 ± 0.3	0.17	3.658	0.56 ± 0.08	3.700	3.1 ± 0.6
3.764		0.018						
3.942		0.036						
4.049	(0)	0.047	3.7 ± 0.6	0.10	4.037	3.1 ± 0.4		
4.057		0.019						
4.142		0.022						
4.189		0.012						
4.441	(0)	0.034	1.5 ± 0.5	0.07				
4.498		0.025						
4.781	(0)	0.078	3.0 ± 0.5	0.16	4.766	2.05 ± 0.35		
4.854		0.033						
4.945		0.026						
4.981		0.031						
4.993	(0)	0.060	3.5 ± 1.5	0.12	4.977	1.96 ± 0.30		
5.211	(0)	0.069	2.4 ± 0.4	0.15	5.196	0.88 ± 0.14		
5.367		0.062	1.9 ± 0.3	0.13				
5.391		0.043						
5.519	(0)	0.098	2.5 ± 0.5	0.21	5.503	1.02 ± 0.16		
5.567		0.005 (0.5)						
5.746		0.030						
5.796	(0)	0.082	3.0 ± 1.0	0.17	5.777	1.5 ± 0.2		
6.077	(0)	0.067	9.0 ± 1.5	0.14	6.060	4.3 ± 0.7		
6.172	(0)	0.078	10.0 ± 1.5	0.16	6.150	5.4 ± 0.9		
6.181		0.028						
6.528		0.013						
6.587		0.056						
6.704		0.050						
6.836	(0)	0.051	2.0 ± 1.5	0.11	6.815	2.1 ± 0.4		
6.861		0.051						
7.042		0.030						
7.115		0.028						
7.188	(0)	0.047	4.0 ± 1.0	0.10				
7.297		0.018 (1)						
7.328		0.015 (1)						
7.406		0.027						
7.474		0.054						
7.530	(0)	0.076	2.0 ± 1.0	0.16	7.505	0.42 ± 0.08		
7.653		0.018 (1)						
7.832		0.042						
8.097		0.062						
8.290		0.018 (1)						
8.362		0.048						
8.465		0.008 (0.7)						
8.484		0.033						

Table 2 (Continued)

E_n (keV)	l^A	Present analysis			Rahn (1971)	
		κ^B (eV)	Γ_n (eV)	Γ_γ^C (eV)	E_n (keV)	$2g\Gamma_n$ (eV)
8.537		0.054				
8.576		0.057				
8.864	(0)	0.053	4.0 ± 1.0	0.11	8.840	0.82 ± 0.14
8.947		0.018 (1)				
8.983		0.027 (2)				
9.120		0.079				
9.160		0.027				
9.182		0.049				
9.251		0.068				
9.612		0.010 (1)				
9.639		0.029				
9.729		0.054				
9.760		0.010 (1)				
9.877		0.035				
10.003		0.069			9.974	0.26 ± 0.04
10.137		0.013 (1)				
10.194		0.064				
10.350		0.018 (2)				
10.400		0.045				
10.510		0.063				
10.657		0.049				
10.896	(0)	0.051	11 ± 2	0.10		
10.919		0.049				
11.538		0.052				
11.652		0.132	< 3	0.29^D		
11.872		0.056				
11.925		0.031 (2)				
12.250		0.031 (2)				
12.428		0.011 (2)				
12.535		0.040 (2)				
12.573		0.044 (2)				
12.615		0.010 (1)				
12.690	(0)	0.208	12 ± 3	0.43		
12.855		0.033 (2)				
12.997		0.040 (2)				
13.177		0.060 (4)				
13.274		0.124	< 4	0.26^D		
13.306		0.104	< 1	0.26^D		
13.369		0.035 (2)				
13.505		0.078				
13.554		0.035 (2)				
13.891	(0)	0.058 (4)	5 ± 3	0.12		
14.039	(0)	0.149	20 ± 5	0.30		
14.129		0.056 (3)				

^A The l values in parentheses are probable assignments.

^B The error in the least significant figure is given in parentheses if > 5%; for example $0.058(5) \equiv 0.058 \pm 0.005$.

^C $\langle g \rangle = 0.5$ is assumed.

^D A maximum value for Γ_n is assumed.

