Aust. J. Phys., 1986, **39**, 505–12

Intensity Correlation Functions for 'Thermalised' TEM$_{n0}$ Laser Beams

R. Hoare,[A] *R. Mardling*[B] *and G. J. Troup*[A]

[A] Physics Department, Monash University,
Clayton, Vic. 3168.
[B] Mathematics Department, Monash University,
Clayton, Vic. 3168.

Abstract

A well-stabilised, single mode laser beam will give zero intensity correlation. However, a laser beam may be 'thermalised' by shining it on a rotating ground glass disc, and then intensity correlation experiments will give nonzero results. We present here calculations of the intensity correlation functions of the TEM$_{10}$, TEM$_{20}$, TEM$_{30}$ and TEM$_{40}$ Gaussian–Hermite beams. We were led to perform these calculations by the results of intensity correlation measurements on a laser with 'butted-on' mirrors nearing the end of its useful life.

1. Introduction

A laser beam emanating from a single TEM$_{n0}$ transverse laser mode can be 'thermalised', or made into a 'pseudothermal source' (Martiennsen and Spiller 1964), by shining it onto a rotating disc of glass which has been finely ground or etched. The 'coherence area' A of such a source is determined by the relative sizes of the disc irregularities and the laser 'spot'. The 'coherence time' τ is determined by the preceding two quantities, and by the speed of rotation of the disc at the laser spot. In a normal thermal source, the mean number $\bar{n}$ of photons in a coherence volume $Ac\tau$ is $\ll 1$, but in the pseudothermal source case $\bar{n} > 1$. The intensity correlation function $G_{12} = \overline{I_1(x_1, t_1)\, I(x_2, t_2)}$, where $I_1(x_1, t_1)$ is the intensity at space and time points x_1 and t_1, and similarly for $I_2(x_2, t_2)$, is quite readily observable for a pseudothermal source (Martiennsen and Spiller 1964; Haner and Isenor 1970), whereas it is very difficult to observe even for narrow-band thermal sources (Hanbury Brown and Twiss 1956). We report here the calculated intensity correlation functions of pseudothermal sources using single *transverse* TEM$_{n0}$ laser modes (unfocussed) as the original laser spots. In a properly stabilised laser, provided that the observation time is much longer than the reciprocal of any beat frequency due to more than one *longitudinal* mode, the intensity correlation function should be zero (see e.g. Armstrong and Smith 1963).

It is a quirk of history that the results we report here have not been presented much earlier. A search of the literature from about the time of the Third Quantum Electronics Conference in 1963 reveals much discussion and confusion about what should be expected as the result of a Hanbury Brown–Twiss experiment for a TEM$_{00q}$ laser (Corcoran and Pao 1962; Mandel and Wolf 1963; Kastler 1964; Bolwijn *et al.* 1964). When the matter was settled in favour of *no* correlation, interest

0004-9506/86/040505$02.00

ceased until the invention of the pseudothermal source by Martiennsen and Spiller (1964). Still, the experiments and accompanying theory remained unreported for higher order modes, even after Haner and Isenor (1970) described an arrangement using a pseudothermal source, suitable for an undergraduate Hanbury Brown–Twiss experiment, and suggested a 'split' source; for example, a double-slit, or a single hair, placed in front of the TEM_{00} laser beam, prior to its impinging on the rotating rough glass screen. A contributing factor may have been the advances in technology which allowed the use of mirrors directly butted onto the plasma tube of He–Ne lasers (the most common), instead of external mirrors used with Brewster-angle windows.

