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The $^{12}C(\alpha,\gamma)^{16}O$ Cross Section at Stellar Energies

F. C. Barker

Department of Theoretical Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601.

Abstract

An earlier calculation of the E1 contribution to the $^{12}C(\alpha,\gamma)^{16}O$ cross section at stellar energies, which used *R*-matrix formulae with parameters chosen to fit $^{12}C+\alpha$ phase shifts and the α spectrum from ^{16}N β decay, is updated by including in the fit $^{12}C(\alpha,\gamma)^{16}O$ cross section data. The E2 contribution is obtained by an *R*-matrix fit to $^{12}C+\alpha$ phase shifts and to data derived from $^{12}C(\alpha,\gamma)^{16}O$ angular distribution measurements. Estimates of these contributions by others are discussed critically. It is suggested that the long-accepted value of the stellar cross section should be increased by a factor of about 2 rather than the factor 3–5 that has been proposed recently.

1. Introduction

The rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction at stellar energies is important in determining not only the $^{12}C/^{16}O$ ratio after helium burning but also the entire future evolution of massive stars (Woosley 1985). Woosley suggests that this rate needs to be determined with an uncertainty of no more than 20%. Its present uncertainty is illustrated by the proposals that have been made over the last few years that the previously accepted rate should be increased by a factor of 3–5 (Kettner *et al.* 1982; Descouvement *et al.* 1984; Langanke and Koonin 1985).

It is generally believed that both E1 and E2 γ transitions are significant at stellar energies. Quite different methods have been used to obtain these two contributions. Since the cross section at laboratory energies is dominated by the peak due to the 9·59 MeV 1^- level of ^{16}O and therefore by E1 capture, the E1 contribution at stellar energies has been obtained by assuming a particular form of energy dependence, with parameters adjusted to fit the measured cross section and other relevant experimental data. On the other hand the E2 contribution has generally been obtained by calculations based on a particular nuclear model chosen to fit some experimental data but none from $^{12}C(\alpha,\gamma)^{16}O$.

Most estimates of the E1 contribution have been based on the most recent measurements of the $^{12}C(\alpha,\gamma)^{16}O$ cross section, made by Dyer and Barnes (1974) and Kettner *et al.* (1982). One estimate (Barker 1971), made before these measurements were available, used an energy dependence given by the three-level approximation of *R*-matrix theory, with parameters adjusted to fit various data including the α spectrum from ^{16}N β decay. In order to put this approach on a comparable basis with other estimates of the low-energy E1 cross section, this fitting procedure is repeated

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here, in Section 2, with the inclusion in the fit of the more recent $^{12}C(\alpha,\gamma)^{16}O$ cross section measurements.

Recently, Redder *et al.* (1985) measured the $^{12}C(\alpha,\gamma)^{16}O$ angular distribution and so determined values of the E2/E1 ratio. This allows the E2 cross section at laboratory energies to be extracted (with considerable uncertainty). In Section 3, we use two-level R-matrix formulae to fit this E2 cross section, together with the $^{12}C+\alpha$ d-wave phase shift and the energy and radiation width of the bound $6\cdot92$ MeV 2^+ level of ^{16}O, and so determine the E2 cross section at stellar energies.

In Section 4, the present results are compared with those obtained by others, and the various methods that have been used are discussed in some detail. There has been little critical discussion in the past, and the choice of the best value of the low-energy cross section for use in astrophysical calculations appears to have been made on a rather arbitrary basis.