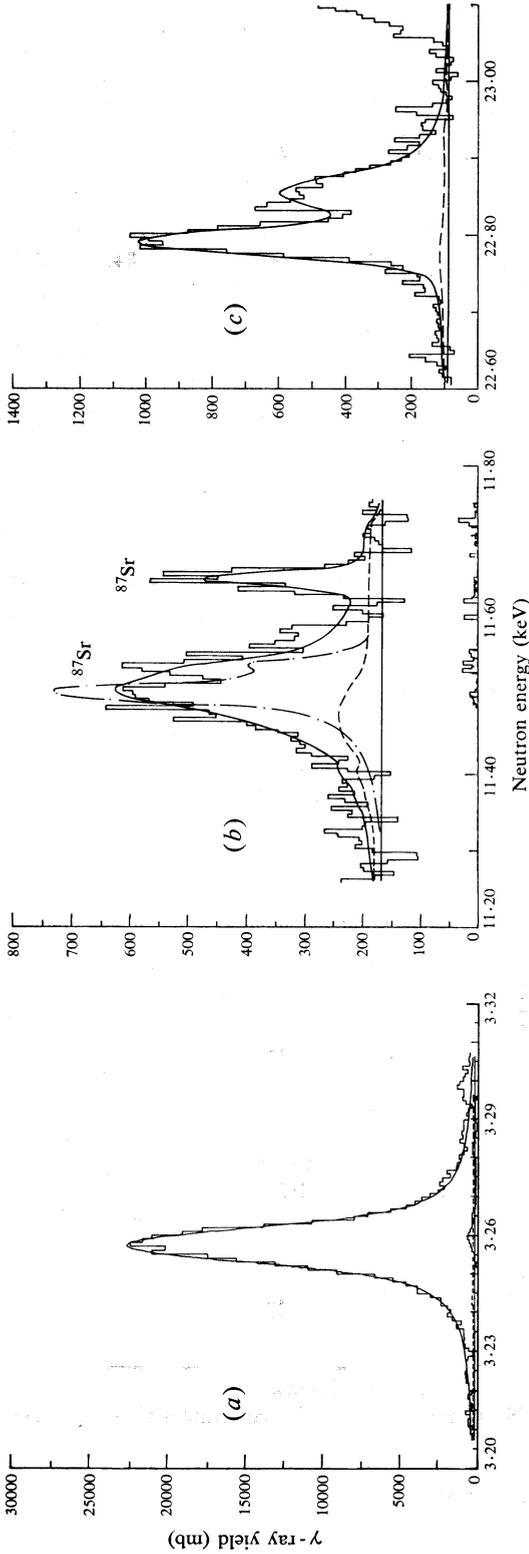


Fig. 1. Resonances in ^{86}Sr :

- (a) The 3.258 keV resonance is equally well fitted by $l = 0, g = 1, \Gamma_n = 3.4$ eV, $\Gamma_\gamma = 0.70$ eV and $l = 1, g = 2, \Gamma_n = 2.4$ eV, $\Gamma_\gamma = 0.43$ eV, the latter parameter set having the larger self-shielding correction. The prompt background (dashed curve) is negligible and a small multiple scattering correction (lower histogram) has been subtracted from the observed yield.
- (b) Comparison of Rahn parameters (dot-dash curve) and our own fit (solid curve) for the 11.510 keV resonance in ^{86}Sr . Isotopic impurities from ^{87}Sr are shown.
- (c) Only one narrow resonance ($\Gamma_n = 1.6$ eV) is given in the Rahn data in the region of this doublet. The 22.85 keV resonance here has $\Gamma_n = 30 \pm 15$ eV.

isomeric state is excited in ^{87}Sr . Because s- and p-wave resonances require a lower multiplicity to decay to the isomeric state ($J_m^\pi = \frac{1}{2}^-$) compared with the ground state ($J_0^\pi = \frac{9}{2}^+$), we have assumed that the isomeric state is preferentially excited. This assumption increases the cross section by 5% but introduces an uncertainty of comparable magnitude.

Resonance Parameters

Resonance parameters are presented for $^{86,87}\text{Sr}$ in Tables 1 and 2 respectively. The resonance energies, although in close agreement with those observed in the measurements of Rahn (1971), are systematically higher by 0.32% (the standard deviation $\text{SD} = 0.04\%$).

The estimated neutron widths of a number of resonances were in considerable disagreement with those obtained by Rahn. For ^{86}Sr , the resolved neutron widths are generally much larger than the lower limit allowed by the fitting techniques, i.e. $\Gamma_n \approx 0.2 \Gamma_R$, and do not depend on an accurate knowledge of Γ_R . However, for ^{87}Sr the resolved neutron widths tend to be smaller and an accurate estimate of the resolution width was required. This was obtained by fitting resonances believed to have small neutron widths. Instances of the discrepancies between neutron widths obtained in the present analysis and those due to Rahn are shown in Fig. 1.

The fitting of the resonances for ^{86}Sr was complicated by the presence of resonances due to a ^{87}Sr impurity in the target material. Where necessary, resonance parameters from the ^{87}Sr analysis were kept constant and the ^{86}Sr resonances were fitted in the usual manner. The presence of these ^{87}Sr resonances can be seen in Fig. 1b.

Nucleus ^{86}Sr

The 3.258 keV resonance in ^{86}Sr is presented in Fig. 1a. This is one of the few cases in which a neutron width obtained in the present analysis compares favourably with the width obtained by Rahn (1971) (assuming $g = 1$). The respective values are $\Gamma_n = 3.4 \pm 0.3$ eV and $g\Gamma_n = 3.6 \pm 0.3$ eV. More importantly, it has a very large radiative width of $\Gamma_\gamma = 0.70$ eV. If this resonance is s-wave, it has the largest known radiative width for $A > 80$, where $\langle \Gamma_\gamma^s \rangle \lesssim 0.2$ eV. However, p-wave radiative widths are larger than s-wave in the mass region $88 \lesssim A \lesssim 100$, and on this basis a p-wave assignment could be made. The neutron width is not so large that it rules out this assignment, although an s-wave assignment is strongly favoured in a Bayes analysis. If we assume $g = 2$ for this resonance, an equally good fit is then obtained for $\Gamma_n = 2.4$ eV and $\Gamma_\gamma = 0.43$ eV. However, the latter value is still unusually large for this mass region.