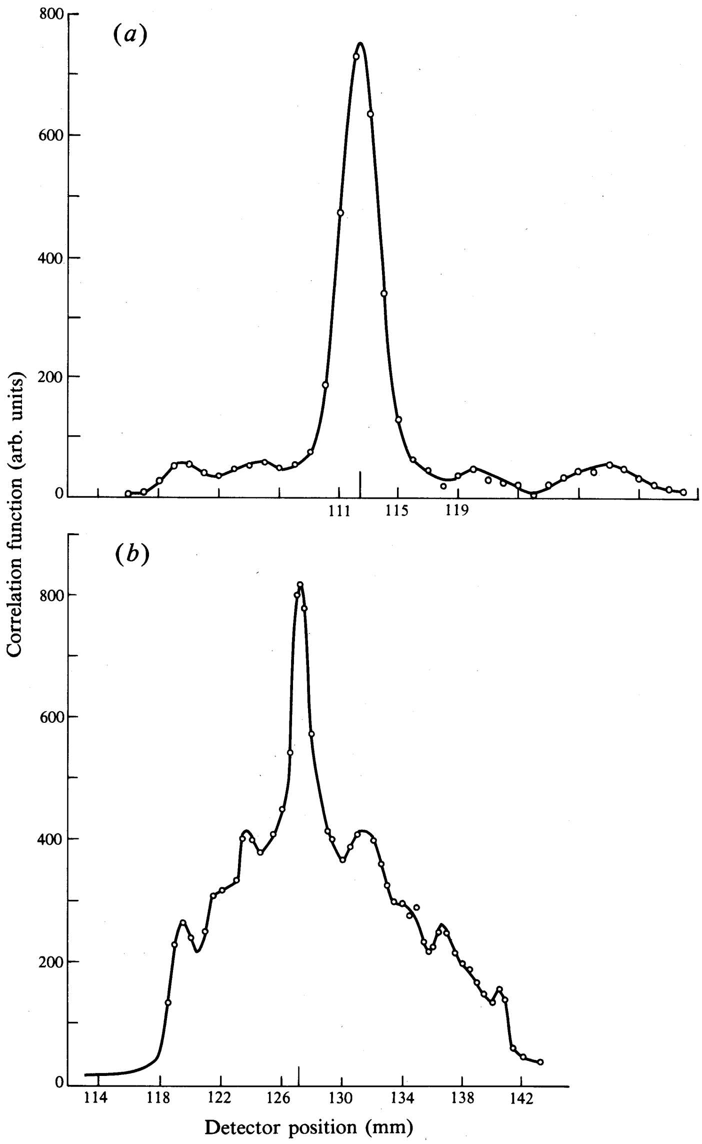

Fig. 1. Experimental intensity correlation function G_{12} for (*a*) a 'normal' unfocussed TEM_{00} laser beam and (*b*) a 'dying' laser beam.

We were led to perform these calculations by the results of two experiments performed with apparatus similar to that described by Haner and Isenor (1970). Fig. 1*a* shows the results of a typical measurement, performed in 1984, of the intensity correlation function with an unfocussed TEM_{00} source, using a Spectra-Physics Model 143 laser, purchased in 1978. Fig. 1*b* shows the results of an identical measurement made some months later in 1985. The laser beam was again unfocussed. The narrowing of the central peak, and the appearance of the subsidiary peaks, well above the noise level, can be seen. When the laser 'spot' was examined by expanding it with a lens, and projecting it onto a screen, it was elongated (rather than circular) with little structure discernible to the eye. An experiment performed some hours later gave similar, but not identical, results, leading to the conclusion that the plasma tube was nearing the end of its life. This particular model has mirrors butted on to the end of the plasma tube. Hence, eventually, as the He leaks out of the tube, strains occur, distorting the mirror alignment. After the tube was left in a bag of He gas at atmospheric pressure for 2 days, the laser spot became circular again.

Table 1. Coefficients a_k of equation (5)

n	a_0	a_1	a_2	a_3	a_4
1	1	1/2			
2	3	1	1/4		
3	15	9/2	3/4	1/8	
4	105	30	9/2	1/2	1/16

2. Calculation of Intensity Correlation Functions

Before giving the mathematical calculations of the TEM_{n0} laser mode intensity, we present some physical arguments. The laser mode electric fields are for example, of the form $E = \exp\{-(x^2+y^2)\}H_n(x)$, where H_n is the Hermite polynomial of order n, and x, y are the coordinate axes. That this must be so can be seen from the following argument. Consider a symmetrical laser, long enough so that the field distribution $F(E_C)$ on one mirror C is the Fraunhofer diffraction pattern of the field distribution $F(E_D)$ on the other mirror D. Then $F(E_C)$ is the Fourier transform (FT) of $F(E_D)$. However, the system is symmetrical, so both field patterns are identical. Hence $F(E_C)$, for example, must be its own FT; the set of functions satisfying this relationship are the Gaussian–Hermite functions defined above.

