*Aust. J. Phys.*, 1984, **37**, 463–74

# Proton Inelastic Scattering in the Interacting Boson Model: Formalism and Application to the Ge Isotopes

*W. Bauhoff*[A,B] *and I. Morrison*[A]

[A] School of Physics, University of Melbourne, Parkville, Vic. 3052.

[B] Permanent address: Theoretische Kernphysik, Universität Hamburg, Luruper Chaussee 149, 2000 Hamburg 50, Federal Republic of Germany.

*Abstract*

The application of the interacting boson model to the coupled channel description of inelastic proton scattering is studied. The radial shape of the transition potentials is determined by analogy to the usual geometrical models, whereas the reduced matrix elements are calculated from the boson Hamiltonian. The general formalism is applied to scattering from the Ge isotopes. We find a better description for the heavier isotopes in terms of an O(6)-symmetric model than for a vibrational model.

## 1. Introduction

The interacting boson model (IBM) of Arima and Iachello (1976, 1978*a*, 1978*b*) has been frequently used in recent years for the description of the structure of medium and heavy nuclei away from closed shells. Energy spectra, quadrupole moments and electromagnetic transition rates have been calculated for a large number of nuclei, and encouraging agreement with experiment has been obtained with only a small number of parameters which vary smoothly with mass number. As limiting cases of symmetry in the boson Hamiltonian, the IBM encompasses the classical geometric models like rotational, vibrational and $\gamma$-unstable nuclei, and describes also situations between these extreme models as symmetry breaking. Thus the IBM describes, for example, the transition from vibrational to rotational nuclei in the Sm isotopes (Scholten *et al.* 1978).

Applications of the IBM to dynamical processes instead of the static quantities mentioned above have been much less frequent. Examples of such applications are inelastic electron scattering (Iachello 1981; Dieperink 1981) and inelastic proton scattering (Morrison and Smith 1980; Cereda *et al.* 1982). In the present paper we will deal with the latter aspect. Previous applications of the IBM to inelastic proton scattering have been confined either to DWBA calculations (Morrison and Smith 1980; Amos *et al.* 1984), which are meaningful only for the lowest $2^+$ state, or to the vibrational limit of the IBM (Cereda *et al.* 1982). As previously stated, the main interest in discussing dynamical problems in the IBM stems, however, from the possibility to describe nuclei of a transitional character which are not well described by one of the classical models.

The description of momentum transfer dependent processes requires a radial dependence for the relevant form factors in addition to the usual parameters of the IBM Hamiltonian. It is of course desirable to derive these form factors from an

0004-9506/84/050463$02.00

underlying shell-model picture as has been attempted by Amos *et al.* (1984). Alternatively, information on them can be obtained from experiment (Moinester *et al.* 1982). This question is central to an application of the IBM to inelastic proton scattering, and it will be discussed at length in the next section.

One set of isotopes which exhibits an apparent structure change with increasing neutron number is provided by the Ge isotopes ($A = 68$ to 76) (Vergnes 1980). Within the IBM, these structure or shape changes correspond to the system moving between the vibrational [SU(5)], $\gamma$-unstable [O(6)] and rotational [SU(3)] limits. Classical geometric models do not provide form factors for $\gamma$-unstable nuclei, in contrast to vibrational and rotational nuclei. Consequently, scattering from $\gamma$-unstable nuclei has almost never been studied experimentally and theoretically. Existing data for Ge are limited to a rather old set of data, covering all stable isotopes, taken at 14·5 MeV with limited resolution (Curtis *et al.* 1970), and new data for $^{74}$Ge taken at 22 MeV (Tamisier *et al.* 1982). It is interesting to note that the latter authors were unable to get a fit to the data for $^{74}$Ge in a vibrational model, but found acceptable agreement in an asymmetric rotational model with state-dependent $\gamma$ deformation. After the calculations for the present paper were finished, a new set of data for $^{76}$Ge was published (Ramstein *et al.* 1983) which corroborates the findings for $^{74}$Ge.

The structure model for Ge in the IBM is discussed in Section 3. Contrary to an earlier model (Duval *et al.* 1983) we use the IBM-1, i.e. we do not distinguish between neutron and proton bosons, and do not consider mixing between different configurations. The application to inelastic proton scattering is discussed in Section 4 with special emphasis on the sensitivity of the results to the radial dependence of the form factors. The conclusions are presented in Section 5.