The fit to the 11.510 keV ^{86}Sr resonance is shown in Fig. 1b together with a fit based on the maximum neutron width (i.e. $g = 1$) found by Rahn (1971). The width obtained in the present analysis is in better agreement with the estimate of Adamchuk *et al.* (1965). Greater disagreement is found for the 22.85 keV resonance where our value ($\Gamma_n = 30 \pm 15$ eV) greatly exceeds that of the 22.78 keV resonance ($\Gamma_n = 1.6 \pm 0.3$ eV) in Rahn's data (Fig. 1c).

In ^{86}Sr , the region between 18.35 and 18.65 keV contains structure which cannot be explained by the presence of ^{87}Sr resonances alone. The structure is ill-defined because of poor statistics but at least one rather broad resonance appears to be present.

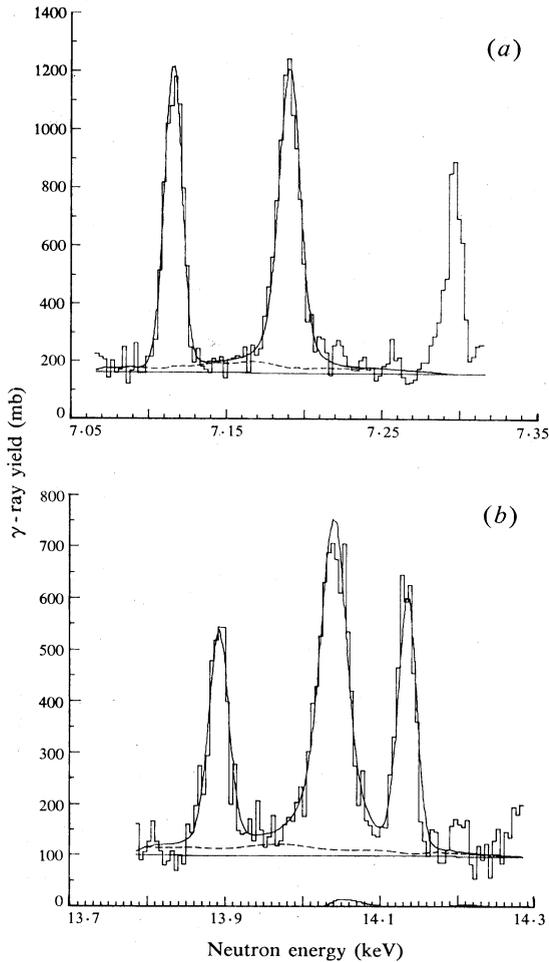


Fig. 2. Resonances in ^{87}Sr : (a) The fit to the 7.188 keV resonance, for which $\Gamma_n \sim 0.3 \Gamma_R$, is compared with the 7.115 keV resonance which is fitted by a gaussian with the resolution width (i.e. $\Gamma_n \ll \Gamma_R$). In (b) the 14.039 keV resonance has the largest neutron width in the ^{87}Sr data. The single resonance fit for $\Gamma_n = 20 \pm 5$ eV is preferred to that of a doublet.

Total cross section parameters show a wide discrepancy here and our data favour the result of Adamchuk *et al.* In the present work only one resonance seen by Rahn is not detected. This is a resonance in ^{86}Sr near 22.03 keV.

Nucleus ^{87}Sr

Resonances in ^{87}Sr at 4.441, 5.367 and 7.188 keV are within the energy range investigated by Rahn but were not seen in that experiment. Furthermore, they have resolved neutron widths. Fig. 2a shows the fit to the 7.188 keV resonance. Here the estimated Γ_n value is only $0.3 \Gamma_R$, but the difference in resonance shape, compared with the 7.115 keV resonance which is adequately fitted by the resonance line shape,

is clearly apparent. The presence of small unresolved resonances could, however, provide an alternative interpretation. If a doublet is mistakenly analysed as a single resonance, a large radiative width will result. This may be the case for the 14.039 keV resonance, shown in Fig. 2*b*. A further three resonances have large values of $\kappa \equiv g\Gamma_n\Gamma_\gamma/\Gamma$. Although it is not possible to measure Γ_n for these resonances, upper limits can be established, and these allow the estimation of lower limits for Γ_γ , which are included in Table 2.

Overlapping resonances in ^{87}Sr in the vicinity of 9.6 keV and also 10.9 keV were analysed as doublets, although the higher energy group could be a triplet. The lower energy side of the 9.6 keV group proved to be especially difficult to fit and a broader resonance may be present. However, there is not evidence for this in the total cross section data.