2. *R*-matrix Fit to the $^{12}C(\alpha,\gamma)^{16}O$ E1 Cross Section

In a previous calculation of the low-energy $^{12}C(\alpha,\gamma)^{16}O$ E1 cross section (Barker 1971; henceforth referred to as I), the $^{12}C+\alpha$ elastic scattering p-wave phase shift and the contribution to the α spectrum following ^{16}N β decay due to 1^- states of ^{16}O were fitted using three-level R-matrix formulae, the three 1^- levels being the $7\cdot12$ MeV level of ^{16}O, which is 45 keV below the $^{12}C+\alpha$ threshold, the broad $9\cdot59$ MeV level, and a higher-energy level representing the background. One of the level parameters that is of vital importance in the low-energy $^{12}C(\alpha,\gamma)^{16}O$ cross section is the α particle reduced width of the $7\cdot12$ MeV level, $\gamma_{11}^{(1)2}$ (in the notation of I), which is approximately equivalent to the more generally used dimensionless reduced width $\theta_\alpha^2(7\cdot12)$. Because $\gamma_{11}^{(1)2}$ is small, relative to the reduced width of the $9\cdot59$ MeV level, its value is not well determined by fitting the p-wave phase shift. For this reason, and because no accurate $^{12}C(\alpha,\gamma)^{16}O$ cross section data were then available, the ^{16}N α spectrum was also fitted in I, this being appropriate both because the spectrum had been measured accurately down to low energies and because the spectrum is sensitive to the value of $\gamma_{11}^{(1)2}$. The latter is due to the β-decay matrix element for the $7\cdot12$ MeV level being large compared with that for the $9\cdot59$ MeV level, which in turn is due to the $7\cdot12$ MeV level being mainly 1p–1h relative to the ^{16}O ground state, while the $9\cdot59$ MeV level is mainly 3p–3h. The γ-ray reduced width of the $7\cdot12$ MeV level, $\gamma_{1\gamma}^{(1)2}$, was then determined by fitting the measured lifetime of the $7\cdot12$ MeV level. The only $^{12}C(\alpha,\gamma)^{16}O$ data used were the value of the E1 cross section at the $9\cdot59$ MeV peak, which was taken as 50 nb, and the assumption of constructive interference below the peak. These determined essentially the magnitude and relative sign of the γ-ray reduced width amplitude of the $9\cdot59$ MeV level, $\gamma_{2\gamma}^{(2)}$. With this procedure, the predicted low-energy $^{12}C(\alpha,\gamma)^{16}O$ E1 cross section is not sensitive to the assumed value of the peak cross section. A disadvantage, however, of making use of the ^{16}N α spectrum is that, since the spectrum has an appreciable contribution due to 3^- levels of ^{16}O, one has also to introduce parameters describing the 3^- levels and determine them by fitting the ^{16}N α spectrum and also the $^{12}C+\alpha$ f-wave phase shift.

The recent $^{12}C(\alpha,\gamma)^{16}O$ measurements confirm the constructive interference assumed in I, but give somewhat different peak cross sections of about 40 nb (Dyer and Barnes 1974) and 55 nb (Kettner *et al.* 1982). We repeat the fitting procedure of

I, including in the fits not only the peak cross section but also the complete measured energy dependence of the E1 cross section. It might seem to be unnecessary to include the ^{16}N α spectrum data in the fit once $^{12}C(\alpha,\gamma)^{16}O$ cross section values are available, since the latter are similarly sensitive to the value of $\gamma_{11}^{(1)2}$; however, the uncertainties in the cross section measurements at low energies are considerably larger than those in the α spectrum at corresponding energies. Also the measured cross section has to be corrected for the E2 contribution. Dyer and Barnes (1974) assumed that the E2 contribution has the energy dependence to be expected for direct E2 radiative capture, and gave the resultant values of the E1 cross section σ_{E1} explicitly; these values are not changed appreciably if the E2 contribution is taken from our best fits (Section 3). We separate out the E1 contribution from the total cross section values given by Kettner *et al.* (1982) by using the E2/E1 ratio measured by Redder *et al.* (1985). The σ_{E1} values of Kettner *et al.* lie consistently above those of Dyer and Barnes, except perhaps at the highest energies. We therefore fit separately the two sets of data.

Table 1. Resultant values from best fits to $^{12}C+\alpha$ phase shifts, α spectrum from ^{16}N β decay and $^{12}C(\alpha,\gamma)^{16}O$ E1 cross section

The cross section data are from either Dyer and Barnes (1974), DB, or Kettner *et al.* (1982), K. In all cases the radiation width of the 7·12 MeV level is taken as 62 ± 5 meV. The differences between cases (*a*)–(*c*) are explained in the text

Case	Data	a (fm)	$\gamma_{11}^{(1)2}$ (MeV)	$\mathscr{R}_\gamma$	X_1	X_3	X_β	X_γ	X_{tot}	$S_{E1}(0\cdot3$ MeV) (MeV b)
(*a*)	DB	5·5	0·065	−0·38	1·06	1·27	0·63	1·50	4·46	0·152
	K	5·5	0·065	−0·41	1·06	1·27	0·63	2·17	5·13	0·155
(*b*)	DB	5·5	0·070	−0·38	1·01	1·28	0·81	1·09	4·19	0·162
	K	5·5	0·068	−0·41	1·06	1·28	0·68	2·07	5·09	0·161
(*c*)	K	6·0	0·042	−0·38	1·17	1·23	0·95	1·64	4·99	0·205