## 2. IBM and Coupled Channel Calculations

A central issue of coupled channel calculations is the specification of the transition operators between the various excited states. In this section we show how the IBM can be used to obtain expressions for these. If we restrict ourselves to two types of correlated pairs, s bosons ($L = 0$) and d bosons ($L = 2$), as is usually done, then the most general form of a one-body quadrupole operator to lowest order is

$$\hat{Q}^{(2)} = \alpha_2(s^+d + d^+s)^{(2)} + \beta_2(d^+d)^{(2)}, \tag{1}$$

where $s$ ($s^+$) are destruction (creation) operators for s bosons and similarly for $d$ ($d^+$). The boson effective charges $\alpha_2$ and $\beta_2$ are not fixed by the IBM Hamiltonian, but have to be found by fits to experimental properties or calculated from an underlying shell-model picture. In this form (1), $\hat{Q}^{(2)}$ can be used to calculate static properties. For dynamic calculations, a radial dependence has to be introduced. This is most naturally done by allowing $\alpha_2$ and $\beta_2$ to depend on $r$. If we now take the matrix element of (1) between states $|\mathrm{i}\rangle$ and $\langle\mathrm{f}|$ which can be connected by a quadrupole transition, then we find the transition density

$$\rho_{\mathrm{if}}^{(2)}(r) = A_{\mathrm{if}}^{(2)}\alpha_2(r) + B_{\mathrm{if}}^{(2)}\beta_2(r), \tag{2}$$

with the reduced matrix elements

$$A_{\mathrm{if}}^{(2)} = \langle \mathrm{f} \| (s^+d + d^+s)^{(2)} \| \mathrm{i} \rangle, \tag{3a}$$

$$B_{if}^{(2)} = \langle f \| (d^+ d)^{(2)} \| i \rangle , \tag{3b}$$

which can be calculated knowing the IBM Hamiltonian. Whereas these matrix elements depend of course on the states $|i\rangle$ and $|f\rangle$, the radial dependence of $\alpha_2(r)$ and $\beta_2(r)$ is the same for all transitions. Similarly, the transition density for monopole and hexadecapole transitions can be found. For hexadecapole transitions, only d bosons can contribute, and we find

$$\rho_{if}^{(4)}(r) = B_{if}^{(4)} \beta_4(r) , \tag{4}$$

with the matrix elements

$$B_{if}^{(4)} = \langle f \| (d^+ d)^4 \| i \rangle . \tag{5}$$

The monopole case is slightly more complicated. The most general form of the monopole operator is

$$\hat{M}^{(0)} = \alpha_0 (s^+ s)^{(0)} + \gamma_0 (d^+ d)^{(0)} . \tag{6}$$

The total number of bosons is, however, conserved which means that

$$(s^+ s)^{(0)} + \sqrt{5} (d^+ d)^{(0)} = N_B . \tag{7}$$

By introducing $\beta_0 = \gamma_0 - \sqrt{5}\alpha_0$, equation (6) can be rewritten as

$$\hat{M}^{(0)} = \alpha_0(r) N_B + \beta_0 (d^+ d)^{(0)} . \tag{8}$$

In addition to this operator, a contribution will come from the core which is not treated in the IBM. So the transition density for monopole transitions is finally found to be

$$\rho_{if}^{(0)}(r) = \{\rho_{core}(r) + N_B \alpha_0(r)\} \delta_{if} + \beta_0(r) B_{if}^{(0)} , \tag{9}$$

with the matrix elements

$$B_{if}^{(0)} = \langle f \| (d^+ d)^{(0)} \| i \rangle . \tag{10}$$

The matrix elements $B_{if}^{(l)}$ ($l = 0, 2, 4$) and $A_{if}^{(2)}$ contain the nuclear structure information and are calculated from the IBM Hamiltonian. This will be discussed in the next section for the special case of the Ge isotopes. Here, we are concerned with the determination of the radial form factors $\beta_l(r)$ ($l = 0, 2, 4$) and $\alpha_2(r)$. Once they are found, the transition potentials can be obtained by a folding procedure with an effective interaction, treating exchange contributions in a zero-range approximation (Petrovich *et al.* 1969) which should be reasonable for collective states of interest in the IBM. Here, we will follow, however, the usual reasoning of the collective model and assume that the transition potentials are related to the optical potential in the same way as the transition densities are related to the ground state density. The diagonal terms $\rho_{core}(r) + N_B \alpha_0(r)$ in the monopole transition are thus given by the optical potential. For increasing mass number, its radius increases as $A^{1/3}$ and thus takes care of the additional bosons.

Information on the densities $\alpha_2(r)$ and $\beta_2(r)$ comes from inelastic electron scattering (Moinester *et al.* 1982) to $2^+$ states in Sm isotopes. By comparing transition densities for various states with differing structure, $\alpha_2(r)$ and $\beta_2(r)$ can be extracted. For