The calculation procedure is as follows. The intensity distribution is given by the square of the field distribution; again, as will be seen, this gives a sum of Gaussian–Hermite terms of different argument. The FT of the intensity distribution gives the coherence function by the van Cittert–Zernike theorem (see Zernike 1938); again, we have a sum of Gaussian–Hermite terms of different argument. The square of the coherence function gives the intensity correlation function, once more a sum of Gaussian–Hermite functions with another argument.

The field distribution is effectively

$$E(x,y) = e^{-(x^2+y^2)/2}\,H_n(x)\,, \tag{1}$$

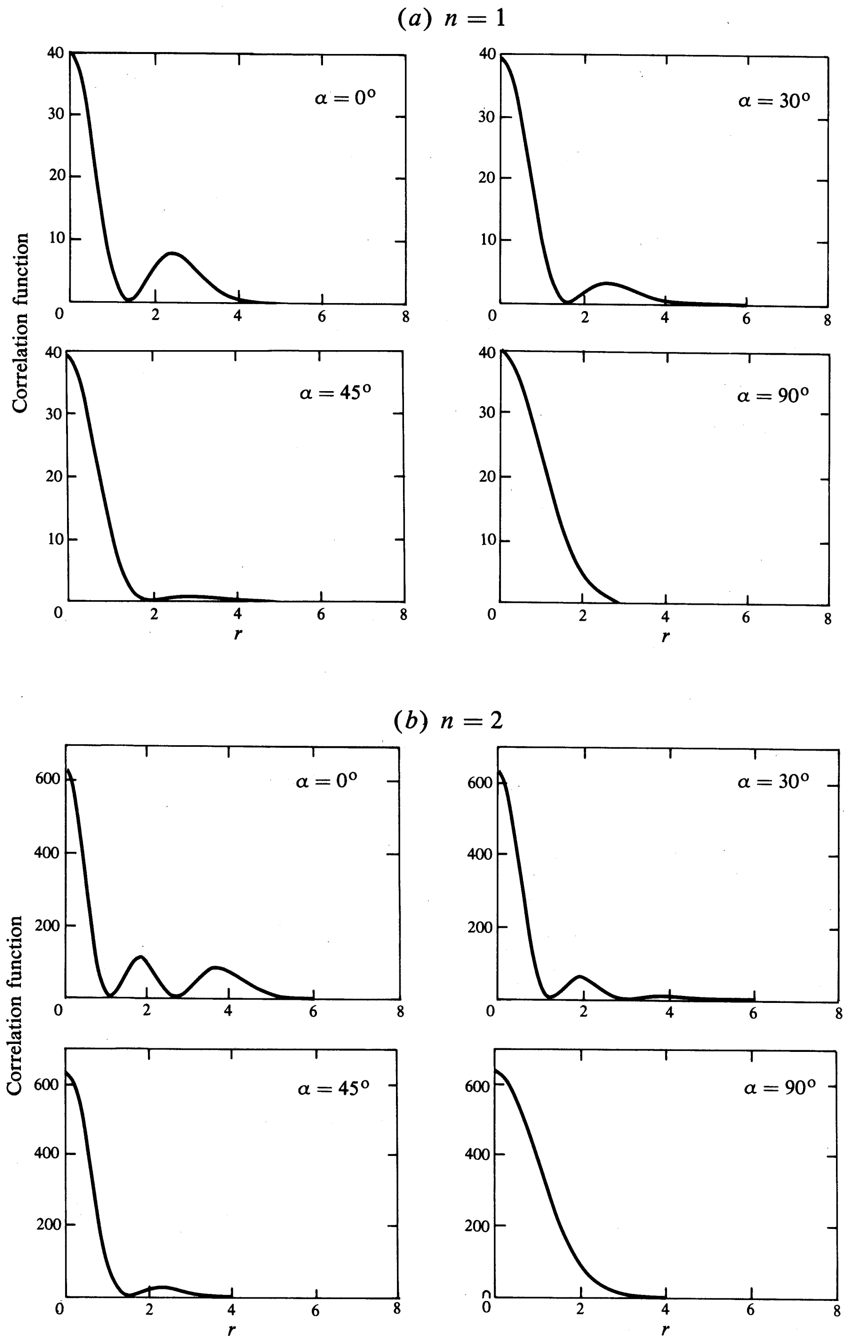

Fig. 2. Calculated intensity correlation function G_{12} for four values of $\alpha = \theta$ and for the four TEM_{n0} modes: (*a*) $n = 1$, (*b*) $n = 2$, (*c*) $n = 3$ and (*d*) $n = 4$.

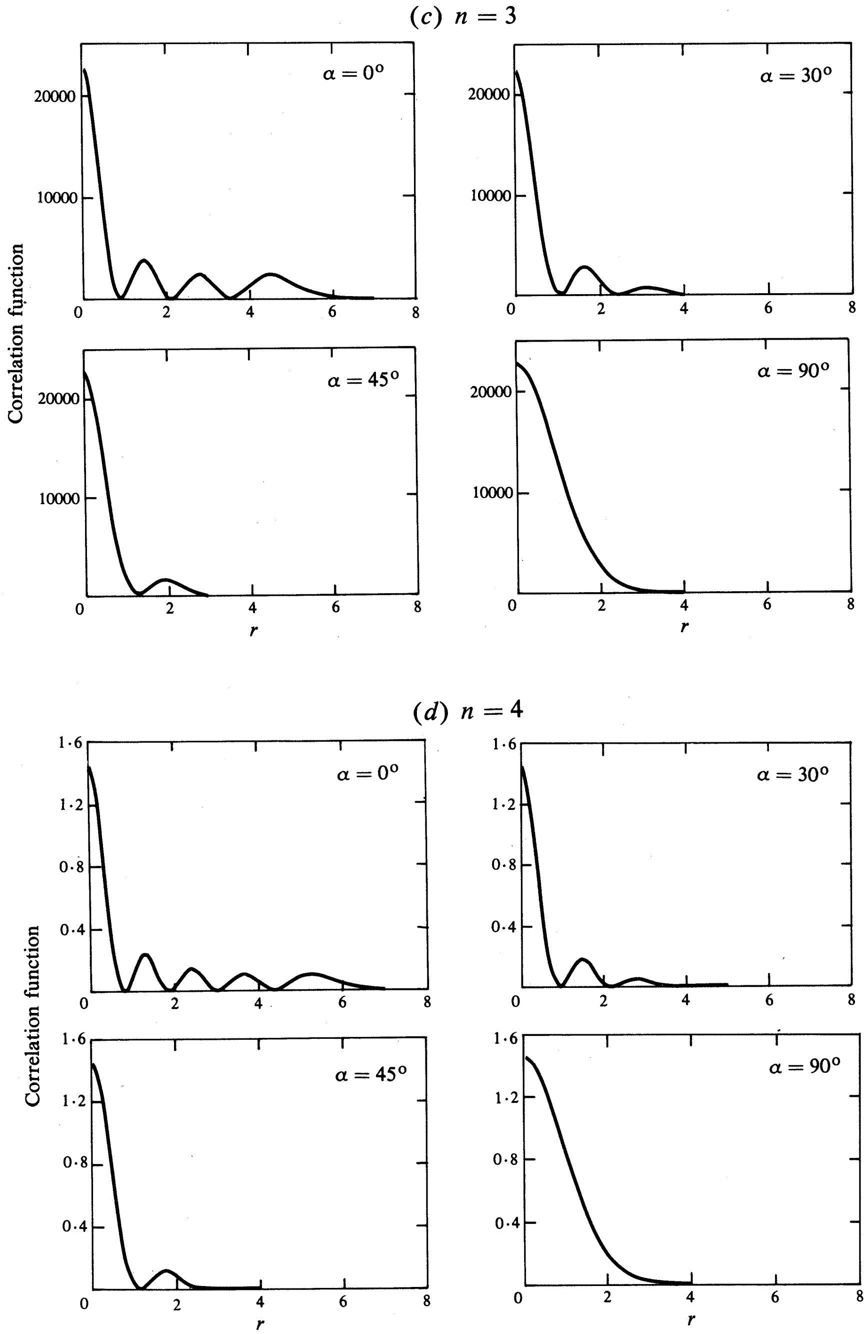

Fig. 2 *(Continued)*

and thus the intensity is

$$I(x, y) = e^{-y^2}\{e^{-x^2} H_n^2(x)\}. \tag{2}$$

We require the double FT, i.e.