Results of these fits are given in Table 1. Most quantities are defined in I; in addition, X_γ gives the quality of fit to the $^{12}C(\alpha,\gamma)^{16}O$ data in analogy with the quantities X_l and X_β, and $S_{E1}(0\cdot3\text{ MeV})$ is the E1 component of the $^{12}C(\alpha,\gamma)^{16}O$ S factor, defined by $S=\sigma E\exp(2\pi\eta)$ where η is the Sommerfeld parameter, evaluated at $E=0\cdot3$ MeV, which is the effective stellar energy. The best fit in I was obtained with a channel radius $a=5\cdot5$ fm. In case (*a*) of Table 1, we use the optimum parameter values from I for $a=5\cdot5$ fm and vary only the parameter $\gamma_{2\gamma}^{(2)}$, or equivalently $\mathscr{R}_\gamma\equiv\gamma_{2\gamma}^{(2)}/\gamma_{1\gamma}^{(1)}$, in order to best fit the $^{12}C(\alpha,\gamma)^{16}O$ data. If $a=5\cdot5$ fm is retained but other level parameters are also allowed to vary, the best overall fit to the data is obtained for the values given in case (*b*); values of other level parameters are little changed. Better fits to the Dyer and Barnes data are not obtained by changing the channel radius, but larger channel radii can give slightly better fits to the Kettner *et al.* data, and the results for $a=6\cdot0$ fm are given in case (*c*). These best fits are shown in Fig. 1, together with the experimental E1 cross sections. Acceptable fits to the data are obtainable for ranges of parameter values about these best fit values. In analogy with Fig. 4 of I, Fig. 2 shows minimum values of X_{tot} as functions of $S_{E1}(0\cdot3\text{ MeV})$ for various values of the channel radius. With the same rather subjective conditions for an acceptable fit as were used in I, namely $X_{tot}\lesssim1\cdot5(X_{tot})_{min}$

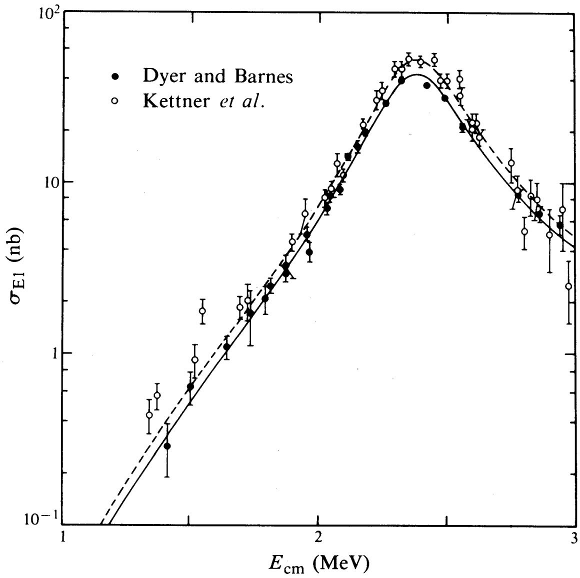

Fig. 1. The $^{12}C(\alpha,\gamma)^{16}O$ E1 cross section as a function of c.m. energy. The experimental points are from Dyer and Barnes (1974) and Kettner *et al.* (1982). The curves are best fits with the parameter values given in Table 1 [solid curve—Dyer and Barnes data, case (*b*); dashed curve—Kettner *et al.* data, case (*c*)].