$\alpha_2(r)$ a surface peaked form was found which can be approximated by the first derivative of a Woods–Saxon shape. In contrast, $\beta_2(r)$ displayed a node in the nuclear surface and thus resembles more the second derivative of a Woods–Saxon form. However, $\beta_2(r)$ is not determined as accurately as $\alpha_2(r)$ which is normally the dominant term. A quantity which is better determined than the radial form is the transition radius $R_\alpha$:

$$R_\alpha^2 = \int \alpha_2(r) r^6 \, \mathrm{d}r \Big/ \int \alpha_2(r) r^4 \, \mathrm{d}r , \tag{11}$$

and correspondingly for $R_\beta$. Experimentally, one finds for their ratio in $^{148}$Sm (Dieperink 1981) $R_\beta/R_\alpha = 1{\cdot}20$, whereas the assumption of a first derivative form for $\alpha_2(r)$ and a second derivative for $\beta_2(r)$ leads to $R_\beta/R_\alpha = 1{\cdot}27$ in good agreement with experiment. It is of course questionable whether this experimental result for Sm is relevant for Ge. A recent measurement of the transition densities for the $2_1^+$ states in the Ge isotopes (Goutte *et al.* 1983) has demonstrated their surface peaked form, thus supporting at least the form of $\alpha_2(r)$. Thus, we will use

$$\alpha_2(r) = \sqrt{\tfrac{1}{5}} k_{202} R \, \mathrm{d}U/\mathrm{d}r , \qquad \beta_2(r) = \sqrt{\tfrac{1}{5}} k_{222} \tfrac{1}{2} R^2 \, \mathrm{d}^2U/\mathrm{d}r^2 , \tag{12a, b}$$

where $U$ is the optical potential of Woods–Saxon shape with radius $R$. The proportionality factors $k$ have to be regarded as generalized deformation parameters or effective charges and must be determined in a phenomenological approach by a fit to the data (cf. Morrison and Smith 1980). In equations (12) we have denoted the transition potentials in the same way as the transition densities above.

Experimental information on $\beta_0(r)$ is much scarcer than for $\beta_2(r)$, and does not exist for $\beta_4(r)$. It is of course tempting to use the same functional form for all $\beta_l(r)$ ($l = 0, 2, 4$), which are all related to operators $d^+d$. In the absence of other information we will do so for $\beta_4(r)$ and use

$$\beta_4(r) = \sqrt{\tfrac{1}{9}} k_{422} \tfrac{1}{2} R^2 \, \mathrm{d}^2U/\mathrm{d}r^2 . \tag{13}$$

Information on $\beta_0(r)$ is obtained rather indirectly from isomer shifts in Sm (Moinester *et al.* 1982). The diagonal term involving $\alpha_0(r)$ in equation (9) contributes in this case and is in fact the dominant term, so that the only information on $\beta_0(r)$ obtained thereby is the existence of a node inside the nucleus, which has of course to be present for a monopole transition. One possible choice for $\beta_0(r)$ seems to be therefore

$$\beta_0(r) = k_{022} \tfrac{1}{2} R^2 \, \mathrm{d}^2U/\mathrm{d}r^2 . \tag{14}$$

As discussed below, this choice corresponds to the normal second order vibrational model. However, $\beta_0(r)$ has to fulfill the constraint

$$\int_0^\infty \beta_0(r) r^2 \, \mathrm{d}r = 0 , \tag{15}$$

because of the orthogonality between states. This is not satisfied by equation (14). Instead, one may use the breathing mode form factor (Satchler 1972):

$$\beta_0(r) = k_{022} \{ 3U(r) + r \, \mathrm{d}U/\mathrm{d}r \} . \tag{16}$$

In order to test the sensitivity of the results to the radial shape of the form factor we have used both forms (14) and (16).

We are thus left with four free parameters, $k_{022}$, $k_{202}$, $k_{222}$ and $k_{422}$, which are to be determined by fitting the experimental data. The relation of these parameters (in a slightly different normalization) to the microscopic shell-model structure has been investigated by Morrison and Smith (1980). They found that

$$k_{L\lambda_1\lambda_2} = \int R_{\lambda_1}(r) R_{\lambda_2}(r) G_L(r) r^2 \, \mathrm{d}r, \tag{17}$$

where $R_{\lambda_1}(r)$ and $R_{\lambda_2}(r)$ are bound state wavefunctions for bosons and $G_L(r)$ is related to the effective interaction. So one expects $k_{L\lambda_1\lambda_2}$ to be constant over a range of nuclei when the bound state wavefunctions do not change. This means essentially that one must not cross a subshell closure if one wants the same parameters $k_{L\lambda_1\lambda_2}$ for a range of nuclei. This condition is not fulfilled for the Ge isotopes where transfer experiments (Rotbard *et al.* 1978) have shown an abrupt change in the proton shell occupation at $^{72}$Ge. Therefore we will treat the $k_{L\lambda_1\lambda_2}$ as free parameters for each isotope.

An attempt to calculate the radial dependence of the form factors microscopically by Amos *et al.* (1984) has confirmed the surface peaked nature of $\alpha_2(r)$. In this paper, a similar shape was found for $\alpha_2(r)$ and $\beta_2(r)$ for the Sn isotopes in contrast to the assumptions made above. This microscopic calculation must, however, assume rather large core polarization corrections to reproduce the experimental transition probabilities. The radial shape of these core polarization contributions is not known, so that microscopic calculations of form factors in the IBM do not seem to be very reliable. Therefore, we prefer to use the phenomenological approach outlined here.