$$\tau(\lambda, \mu) = \int_{-\infty}^{\infty} e^{-y^2} e^{i\mu y}\, dy \int_{-\infty}^{\infty} e^{-x^2} H_n^2(x)\, e^{i\lambda x}\, dx, \tag{3}$$

where λ and μ are proportional to the separation of the detectors in the Fourier plane. According to Gradshtein and Ryzhik (1980) we have

$$\int_{-\infty}^{\infty} e^{irs} e^{-s^2/2} H_n(s)\, ds = (2\pi)^{\frac{1}{2}} e^{-r^2/2} H_n(r)\, i^n. \tag{4}$$

Since we can write

$$H_n^2(x) = \sum_{k=0}^{n} a_k H_{2k}(\sqrt{2}\, x), \tag{5}$$

where the a_k are given in Table 1, then equation (4) can be used with the transformations $s = \sqrt{2}\, x$ and $r = \lambda/\sqrt{2}$ to write (3) as

$$\begin{aligned} \tau(\lambda, \mu) &= \sqrt{\pi}\, e^{-\mu^2/4} \sum_{k=0}^{n} a_k \sqrt{\pi}\, e^{-\lambda^2/4} H_{2k}(\lambda/\sqrt{2}) i^{2k} \\ &= \pi\, e^{-(\lambda^2+\mu^2)/4} \sum_{k=0}^{n} a_k H_{2k}(\lambda/\sqrt{2})(-1)^k. \end{aligned}$$

The correlation function is thus

$$\begin{aligned} G_{12} &= \{\tau(\lambda, \mu)\}^2 \\ &= \pi^2 e^{-(\lambda^2+\mu^2)/2} \left(\sum_{k=0}^{n} a_k H_{2k}(\lambda/\sqrt{2})(-1)^k \right)^2. \end{aligned}$$

3. Results and Discussion

Our results are shown in Fig. 2 for various angles α in the Fourier plane, where the coordinate change is given by

$$r = 2(\mu^2 + \lambda^2)^{\frac{1}{2}}, \qquad \tan\alpha = \mu/\lambda,$$

and the radial coordinate r is now proportional to the detector separation. The angle α is also effectively defined in Fig. 3, where it is shown with respect to the *field* distributions in the laser modes; i.e. $\alpha \equiv \theta$ in Figs 2 and 3. We must remember that we are taking the FT of the *square* of the field distribution.

The intensity correlation for the TEM_{00} mode is clearly a Gaussian function, whatever the value of θ, so this is not shown in Fig. 2. The narrowing of the central peak and the appearance of sidelobes, as r increases, for $\alpha = 0$ should be noticed. The TEM_{00} Gaussian function is effectively that for $\alpha = 90°$, for all values of n. The narrowing for $\alpha = 0$ is then clearly seen.