Table 3. Average resonance parameters
Parentheses designate uncertain values and square brackets assumed values

Nuclide	l^A	$\langle D_l \rangle^B$ (keV)	$10^4 S_l$	$\langle g\Gamma_\gamma \rangle / \langle g \rangle^C$ (meV)	N_{obs}^A	E_{max} (keV)
^{86}Sr	0	(1.9)	1.0 ± 0.4	220 ± 105	(8)	37.2
	1	(0.63)	[3.5]	130 ± 45	(66)	37.2
	2		[1.0]	[200]		
^{87}Sr	0	(0.37)	0.40 ± 0.10	160 ± 85	(24)	14.2
	1	(0.18)	[3.0]	(140)	(91)	14.2
	2		[1.5]	[100]		

^A The $l = 0$ resonances are probable assignments based on neutron width statistics. Resonances below 3 keV from Mughabghab and Garber (1973) are included here.

^B A $2J+1$ level density dependence is assumed.

^C The p-wave values are based on the distribution of κ ; standard deviations are given.

Average Resonance Parameters

The results for $^{86,87}\text{Sr}$ are presented in Table 3 and supersede the previous values of Musgrove *et al.* (1978).

For completeness, the parameters of resonances below 3 keV, as listed in the BNL compilation of resonance parameters (Mughabghab and Garber 1973) were included with those obtained in the present analysis. In the absence of any definite spin and parity assignments the division between $l = 0$ and $l > 0$ resonances was investigated in the following way. A division was assumed and the average parameters were used to generate Wigner and Porter–Thomas distributions. In turn, these allowed a Bayes analysis to be made and a new division was obtained on this basis. Essentially, this process designates the majority of resonances with large neutron widths (i.e. Γ_n measurable in this analysis) as s-wave resonances. While a reasonably reliable division of resonances can be obtained in this way, average parameters, such as the level spacings, are not necessarily determined unambiguously by this division.

If we assume that the level densities are proportional to $2J+1$, the expected ratios of the number of p- to s-wave resonances for $^{86,87}\text{Sr}$ are 3 : 1 and 2 : 1 respectively. For ^{87}Sr the result obtained is in fair agreement with this ratio. The discrepancy can be attributed to missed resonances at the higher energies. This is apparent from the change in slope, above 8 keV, of the staircase plot of a number of s-wave resonances against neutron energy, as shown in Fig. 3. For ^{86}Sr the agreement is not so

good. As the number of s-wave resonances is significantly smaller, it is difficult to establish whether the situation is similar to that for ^{87}Sr . It can only be presumed that a greater proportion of the s-wave resonances remain unidentified. Consequently, the level spacings have been deduced directly from the $2J+1$ level density rule and the total number of resonances observed.

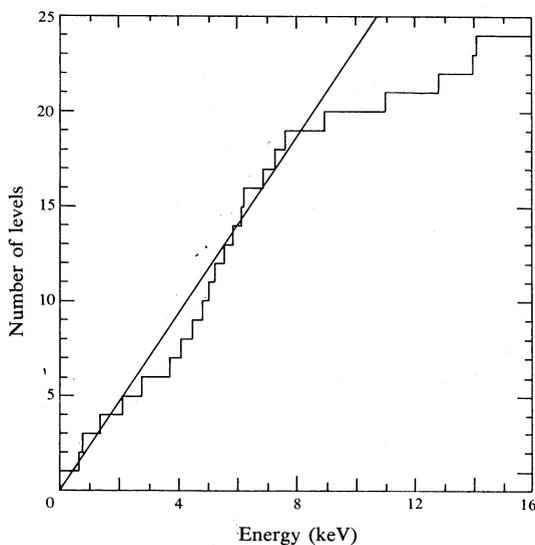


Fig. 3. Staircase plot for the assumed s-wave level sequence for ^{87}Sr . Resonance parameters below 3 keV were taken from Mughabghab and Garber (1973).

The average radiative widths for the assumed s-wave resonances are calculated from the neutron widths and κ values. From these parameters the estimation of each of the radiative widths, and hence the average width $\langle\Gamma_{\gamma}^s\rangle$, follows in a straightforward manner. A difficulty occurs for ^{87}Sr where s-wave neutrons populate both 4^+ and 5^+ states. Here it is only possible to calculate $\langle g\Gamma_{\gamma}\rangle^s/\langle g\rangle$, assuming $\langle g\rangle = 0.5$. It should be noted that those radiative widths which have previously been referred to as large, in fact lie significantly outside the distribution of the radiative widths of the remaining assumed s-wave resonances. For ^{86}Sr the radiative width (for $g = 1$) of the 3.258 keV resonance lies 4.6 standard deviations from the mean of the distribution formed by the remainder. In view of the large absolute magnitude of this width and the possibility of a $g = 2$ assignment, this resonance has been excluded from the s-wave population in calculating $\langle\Gamma_{\gamma}^s\rangle$. The 12.690 and 14.039 keV resonances for ^{87}Sr also have radiative widths which lie well away from the mean of the distribution formed by the other 17 resonances, by 8.2 and 4.6 standard deviations respectively. The assumption $\langle g\rangle = 0.5$ cannot cause these large values (since $g = 0.45$ or 0.55). It is possible they might indicate unresolved groups of resonances as the neutron widths are also large. Nevertheless, single level fits appear to give the best results. Although these radiative widths are relatively large and clearly non-statistical, there are no clear grounds for removing them from the s-wave population and they have been included in calculating $\langle g\Gamma_{\gamma}\rangle^s/\langle g\rangle$.