Transition potentials given as first and second derivatives of the optical potential are well known from traditional collective models. The correspondence to the IBM approach presented here is most readily recognized in the vibrational limit of the IBM. In this case, no d bosons are present in the ground state, and one- and two-phonon states contain one and two d bosons respectively. So transitions from the ground state to the one-phonon state have to proceed with the operator $(s^+d+d^+s)^{(2)}$, which changes the number of d bosons, and similarly from the one-phonon state to the two-phonon triplet. The corresponding radial form factor is $\alpha_2(r)$ which we have assumed to be given as the first derivative of the optical potential. This coincides with the customary first-order vibrational model where these transitions are the only ones considered. The reduced matrix elements will be different in the IBM, however, because of the finite boson number. The operator $(d^+d)^{(2)}$ induces recoupling terms in the one-phonon state and transitions among the members of the two-phonon triplet, because it does not change the number of d bosons. The associated form factors $\beta_l(r)$ $(l = 0, 2, 4)$ are represented by second derivatives of the optical potential. This corresponds to a second-order vibrational model in the classical picture. Again the reduced matrix elements will be different in the IBM. A similar correspondence can be established in the rotational limit, but not so easily because the number of d bosons no longer has a simple relationship to the nuclear state, so that d-boson number changing and preserving operators contribute in all cases. Our choice of radial dependences is thus a natural generalization of the classical models.

Finally, we discuss the relation of the parameters $k_{L\lambda_1\lambda_2}$ to electromagnetic transition rates. For an isoscalar projectile and a first order derivative form factor, the

question of obtaining $B(EL)$ values from deformation parameters has recently been clarified by Wagner *et al.* (1982). Their work can readily be generalized to second order derivatives, and the result can be summarized as follows: If the potential for the transition with multipolarity $L \geqslant 2$ from a state with spin $I_i$ to a state with spin $I_f$ is given by

$$F_L(r) = \frac{1}{(2L+1)^{\frac{1}{2}}} \beta_L^{(1)} R \frac{dU}{dr} + \frac{1}{(2L+1)^{\frac{1}{2}}} \beta_L^{(2)} \tfrac{1}{2} R^2 \frac{d^2U}{dr^2}, \tag{18}$$

then the (isoscalar) associated transition probability is

$$B(L, I_i \to I_f) = \frac{2I_f+1}{2I_i+1} \frac{1}{2L+1} \frac{A^2}{(4\pi)^2} \times \{-(L+2)\beta_L^{(1)} R\langle r^{L-1}\rangle + \tfrac{1}{2}(L+1)(L+2)\beta_L^{(2)} R^2 \langle r^{L-2}\rangle\}. \tag{19}$$

Here $\langle r^L \rangle$ is the radial moment of the (real part of) the optical potential. For monopole transitions, a related formalism (Bauhoff 1984) can be used to obtain the $\langle r^2 \rangle_{tr}$ transition matrix element. For the breathing mode form factor (16), the result reads

$$\langle r^2 \rangle_{tr} = 2\beta A \langle r^2 \rangle. \tag{20}$$

The deformation parameter $\beta$ is here the product of the reduced matrix element and the appropriate parameter $k_{L\lambda_1\lambda_2}$. Note that a formula corresponding to (20) cannot be derived for monopole form factors of second derivative form because it does not fulfill (15).

If one wants to apply these formulas to low-energy proton scattering from Ge isotopes one faces two problems. First, the incident protons do not interact with equal strength with target neutrons and protons. At low energies, the neutron–proton interaction is about three times as strong as the proton–proton interaction. Second, even if one assumes equal strength for both types, the resulting transition rates cannot be easily compared with results obtained from electromagnetic measurements because of uncertainty on how to scale the results. The geometric collective model suggests scaling with $(Z/A)^2$, which varies from 0·21 for $^{70}$Ge to 0·18 for $^{76}$Ge. For the IBM, on the other hand, an inert core of $^{56}$Ni is assumed, which is not changed in the excited states. So the transition affects only the valence nucleons, and the scaling factor should be $\{n_\pi/(n_\pi+n_\nu)\}^2$, where $n_\pi$ ($n_\nu$) is the number of proton (neutron) bosons. This leads to scaling factors between 0·082 for $^{70}$Ge and 0·040 for $^{76}$Ge. So the factors are not only much smaller than those from the collective models, but change also much more rapidly among the isotopes. For these two reasons, extractions of $B(EL)$ values are very uncertain and give at best an order of magnitude estimate.