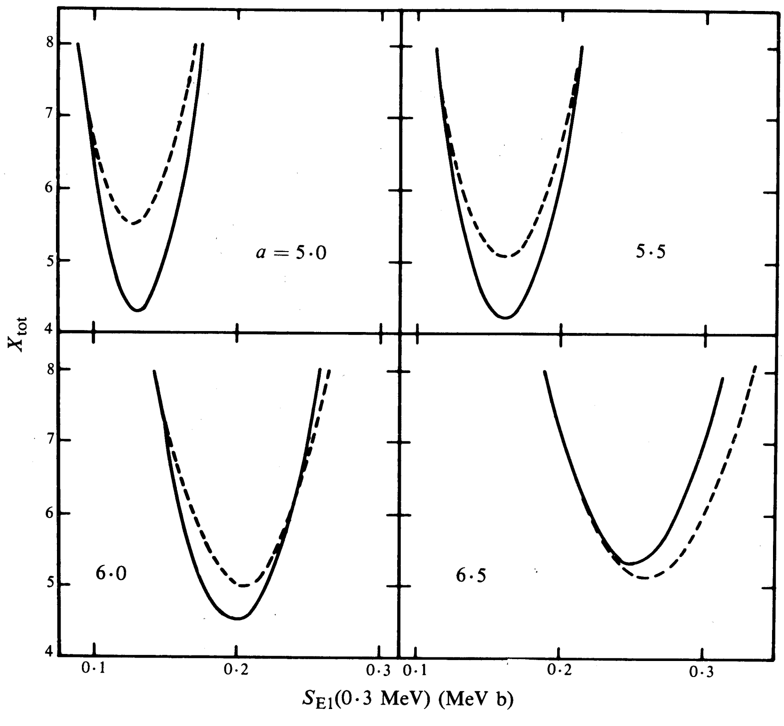

Fig. 2. Minimum values of X_{tot} as functions of $S_{E1}(0\cdot3$ MeV) for the indicated values of a (in fm) (solid curves—Dyer and Barnes data; dashed curves—Kettner *et al.* data).

and $5\cdot0$ fm $\lesssim a \lesssim 6\cdot8$ fm, one finds $S_{E1}(0\cdot3\ \text{MeV}) = 0\cdot16^{+0\cdot15}_{-0\cdot06}$ MeV b from fits to the Dyer and Barnes data, and $0\cdot20^{+0\cdot18}_{-0\cdot11}$ MeV b for the Kettner *et al.* data. These are not very different from the values given in I, the reason being that $\gamma_{11}^{(1)2}$ is still determined mainly by the fit to the ^{16}N α spectrum, due to the high precision of the data, whereas the particular $^{12}C(\alpha,\gamma)^{16}O$ data fitted has relatively little effect. The smaller values of X_{tot} for the fits to the Dyer and Barnes data show that these are more consistent with the ^{16}N α spectrum than those for the Kettner *et al.* data.

So far, the fits in I have been modified only by the introduction of new $^{12}C(\alpha,\gamma)^{16}O$ data. There are, however, new values for some of the other quantities used in I, including ^{16}O level energies, Q values, branching ratios and lifetimes (Ajzenberg-Selove 1982, 1986). In general the use of the new values has little effect on either the quality of fits to the data or on the resultant parameter values. One change of significance is that the accepted value for the radiation width of the $7\cdot12$ MeV 1^- level of ^{16}O is now 55 ± 3 meV (Ajzenberg-Selove 1986), compared with 62 ± 5 meV used in I. This leads to a reduction of the predicted values of $S_{E1}(0\cdot3\ \text{MeV})$ by about 10%, giving

$$S_{E1}(0\cdot3\ \text{MeV}) = 0\cdot14^{+0\cdot13}_{-0\cdot05}\ \text{MeV b}\quad \text{(Dyer and Barnes data)}, \tag{1a}$$

$$S_{E1}(0\cdot3\ \text{MeV}) = 0\cdot18^{+0\cdot16}_{-0\cdot10}\ \text{MeV b}\quad (\text{Kettner } et\ al.\ \text{data}). \tag{1b}$$

3. *R*-matrix Fit to the $^{12}C(\alpha,\gamma)^{16}O$ E2 Cross Section

Contributions to the $^{12}C(\alpha,\gamma)^{16}O$ cross section due to E2 transitions are expected to come from 2^+, $T = 0$ levels of ^{16}O and possibly from direct capture. The 2^+ level at $6\cdot92$ MeV, which is 245 keV below the $^{12}C+\alpha$ threshold, should dominate the low-energy cross section; the known 2^+ levels above threshold, starting with those at $9\cdot85$ MeV ($\Gamma = 0\cdot625$ keV) and $11\cdot52$ MeV ($\Gamma = 74$ keV), are too narrow to affect the cross section except in their immediate neighbourhoods. Thus we fit the $^{12}C(\alpha,\gamma)^{16}O$ E2 cross section over the measured energy range, with the omission of regions near narrow levels, using a two-level *R*-matrix approximation, and extrapolate this to find the cross section at stellar energies. The lower of the two levels corresponds to the $6\cdot92$ MeV level and the other gives a background term representing all high-lying 2^+ levels of ^{16}O as well as direct capture. The energy of this background level is fixed at $E_{22}^{(2)} = 15$ MeV, although the results are not sensitive to this particular choice. Additional restrictions on the values of the level parameters are obtained by fitting the $^{12}C+\alpha$ d-wave phase shift and the energy and radiation width of the $6\cdot92$ MeV level.