Average Capture Cross Sections

The average capture cross sections for $^{86,87}\text{Sr}$ are given in Table 4. In the resolved resonance region, few resonances are missed and the capture cross section is fully accounted for by the contributions from the observed resonances. At higher energies, the capture cross section was obtained by fitting a time-dependent background in the resolved resonance region and adding the estimated contributions to the capture yield of the minor isotopic impurities. These backgrounds were then extrapolated to higher energies assuming a $1/v$ behaviour. The average prompt backgrounds from resonance scattered neutrons (19% for ^{86}Sr and 4% for ^{87}Sr) and multiple scattering correction ($\sim 2\%$) were calculated by the Monte Carlo method using average resonance parameters to generate the appropriate cross sections. An average self-shielding correction (~ 0.93) was then applied to obtain the average capture cross sections.

Table 4. Radiative capture cross sections

The 30 keV Maxwellian averaged cross sections for $^{86,87}\text{Sr}$ are 70 ± 8 and 74 ± 10 mb respectively

E (keV)	$\sigma(^{86}\text{Sr})$ (mb)	$\sigma(^{87}\text{Sr})$ (mb)	E (keV)	$\sigma(^{86}\text{Sr})$ (mb)	$\sigma(^{87}\text{Sr})$ (mb)
3-4	735 ± 84	232 ± 25	30-40	80 ± 11	66 ± 13
4-5	266 ± 26	353 ± 40	40-50	65 ± 11	62 ± 12
5-6	275 ± 26	296 ± 30	50-60	53 ± 11	57 ± 11
6-8	116 ± 13	228 ± 25	60-80	42 ± 9	42 ± 9
8-10	89 ± 9	173 ± 20	80-100	36 ± 8	39 ± 8
10-15	140 ± 16	124 ± 20	100-150	37 ± 8	32 ± 6
15-20	98 ± 16	97 ± 20	150-200	36 ± 8	28 ± 6
20-30	64 ± 6	82 ± 16			

Uncertainties attributed to the time-dependent background and the prompt background were 10% and 15% respectively. The $1/v$ component of the capture yield at ~ 35 keV was $\sim 50\%$. The normalization uncertainty for both $^{86,87}\text{Sr}$ was 10%.

The average cross sections for $^{86,87}\text{Sr}$ are shown in Fig. 4. Partial capture cross sections calculated from the average resonance parameters of Table 3 adequately reproduce the experimental results to 100 keV.

Maxwellian averaged cross sections for the isotopes of strontium are of astrophysical interest (Vanpraet *et al.* 1972). The average cross section data have therefore been weighted by the Maxwellian distribution for $kT = 30$ keV to obtain cross sections pertinent to s-process synthesis in red giant stars. Our results (Table 4) supersede those of Vanpraet *et al.*

Valence Model

The valence capture contribution to the ^{86}Sr p-wave radiative widths has been calculated using the optical model formulation (Lane and Mughabghab 1974; Barrett and Terasawa 1975). The valence contribution is given by the summation over final states

$$\Gamma_{\lambda\gamma}^v = \sum_{\mu} q_{\lambda\mu} E_{\lambda\mu}^3 \theta_{\mu}^2 (Z/A)^2 \Gamma_{\lambda n}^l = Q_{\lambda} \Gamma_{\lambda n}^l.$$

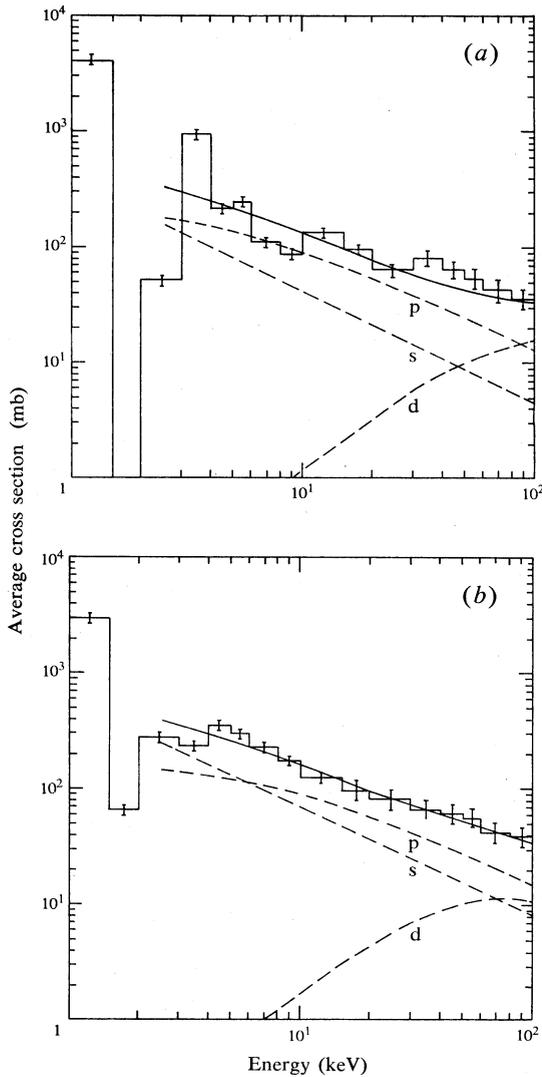


Fig. 4. Average capture cross sections for (a) ^{86}Sr and (b) ^{87}Sr . The partial wave (dashed curves) and total (solid curve) capture cross sections have been calculated from the average resonance parameters of Table 3 and are compared with the measured average capture cross sections (histogram).