## 3. IBM Calculations for Ge Isotopes

The structure of the $^{68-76}$Ge isotopes is described in the IBM-1 framework (Arima and Iachello 1976) using an uncoupled basis representation (Morrison and Smith 1980). The parameters of the empirical boson Hamiltonian are determined by a $\chi^2$ minimization of the calculated energy level spectra of these isotopes, assuming a $^{56}$Ni closed core. The IBM-1 formalism is used in preference to IBM-2 because of the need to limit the plethora of parameters both in the reaction analysis (separate

$k$ values for both proton and neutron valence bosons) and in the structure model where up to six boson types and ten interaction parameters have been used (Duval *et al.* 1983). This restriction is not important for our purposes because a boson Hamiltonian retaining p–n symmetry automatically ensures that the predictions of IBM-1 and IBM-2 are exactly equivalent for the lowest (maximal symmetry) eigenstates, their predictions merely being scaled by the standard $Z/A$ and $N/A$ of the geometric models (Morrison 1980). In typical IBM-2 parametrizations (Bijker *et al.* 1980), the boson Hamiltonian used only slightly breaks $F$-spin p–n symmetry for the lowest states and only marginally improves energy and $B$(E2) fits over an equivalent IBM-1 approach. We therefore use IBM-1 as an accurate approximation to the structure of these states, limiting the number of parameters. Our intention is to discover whether an empirical analysis of reaction data within the IBM and coupled channel model retains sufficient sensitivity to, *by itself*, discriminate between different nuclear structure models ($H_B$) and therefore be a valid tool. It is crucial therefore to sensibly limit the number of free parameters. Within IBM-1, the excitation energy spectrum in a nucleus is a function of six one-body and two-body matrix elements of the boson Hamiltonian. These are determined by a $\chi^2$ search applied to the even $^{68-76}$Ge isotopes, both as a whole and as clusters of two to three isotopes, with weak constraints on the $0_2^+$ levels which are expected to be anomalous (Vergnes 1980; Duval *et al.* 1983). The best fit was obtained for $^{68-72}$Ge and $^{72-76}$Ge separately with the interaction parameters given in Table 1.

**Table 1. Matrix elements (in MeV) of the IBM-1 boson Hamiltonian**

Our notation follows that of Arima–Iachello (1976)

| Matrix element | $A$ = 68–72 | $A$ = 72–76 |
|---|---|---|
| $\langle s^2 \mid V \mid d^2\rangle_{L=0}$ | −0·164 | −0·465 |
| $\langle d^2 \mid V \mid d^2\rangle_{L=0}$ | −0·608 | −0·406 |
| $\langle sd \mid V \mid d^2\rangle_{L=2}$ | ≈0 | −0·019 |
| $\langle d^2 \mid V \mid d^2\rangle_{L=2}$ | −0·289 | −0·391 |
| $\langle d^2 \mid V \mid d^2\rangle_{L=4}$ | 0·017 | −0·020 |
| $e_d$ | 1·101 | 0·864 |

For the lighter isotopes, the fitted Hamiltonian is similar to a spherical vibrator [SU(5) limit] with a '$u_0$' perturbation (Arima and Iachello 1976), whereas the heavier isotopes are best fitted by a perturbed $\gamma$-unstable system [O(6) limit]. If we fold back the interaction parameters to define a potential energy surface (Ginnochio and Kirson 1980), it appears that a nuclear shape change occurs with $^{72}$Ge as the 'pivot'. In both regions, $^{72}$Ge is least well 'fit', and we shall concentrate on $^{68,70}$Ge and $^{74,76}$Ge as systems where the two different shapes are well established. This change of shape is well supported by other experimental evidence, and although IBM-1 cannot support a permanent triaxial shape, as has been suggested (Duval *et al.* 1983), there is no empirical evidence for this (Vergnes 1980) and the $\gamma$-unstable vibrator [O(6) limit] will therefore be a good approximation for weak triaxiality (Ginnochio and Kirson 1980). As a consequence of the change in shape, the structure factors (3) will change in these nuclei. We address in the next section the question of whether a standard reaction analysis is sufficiently sensitive to these changes to be a valid

discriminatory tool. To do this for a range of levels in these SU(5) and O(6) systems, extension beyond the direct reaction mechanism (Morrison and Smith 1980) is required.