Experimental values of the $^{12}C(\alpha,\gamma)^{16}O$ E2 cross section σ_{E2} are obtained from the measured values of σ_{E2}/σ_{E1} given by Dyer and Barnes (1974) at four energies* and by Redder *et al.* (1985) at 18 energies, together with those of σ_{E1} given by Dyer and Barnes (1974) or of $\sigma_{tot} = \sigma_{E1}+\sigma_{E2}$ given by Kettner *et al.* (1982). Since the different measurements were made at different energies, and since the errors in

* Langanke and Koonin (1985) claimed that the σ_{E2}/σ_{E1} values of Dyer and Barnes (1974) should be multiplied by $\frac{3}{5}$, but refitting the angular distributions shows that this is not correct. Consequently the experimental data in Fig. 5 of Langanke and Koonin (1983) and in Fig. 4 of Langanke and Koonin (1985) should be multiplied by $\frac{5}{3}$.

σ_{E2}/σ_{E1} are in general appreciably larger than those given for σ_{E1} or σ_{tot}, our procedure is to use $\sigma_{E2}^{exp} = (\sigma_{E2}/\sigma_{E1})^{exp}\sigma_{E1}$, where the values of σ_{E1} at the energies at which σ_{E2}/σ_{E1} has been measured are calculated from the best fits to the data as obtained in the preceding section. Since we there fitted separately the σ_{E1} values from Dyer and Barnes and from Kettner *et al.*, so here we make separate fits to the resultant σ_{E2}^{exp} values. Values of the d-wave phase shift δ_2^{exp} are taken from Jones *et al.* (1962), with assumed errors of 1°, and from Clark *et al.* (1968). The radiation width of the 6·92 MeV level, $\Gamma_\gamma^{obs}(6\cdot92) = 99\pm3$ meV (Ajzenberg-Selove 1986), is fitted exactly because of the small error.

Formulae and notation are similar to those given in I. One modification is that the factor 6π in equation (21) of I is replaced by 10π (see equation 4 below). The results of fitting simultaneously the values of σ_{E2}^{exp}, $\delta_2^{exp'}$ and $\Gamma_\gamma^{obs}(6\cdot92)$ are summarised in Fig. 3. This shows the minimum values of $X_{tot} = X_2 + X_\gamma$ as a function of $S_{E2}(0\cdot3\text{ MeV})$ for various values of the channel radius a_2 from 4·5 to 7·0 fm. For all channel radii, the best fits give $S_{E2}(0\cdot3\text{ MeV}) \approx 0\cdot03$ MeV b. In the spirit of I, we take for the overall best fit to the data, using the Dyer and Barnes σ_{E1} values, the parameter values

$$a_2 = 5\cdot5\text{ fm}, \qquad \gamma_{12}^{(1)2} = 0\cdot063\text{ MeV}, \qquad \mathscr{R}_\gamma = 0\cdot17,$$