Here the reduced valence width $q_{\lambda\mu}$ is an energy-dependent parameter containing radial integration and geometrical factors. The average total valence width for resonances of spin J and angular momentum l is given by

$$\langle \Gamma_{\gamma}^{\nu} \rangle_{lJ} = Q_{lJ} S_l \langle D_{lJ} \rangle,$$

where S_l is the l -wave neutron strength function and $\langle D_{lJ} \rangle$ is the average spacing of levels with the same J . The calculation of p-wave capture in ^{86}Sr considered final states up to 3.12 MeV; the spectroscopic factors θ_{μ}^2 were taken from Bercaw and

Warner (1970) and Morton *et al.* (1971). The values of $q_{\lambda\mu}$ were obtained from Allen and Musgrove (1978) and the calculation yielded the results $Q_{1,1/2^-} = 0.7 \times 10^{-2}$ and $Q_{1,3/2^-} = 1.8 \times 10^{-2}$. These are very similar in magnitude to $Q_{IJ\pi}$ values tabulated in this same reference for other nuclides in the mass region with $N < 50$, i.e. Ge through to Rb.

If we use the values of S_I and $\langle D_{IJ} \rangle$ obtained in the present analysis, the average contribution to the radiative widths of both $p_{1/2}$ and $p_{3/2}$ resonances is approximately 6 meV. Thus the average valence contribution is small, being only a few percent of the average total radiative width. The valence contribution will be larger for individual resonances with large Γ_n and it is interesting to calculate its magnitude for the 3.258 keV resonance for ^{86}Sr assuming it to be a p-wave ($g = 2$) resonance. The valence contribution of 100 meV cannot account for the anomalous magnitude of the radiative width of this resonance.

Statistical Calculations

The statistical contribution and its standard deviation are derived from the summation of the E1 and M1 γ -ray transitions to allowed final states. The summation has the form

$$\Gamma_{\lambda\gamma}^s = \sum_{\mu} \Gamma_{\lambda\mu}^s, \quad \text{with} \quad \Gamma_{\lambda\mu}^s = k D_{J\pi} \{ E_{\gamma\mu}^n(E1) + R E_{\gamma\mu}^n(M1) \}.$$

The average ratio of M1 and E1 transition strengths is assumed to be $R = 0.14$ and the exponent n determines the γ -ray strength function (Axel 1962; Bollinger 1973). For the single particle model $n = 3$, and the effect of the giant dipole resonance can be taken into account with $n = 5$. The calculation uses the known low lying final states and, at higher energies where these states are not sufficiently well known, the level density as given by Gilbert and Cameron (1965) is used. In the present calculation final states in ^{87}Sr , taken from the same references (Bercaw and Warner 1970; Morton *et al.* 1971) as for the valence calculation, were included up to 2.17 MeV. The final states in ^{88}Sr , which were included up to 4.04 MeV, were obtained from Bunting and Kraushaar (1976). Further general details relating to the statistical contribution, in particular calculation of the standard deviation, have been discussed by Taylor *et al.* (1979).

Nucleus ^{86}Sr

Relative widths and standard deviations were obtained for transitions from the allowed initial states. For p-wave capture in ^{86}Sr , these initial states are $\frac{1}{2}^-$ and $\frac{3}{2}^-$. The statistical calculation predicts $\langle g\Gamma_{\gamma} \rangle^{3/2^-} / \langle g\Gamma_{\gamma} \rangle^{1/2^-} \approx 2$ for both $n = 3$ and $n = 5$; i.e. it predicts equal radiative widths for each spin state. For $n = 3$ the relative standard deviation in $\langle g\Gamma_{\gamma} \rangle$ is about 15%, but for $n = 5$ it is 30%. The distribution of $g\Gamma_{\gamma}$ should therefore be a double peaked distribution.

It is worth noting that for the distribution of κ values of the assumed ^{86}Sr p-wave resonances, the bulk of the values lies in the region 80–300 meV. As shown in Fig. 5a, this region contains two peaks—a well defined peak at $\kappa = 120$ meV and a broader peak at $\kappa = 210$ meV. This is not surprising since the level width distributions generated on the basis of the adopted average parameters indicate that the p-wave neutron widths are, on average, an order of magnitude greater than the radiative

widths. If $\Gamma_n \gg \Gamma_\gamma$, then $\kappa \approx g\Gamma_\gamma$ and the distribution of κ values thus reflects the distribution of the $g\Gamma_\gamma$, but with the peaks slightly broadened and occurring at lower values. Consequently, it seems that the majority of the radiative widths are explained by a statistical capture mechanism. The average value of κ for those resonances which can be reasonably associated with the peaks, i.e. between 80 and 300 meV, is $\langle \kappa \rangle = 180$ meV (SD = 65). If all κ values are taken into account, the standard deviation is of course increased, but the magnitude is unchanged, i.e. $\langle \kappa \rangle = 180$ meV (SD = 90). It is considered that the standard deviation of the restricted sample more properly estimates the extent of the distribution of those κ values of predominantly statistical origin. Because of the order of magnitude difference, on average, between Γ_n and Γ_γ it is not unreasonable to suggest that $\langle g\Gamma_\gamma \rangle$ is 10–20% greater than $\langle \kappa \rangle$, i.e. $\langle g\Gamma_\gamma \rangle \approx 210$ meV (SD = 75).