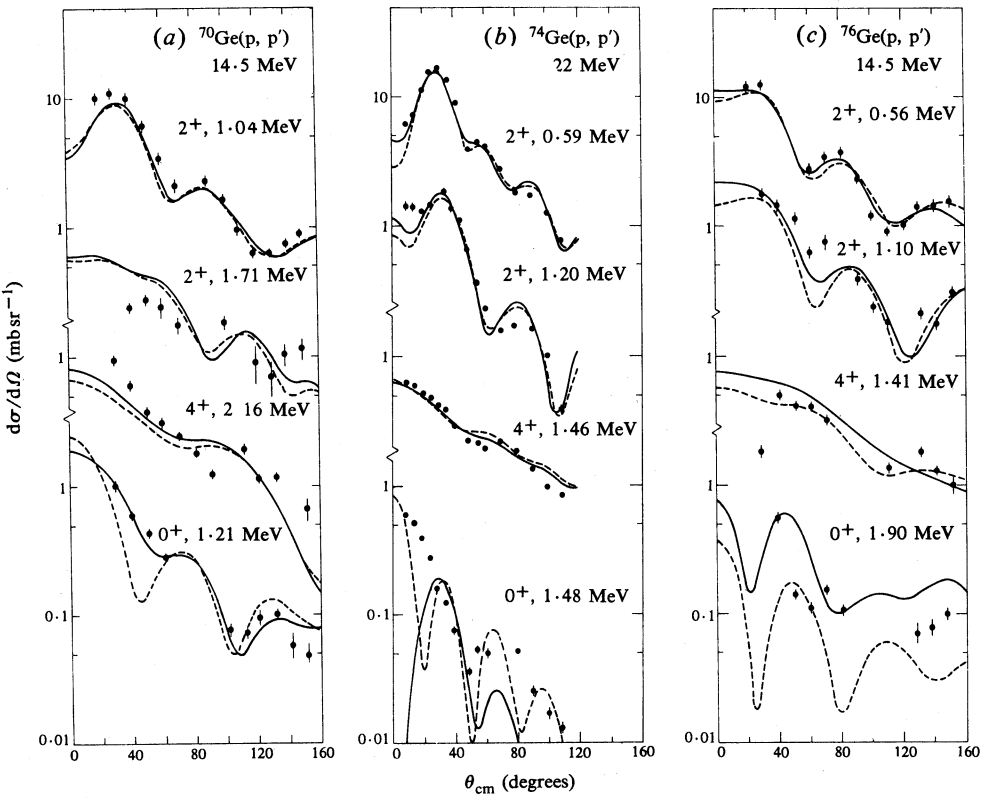


**Fig. 1.** Results in the IBM for proton inelastic scattering from (*a*) $^{70}$Ge at 14·5 MeV, (*b*) $^{74}$Ge at 22 MeV and (*c*) $^{76}$Ge at 14·5 MeV. For the monopole transition potential, a breathing mode form factor (solid curves) and the second derivative of the optical potential (dashed curves) are used. In Figs 1*a* and 1*c* the data are from Curtis *et al.* (1970), and in Fig. 1*b* from Tamisier *et al.* (1982).

## 4. Application to Scattering from the Ge Isotopes

As an example of the general formalism developed in Section 2, we consider now inelastic proton scattering from the Ge isotopes $A = 70$, 74 and 76. The isotope $^{68}$Ge is unstable, so no proton scattering data exist, and $^{72}$Ge is left out because of the uncertainties in its structure. For the remaining three isotopes, we consider the $2_1^+, 2_2^+, 0_2^+$ and $4_1^+$ states, in addition to the ground state. With the boson Hamiltonian given in the previous section, we have calculated the reduced matrix elements for transitions between these five states for angular momentum transfers of $L = 0, 2$ and 4. Higher angular momenta are not possible in the IBM. Additional states of higher excitation energy can be included in the coupling scheme. We have not done it here because one might anticipate sizable quasiparticle admixtures in these higher states which are not described by the IBM.

Specifically, we consider the 14·5 MeV data of Curtis *et al.* (1970) for all three isotopes, and in addition for $^{74}$Ge the more recent higher quality data of Tamisier

*et al.* (1982), taken at 22 MeV. In the latter experiment, the $0_2^+$ and $4_1^+$ states were resolved from each other, in contrast to the results of Curtis *et al.* The optical potential is taken from the original publications. In the coupled channel calculations, real and imaginary parts were both deformed with the same deformation parameter. We have not included a spin–orbit deformation because its influence is found to be small for cross-section data. It has of course to be considered when polarization data are available. The calculations are performed with the coupled channel code ECIS79 by J. Raynal (unpublished, 1979), which allows the input of arbitrary reduced matrix elements and externally defined transition potentials. The 'deformation parameters' $k_{022}$, $k_{202}$, $k_{222}$ and $k_{422}$ are determined by a $\chi^2$ fit to the existing data. We have put strongest emphasis on a fit to the $2_1^+$ data which have been measured most accurately.

**Table 2. Best-fit values for the 'deformation parameters' $k_{L\lambda_1\lambda_2}$**

The parameters are given for all nuclei considered for monopole form factors of either breathing mode or second derivative form

| $k_{L\lambda_1\lambda_2}$ | $^{70}$Ge (14·5 MeV) | $^{74}$Ge (14·5 MeV) | $^{74}$Ge (22 MeV) | $^{76}$Ge (14·5 MeV) |
|---|---|---|---|---|
| | | *Breathing mode* | | |
| $k_{022}$ | −0·012±0·005 | 0·0022±0·0041 | −0·012±0·001 | 0·015±0·005 |
| $k_{202}$ | 0·040±0·001 | 0·029±0·001 | 0·027±0·0005 | 0·028±0·001 |
| $k_{222}$ | 0·019±0·004 | 0·0027±0·0006 | 0·00097±0·00076 | −0·0042±0·0014 |
| $k_{422}$ | 0·015±0·004 | 0·0020±0·0014 | 0·0027±0·0015 | 0·0029±0·0018 |
| | | *Second derivative* | | |
| $k_{022}$ | −0·0025±0·0073 | 0·0011±0·0011 | 0·032±0·002 | −0·025±0·004 |
| $k_{202}$ | 0·038±0·001 | 0·029±0·001 | 0·027±0·0004 | 0·024±0·001 |
| $k_{222}$ | 0·016±0·004 | 0·0026±0·0010 | 0·0023±0·0007 | −0·0011±0·0010 |
| $k_{422}$ | 0·017±0·004 | 0·0018±0·0018 | 0·0029±0·0010 | 0·0006±0·0010 |