$$X_2 = 1\cdot61, \qquad X_\gamma = 1\cdot29, \qquad X_{tot} = 2\cdot90;$$

$$S_{E2}(0\cdot3\text{ MeV}) = 0\cdot029\text{ MeV b}. \tag{2}$$

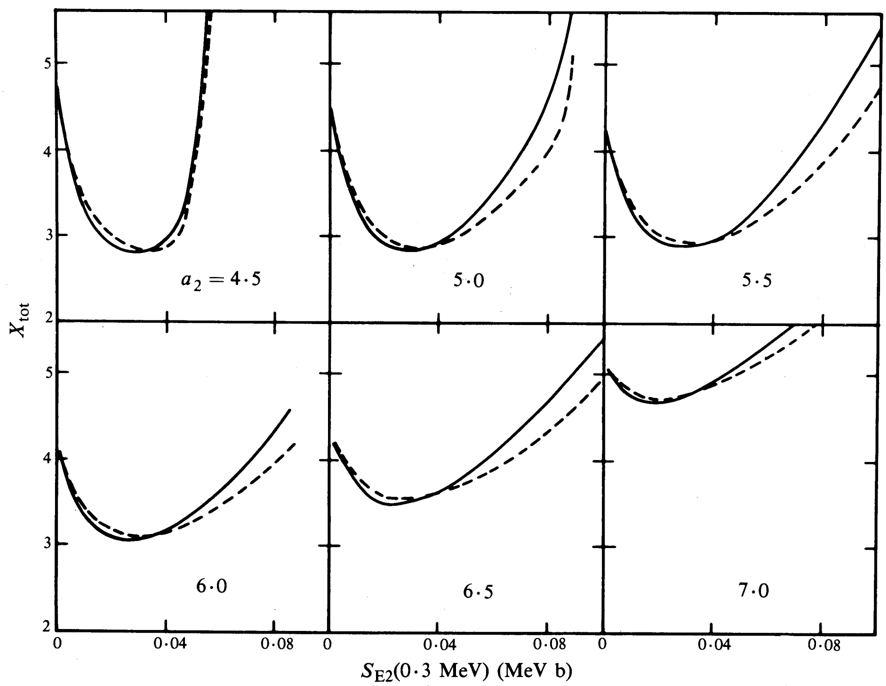

Fig. 3. Minimum values of X_{tot} as functions of $S_{E2}(0\cdot3\text{ MeV})$ for the indicated values of a_2 (in fm) (solid curves—Dyer and Barnes σ_{E1} data; dashed curves—Kettner *et al.* σ_{E1} data).

This best fit is shown in Figs 4 and 5 for δ_2 and σ_{E2} respectively. Fits using the Kettner *et al.* σ_{E1} values give very similar results, including $\gamma_{12}^{(1)2} = 0{\cdot}071$ MeV and $S_{E2}(0{\cdot}3\ \text{MeV}) = 0{\cdot}033$ MeV b. If fits with $X_{tot} \lesssim 1{\cdot}5(X_{tot})_{min}$ are regarded as acceptable, then one finds that values of a_2 greater than 7 fm are excluded and

$$S_{E2}(0{\cdot}3\ \text{MeV}) = 0{\cdot}03^{+0{\cdot}05}_{-0{\cdot}03}\ \text{MeV b} \quad (\text{Dyer and Barnes } \sigma_{E1} \text{ data}), \tag{3a}$$

$$S_{E2}(0{\cdot}3\ \text{MeV}) = 0{\cdot}03^{+0{\cdot}06}_{-0{\cdot}03}\ \text{MeV b} \quad (\text{Kettner } et\ al.\ \sigma_{E1} \text{ data}). \tag{3b}$$

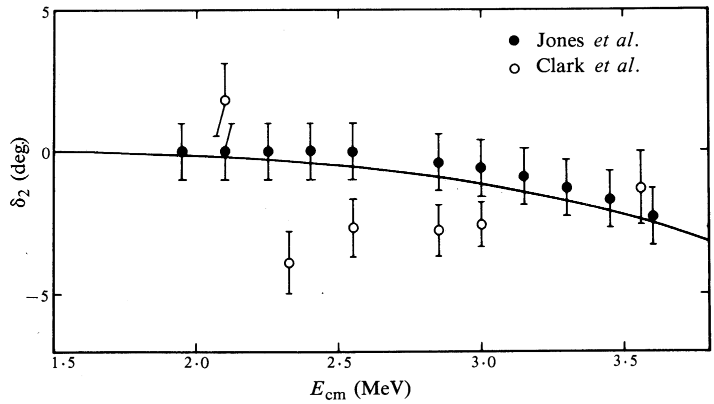

Fig. 4. The $^{12}C+\alpha$ d-wave phase shift as a function of c.m. energy. The experimental points are from Jones *et al.* (1962) and Clark *et al.* (1968). The best fit curve corresponds to the parameter values given in equations (2).