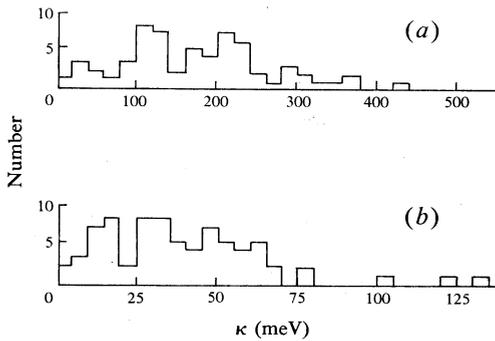


Fig. 5. Distribution of κ values for (a) ^{86}Sr and (b) ^{87}Sr . In the former, the $g = 1$ and 2 peaks are seen at ~ 120 and 210 meV respectively. Anomalous values are observed in both data sets.

Nucleus ^{87}Sr

For s-wave capture in ^{87}Sr there are two initial states and for p-wave capture there are four. The calculation indicates that the values of $\langle g\Gamma_\gamma \rangle$ for transitions from each of these states are very similar. For $n = 3$ there is a 22% variation with the standard deviations of each ranging from 8 to 9%; for $n = 5$ the variation is 26% and standard deviations are in the range 17–21%. In particular, for p-wave capture this means that all values of $g\Gamma_\gamma$ will be distributed in a well defined group having a maximum standard deviation of $\approx 30\%$ (the $n = 5$ case). This grouping will not be reflected in the distribution of κ values, however, since on average the neutron widths are no longer significantly greater than the radiative widths. Calculations using the average level spacings and strength functions obtained in this analysis suggest that $\langle \Gamma_n \rangle \gtrsim \langle \Gamma_\gamma \rangle$ can be expected, especially for resonances occurring at energies above 5 keV. Thus, relative to the distribution of $g\Gamma_\gamma$, the κ distribution will be considerably distorted and spread in the direction of lesser values. Nevertheless, for the higher values of κ in the distribution, the Γ_n are still expected to be considerably larger than Γ_γ , with the result that the upper cut-off of the distribution provides a rough estimate of $\langle g\Gamma_\gamma \rangle$. The experimental distribution is shown in Fig. 5b and is consistent with expectations. The apparent dip in the vicinity of 23 meV is a consequence of the way in which the data have been grouped. The main portion of the distribution has its upper limit in the vicinity of 80 meV and the estimated value of $\langle g\Gamma_\gamma \rangle$ is taken as 70 meV. The three higher values of κ correspond to those ^{87}Sr p-wave resonances which have already been noted to have anomalously large radiative widths.

A comparison between theory and experiment is made in terms of the ratios $\langle g\Gamma_\gamma \rangle^s / \langle g\Gamma_\gamma \rangle^p$, $\sigma^s / \langle g\Gamma_\gamma \rangle^s$ and $\sigma^p / \langle g\Gamma_\gamma \rangle^p$ listed in Table 5. Theoretical values are calculated for both $n = 3$ and 5, and we have excluded those assumed s-wave radiative widths which lie well outside the distribution of the remainder. The experimental values of $\langle g\Gamma_\gamma \rangle^p$ used are those estimated from the cut-off of the κ distribution of assumed p-wave resonances. The experimental results are consistent with theory and, for ^{86}Sr , better agreement is obtained with $n = 5$ for the ratio of s- to p-wave values of $g\Gamma_\gamma$.

Table 5. Comparison of statistical theory results with experiment

Nuclide		$\langle g\Gamma_\gamma \rangle^s / \langle g\Gamma_\gamma \rangle^p$	$\sigma^s / \langle g\Gamma_\gamma \rangle^s$	$\sigma^p / \langle g\Gamma_\gamma \rangle^p$
^{86}Sr	Theory $n = 3$	0.68	0.29	0.33
	$n = 5$	0.86	0.53	0.45
	Exp.	1.05	0.48	0.35
^{87}Sr	Theory $n = 3$	1.01	0.09	—
	$n = 5$	0.97	0.20	—
	Exp.	0.96	0.27	—

Discussion

The extraction of s- and p-wave average parameters depends in the first instance on an acceptable division of the observed resonances into these two classes. Unfortunately, spin and parity data are unavailable for $^{86,87}\text{Sr}$, and the division has been accomplished by seeking self-consistency as outlined earlier.

Calculations of valence radiative widths have been made for p-wave resonances, but only a small contribution to the observed widths can arise from this mechanism.