Results are shown in Figs 1*a*, 1*b* and 1*c* for $^{70}$Ge, $^{74}$Ge and $^{76}$Ge respectively, and the corresponding best-fit values of $k_{L\lambda_1\lambda_2}$ are given in Table 2. We have also included in Table 2 the results for $^{74}$Ge at 14·5 MeV, which are not shown in Fig. 1. As discussed in Section 2, the form of the monopole transition potential is uncertain, and we have used both the breathing mode form and the second derivative of the optical potential. Results for both cases are shown in Fig. 1 and Table 2. The quoted uncertainties for the $k_{L\lambda_1\lambda_2}$ are those given by the ECIS code and correspond to the change in any $k_{L\lambda_1\lambda_2}$ which will increase the $\chi^2$ by the inverse of the number of parameters.

In all nuclei, the fit to the $2_1^+$ data is the best of all states considered. This is not surprising since it is well described by a standard DWBA vibrational model calculation. The $s^+d+sd^+$ term is the dominant one in the excitation of the $2_1^+$ state in the IBM in all cases, and the first derivative form of the form factor $\alpha_0(r)$ reproduces the conventional radial shape. For the other states, the fits for the $2_2^+$ and $4_1^+$ states are reasonable in all cases and, in fact, better than in the standard second order vibrational model (Curtis *et al.* 1970; Tamisier *et al.* 1982). The results for the $0_2^+$ state differ strongly from nucleus to nucleus and depend sensitively on the radial form of the monopole form factor which mediates the one-step transitions to this state. The influence of changes in the monopole form factor on other states is visible, but not as dramatic as for the $0_2^+$ state. For $^{70}$Ge, the use of the breathing mode form (solid

curve in Fig. 1*a*) leads to an obvious improvement compared with the normal second derivative form (dashed curve). In the other two isotopes, the quality of the $0_2^+$ fits is generally inferior and does not discriminate between the two forms.

Table 2 reveals that $k_{202}$ is best determined by the experimental data. This stems from the good fit to the $2_1^+$ state. For the other $k_{L\lambda_1\lambda_2}$ values sizable uncertainties in their magnitude are found. They are largest for $k_{022}$, i.e. the monopole form factor. This reflects of course the unsatisfactory fits for the $0_2^+$ states. For $^{74}$Ge, we find good agreement for the two different energies in all cases except for $k_{022}$. This is not surprising since the $4_1^+$ and $0_2^+$ states are not resolved at 14·5 MeV. The determination of the parameters from the sum of the two cross sections necessarily introduces uncertainties, and since the fit to the $0_2^+$ data at 22 MeV is bad, one gets an unreliable result at 14·5 MeV. If we compare the results for the different isotopes, we find that they are different for $^{70}$Ge on one side and $^{74}$Ge and $^{76}$Ge on the other side. This can be related to the expression (17) for the $k_{L\lambda_1\lambda_2}$ in terms of the underlying wavefunctions. Since the proton occupation number for the $f_{5/2}$ shell and the 2p shells changes abruptly at $^{72}$Ge (Rotbard *et al.* 1978), the contributing shells are different for $^{70}$Ge compared with the heavier isotopes, and therefore the overlap integrals (17) will be different in the two cases.

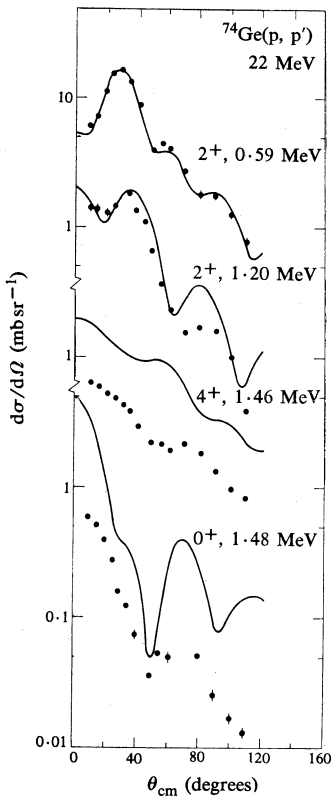


**Fig. 2.** As in Fig. 1*b*, but the vibrational limit of the IBM is used. The monopole form factor is of the second derivative form.