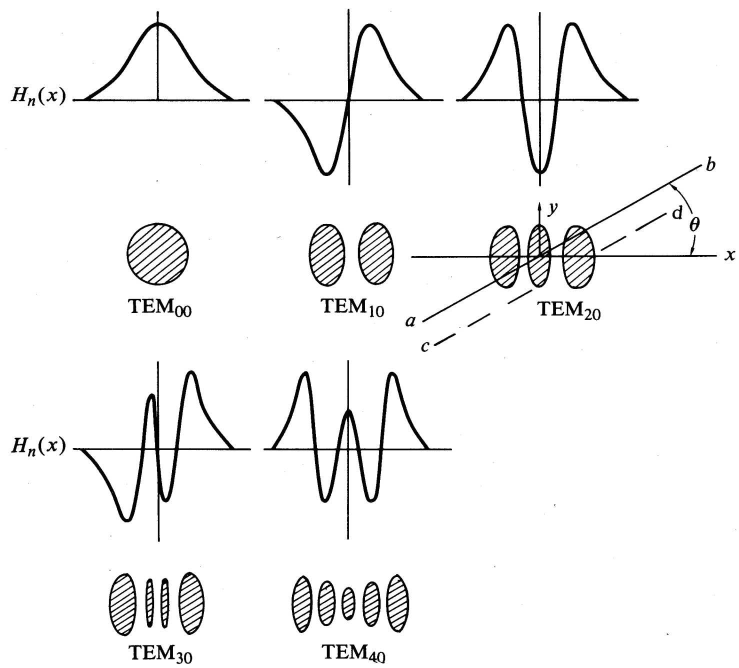

Fig. 3. Schematic representation of the field distributions for the TEM$_{00}$–TEM$_{40}$ Gaussian–Hermite laser modes, showing the angle θ.

Suppose we have a superposition of a number of Gaussian modes in a laser, with *no* phase-locking mechanism. Then the intensity distribution will be of the form

$$\sum_{k=0}^{n} a_k[\exp\{-(x^2+y^2)\}\,H_k(x)]^2\,,$$

and there will be no interference terms between the modes. We must then take the square of its FT to obtain the intensity correlation function. We attempted to find a 'disentangling' theorem to enable us to obtain the a_n so defined from the intensity superposition, but could not do so. It is a matter of trial and error involving many computer hours.

We believe that our results for the *calculated* correlation functions are verified in part by the experimental result given in Fig. 1*b*. The central peak is narrowed, and several sidelobes appear well above noise level. Given that strain will distort the butted on mirror alignment in a 'dying' laser tube, there is no need for the lowest order mode even to be present. There was no means of measuring accurately the angle $\theta \equiv \alpha$ for the detector separation with respect to the mode axes, but it must have been reasonably close to $\alpha = 0$. The laser was rigidly mounted, and so was the plasma tube inside the laser housing.

A full verification of our calculations would require a laser source with at least one Brewster-angle window, and at least one adjustable mirror. In order to preserve power output onto the rotating rough disc, it might be necessary to place an extremely fine fibre, or fibre arrangement, in between the Brewster-angle window and the adjustable mirror, to force the laser into a higher order mode, rather than relying on 'misalignment' of the laser mirrors.

Acknowledgments

The measurements shown in Fig. 2*c* were taken from a third year physics report by Messrs G. Murphy, M. Joyce and P. Willis of Monash University. We are indebted to Mr Robin Turner, Physics Department, Monash University, for helpful discussions.

References

Armstrong, J. A., and Smith, A. W. (1963). *Phys. Rev. Lett.* **14**, 68.

Bolwijn, P. T., Alkemade, C. Th., and Boschloo, G. A. (1964). *In* 'Quantum Electronics III', 1963 Paris Conf. Proc. (Eds N. Bloembergen and P. Grivet), p. 111 (Columbia Univ. Press).

Corcoran, V. J., and Pao, Y. (1962). *J. Opt. Soc. Am.* **52**, 1341.

Gradshtein, I. S., and Ryzhik, I. M. (1980). 'Table of Integrals, Series and Products' (Academic: New York).

Hanbury Brown, R., and Twiss, R. Q. (1956). *Nature* **178**, 1046.

Haner, A. B., and Isenor, N. R. (1970). *Am. J. Phys.* **33**, 748.

Kastler, A. (1964). *In* 'Quantum Electronics III', 1963 Paris Conf. Proc. (Eds N. Bloembergen and P. Grivet), p. 3 (Columbia Univ. Press).

Mandel, L., and Wolf, E. (1963). *J. Opt. Soc. Am.* **53**, 1315.

Martiennsen, W., and Spiller, E. (1964). *Am. J. Phys.* **32**, 919.

Zernike, F. (1938). *Physica* **5**, 785.

Manuscript received 28 October 1985, accepted 6 June 1986