The majority of resonances have radiative widths consistent with a statistical mechanism. The double peaked nature of the distribution of κ values for ^{86}Sr and the ratios $\langle g\Gamma_\gamma \rangle^s / \langle g\Gamma_\gamma \rangle^p$ and associated standard deviations for both isotopes exhibit qualitative agreement between theory and experiment. In the mass region $88 \lesssim A \lesssim 100$, we have $\langle \Gamma_\gamma^s \rangle \approx 0.15$ eV (Musgrove *et al.* 1978), and $\langle \Gamma_\gamma^p \rangle$ is usually significantly greater than $\langle \Gamma_\gamma^s \rangle$. However, on the basis of our estimates of $\langle \Gamma_\gamma^p \rangle$ made for $^{86,87}\text{Sr}$, a similar tendency is not found here. The s- and p-wave average radiative widths appear to be of similar magnitude.

In summary, it is seen that resonances in $^{86,87}\text{Sr}$ have properties consistent with a statistical radiative capture mechanism. Despite the difficulties in denoting resonances as either s- or p-wave, and the qualitative nature of the comparison between experiment and theory, it is difficult to avoid this conclusion. However, there are also several resonances which, with large radiative widths, have been excluded from our analysis because of experimental uncertainties. These widths are anomalous with respect to the statistical model irrespective of l -wave assignment and cannot be ascribed to valence contributions.

Acknowledgments

G. C. Hicks acknowledges the financial assistance of a grant from the Australian Institute of Nuclear Science and Engineering.

References

- Adamchuk, Yu. V., Danelyan, L. S., Moskalev, S. S., Muradyan, G. V., and Shchepkin, Yu. G. (1965). *Euronuclear* **2**, 183.
- Allen, B. J., Macklin, R. L., Winters, R. R., and Fu, Y. C. (1973). *Phys. Rev. C* **8**, 1504.
- Allen, B. J., and Musgrove, A. R. de L. (1978). *Adv. Nucl. Phys.* **10**, 129.
- Allen, B. J., Musgrove, A. R. de L., Macklin, R. L., and Winters, R. R. (1977). Specialist Meeting on Neutron Data of Structural Materials for Fast Reactors, CBNM, Geel (Ed. K. H. Böckhoff) (Pergamon: London).
- Axel, P. (1962). *Phys. Rev.* **126**, 671.
- Barrett, R. F., and Terasawa, T. (1975). *Nucl. Phys. A* **240**, 445.
- Bercaw, R. W., and Warner, R. E. (1970). *Phys. Rev. C* **2**, 297.
- Boldeman, J. W., Allen, B. J., Musgrove, A. R. de L., and Macklin, R. L. (1975). *Nucl. Phys. A* **246**, 1.
- Boldeman, J. W., Allen, B. J., Musgrove, A. R. de L., Macklin, R. L., and Winters, R. R. (1976a). *Nucl. Phys. A* **269**, 397.
- Boldeman, J. W., Musgrove, A. R. de L., Allen, B. J., Harvey, J. A., and Macklin, R. L. (1976b). *Nucl. Phys. A* **269**, 31.
- Bollinger, L. M. (1973). Proc. Asilomar Conf. on Photonuclear Reactions and Applications (Ed. B. L. Berman) CONF-730301, p. 783 (Lawrence Livermore Lab.).
- Bunting, R. L., and Kraushaar, J. J. (1976). *Nucl. Data Sheets* **18**, 87.
- Gilbert, A., and Cameron, A. G. W. (1965). *Can. J. Phys.* **43**, 1446.
- Lane, A. M., and Mughabghab, S. F. (1974). *Phys. Rev. C* **10**, 412.
- Macklin, R. L., and Allen, B. J. (1971). *Nucl. Instrum. Methods* **91**, 565.
- Macklin, R. L., Halperin, J., and Winters, R. R. (1975). *Phys. Rev. C* **11**, 1270.
- Macklin, R. L., Hill, N., and Allen, B. J. (1971). *Nucl. Instrum. Methods* **96**, 509.
- Morton, J. M., Davies, W. G., McLatchie, W., Darcey, W., and Kitching, J. E. (1971). *Nucl. Phys. A* **161**, 228.
- Mughabghab, S. F., and Garber, D. I. (1973). Brookhaven National Laboratory Rep. No. BNL-325.
- Musgrove, A. R. de L., Allen, B. J., Boldeman, J. W., and Macklin, R. L. (1978). Proc. Int. Conf. on Neutron Physics and Nuclear Data for Reactors and Other Applied Purposes, Harwell, p. 449 (OECD Nuclear Energy Agency: Paris).
- Rahn, F. (1971). Personal communication cited in Mughabghab and Garber (1973).
- Sullivan, J. G., Warner, R. G. G., Block, R. C., and Hockenbury, R. W. (1969). Rensselaer Polytechnic Institute Rep. No. RPI-328-155.
- Taylor, R. B., Allen, B. J., Musgrove, A. R. de L., and Macklin, R. L. (1979). *Aust. J. Phys.* **32**, 551.
- Toohy, R. W., and Jackson, H. E. (1974). *Phys. Rev. C* **9**, 346.
- Vanpraet, G. J., Macklin, R. L., Allen, B. J., and Winters, R. R. (1972). Proc. Int. Conf. on Nuclear Structure Study with Neutrons, Budapest, p. 482 (Eds J. Erö and J. Szücs) (Plenum: New York, 1974).