In order to test how much the proton scattering results depend on the underlying boson structure, we have performed a calculation for $^{74}$Ge in the vibrational limit of the IBM. The best-fit result is shown in Fig. 2. As is customary in the vibrational

model, we have used a monopole form factor of the second derivative form. It is evident that the results are worse than those in Fig. 1*b* which are obtained with a boson Hamiltonian close to the O(6) limit. It is not possible to improve the result for the $4_1^+$ and $0_2^+$ states by changing $k_{022}$ and $k_{422}$ as one might naively expect. The magnitude of the two-step contributions does not allow it. So we find that the proton scattering results are in fact sensitive to the nuclear structure input, and the free parameters $k_{L\lambda_1\lambda_2}$ do not mask this dependence. Our result confirms the findings of Tamisier *et al.* (1982) and Ramstein *et al.* (1983) that proton scattering from the heavier Ge isotopes cannot be described in the vibrational model.

## 5. Conclusions

In this paper, we have investigated the description of proton inelastic scattering in the IBM. The discussion was based entirely on a macroscopic collective approach by using analogies with the classical geometric models. The radial shapes of the transition form factors were given in terms of the optical potential and its derivatives. The IBM provided the reduced matrix elements for the coupling between the states of interest. The application to the Ge isotopes has demonstrated the sensitivity to the nuclear structure input. The fits obtained were of comparable or better quality than those of the usual vibrational model.

Obviously we have not presented an exhaustive discussion of the subject. Further investigations are necessary in two different areas. The first one concerns the application to a wider range of nuclei in a phenomenological way. Of particular interest in this respect are the Sm isotopes which exhibit a transition from vibrational to rotational character. On the other hand, the microscopic basis for the approach has to be investigated. The first steps in this direction have already been taken (Amos *et al.* 1984). Ultimately, one would like to calculate the radial shape of the form factors and the scaling parameters $k_{L\lambda_1\lambda_2}$ from an underlying fermion model. This will then require an explicit treatment of exchange effects. A coupled channel code which includes antisymmetrization has still to be written. Therefore, a large amount of work must be done before a completely microscopic description can be given. On a more positive note, we have shown that an analysis of inelastic proton scattering based on a phenomenological coupled channel and IBM approach *does* retain sufficient sensitivity to be a valid test of the underlying nuclear structure. Additionally, the $0_2^+$ levels in $^{74,76}$Ge are shown to be beyond the scope of the standard collective IBM prescription.

## Acknowledgment

This work has been supported by the Australian Research Grants Scheme.

## References

Amos, K., Morrison, I., and Smith, R. (1984). *J. Phys.* G **10**, 331.
Arima, A., and Iachello, F. (1976). *Ann. Phys. (New York)* **99**, 253.
Arima, A., and Iachello, F. (1978*a*). *Ann. Phys. (New York)* **111**, 201.
Arima, A., and Iachello, F. (1978*b*). *Phys. Rev. Lett.* **40**, 385.
Bauhoff, W. (1984). *Z. Phys.* (in press).
Bijker, R., Dieperink, A. E. L., Scholten, O., and Spanhoff, R. (1980). *Nucl. Phys.* A **344**, 207.
Cereda, E., *et al.* (1982). *Phys. Rev.* C **26**, 1941.
Curtis, J. H., Lutz, H. F., and Bartolini, W. (1970). *Phys. Rev.* C **1**, 1418.

Dieperink, A. E. L. (1981). *Nucl. Phys.* A **358**, 189c.
Duval, P. D., Goutte, D., and Vergnes, M. (1983). *Phys. Lett.* B **124**, 297.
Ginnochio, J., and Kirson, M. W. (1980). *Nucl. Phys.* A **350**, 31.
Goutte, D., *et al.* (1983). Int. Conf. on Nuclear Physics, Florence, 1983, p. 329 (Tipografia Compositori: Bologna).
Iachello, F. (1981). *Nucl. Phys.* A **358**, 89c.
Moinester, M. A., Alster, J., Azuelos, G., and Dieperink, A. E. L. (1982). *Nucl. Phys.* A **383**, 264.
Morrison, I. (1980). *Aust. J. Phys.* **33**, 801.
Morrison, I., and Smith, R. (1980). *Nucl. Phys.* A **350**, 89.
Petrovich, F., McManus, H., Madsen, V. A., and Atkinson, J. (1969). *Phys. Rev. Lett.* **22**, 895.
Ramstein, B., Tamisier, R., Rosier, L. H., Avignon, P., and Delaroche, J. P. (1983). *Nucl. Phys.* A **411**, 231.
Rotbard, G., *et al.* (1978). *Phys. Rev.* C **18**, 86.
Satchler, G. R. (1972). *Part. Nucl.* **5**, 105.
Scholten, O., Iachello, F., and Arima, A. (1978). *Ann. Phys. (New York)* **115**, 325.
Tamisier, R., *et al.* (1982). *Nucl. Phys.* A **385**, 430.
Vergnes, M. (1980). 'Structure of Medium-heavy Nuclei', Int. Conf. Series No. 49, p. 25 (Institute of Physics: London).
Wagner, G. J., Grabmayr, P., and Schmidt, H. R. (1982). *Phys. Lett.* B **113**, 447.

Manuscript received 30 May, accepted 3 August 1984