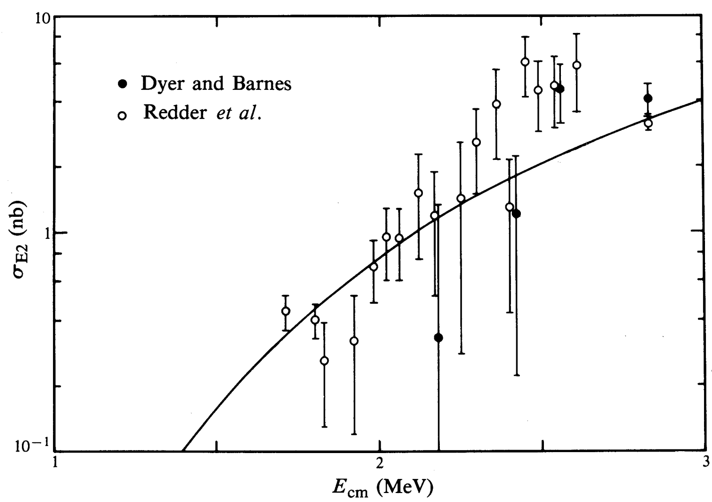

Fig. 5. The $^{12}C(\alpha,\gamma)^{16}O$ E2 cross section as a function of c.m. energy. The experimental points are from the σ_{E2}/σ_{E1} values of Dyer and Barnes (1974) and Redder *et al.* (1985), and the σ_{E1} values of Dyer and Barnes. The best fit curve corresponds to the parameter values given in equations (2).

In addition to obtaining values of σ_{E2}/σ_{E1} from their $^{12}C(\alpha,\gamma)^{16}O$ angular distributions, Dyer and Barnes (1974) and Redder *et al.* (1985) also extracted magnitudes of the relative phase ϕ_{12} between the E1 and E2 amplitudes. Dyer and Barnes compared their values with those obtained from a relation based on single-level approximations for the two amplitudes, which would seem to be inappropriate here since we use a three-level approximation for the E1 amplitude and a two-level approximation for the E2 amplitude. We note, however, that equation (21) of I can be written for the general case of E1 or E2 capture as (Lane and Thomas 1958)

$$\sigma_{El} = (\pi/k_a^2)(2l+1)|U_l|^2, \tag{4}$$

with

$$U_l = \mathrm{i}\exp\{\mathrm{i}(\omega_l-\phi_l)\}(2P_l)^{\frac{1}{2}}\frac{\sum_{\lambda=1}^{q_l}\{\gamma_{\lambda l}\Gamma^{\frac{1}{2}}_{\lambda\gamma l}/(E_{\lambda l}-E)\}}{1-(S_l-B_l+\mathrm{i}P_l)\sum_{\lambda=1}^{q_l}\{\gamma^2_{\lambda l}/(E_{\lambda l}-E)\}}. \tag{5}$$

Here ω_l is the relative Coulomb phase shift and $-\phi_l$ the hard sphere phase shift for the $^{12}C+\alpha$ channel. The summation in the numerator of equation (5) is real, while the denominator is complex with phase ζ_l which, from equation (3) of I, can be written

$$\zeta_l = -\delta_l-\phi_l. \tag{6}$$

Thus we write

$$U_2/U_1 = |U_2/U_1|\exp(\mathrm{i}\phi_{12}), \tag{7}$$

with

$$\begin{aligned}\phi_{12} &= \delta_2+\omega_2-\delta_1-\omega_1\\ &= \delta_2-\delta_1+\arctan(\tfrac{1}{2}\eta).\end{aligned} \tag{8}$$

The relation (8) is the one used by Dyer and Barnes. Its validity is independent of the number of levels used to approximate the El amplitude and the phase shift δ_l, also it does not involve the channel radius. We therefore calculate ϕ_{12} from (8) using values of δ_l given by the *R*-matrix best fits, which well represent the experimental phase shifts. These predictions are compared in Fig. 6 with the values derived by Dyer and Barnes and Redder *et al.* from their measurements. The agreement is satisfactory; in fact, it suggests that the angular distribution data should be analysed with ϕ_{12} fixed at the value (8), so that the resultant values of σ_{E2}/σ_{E1} would have higher precision.

4. Discussion

(a) The $^{12}C(\alpha,\gamma)^{16}O$ E1 Cross Section

The E1 contribution to the $^{12}C(\alpha,\gamma)^{16}O$ cross section at stellar energies has previously been obtained by a variety of extrapolations of the measured values, and results are given in Table 2 together with the present results. In all of these cases, fits were made to measured values of the $^{12}C(\alpha,\gamma)^{16}O$ cross section, the $^{12}C+\alpha$ p-wave phase shift, and the energy and radiation width of the 7·12 MeV level.

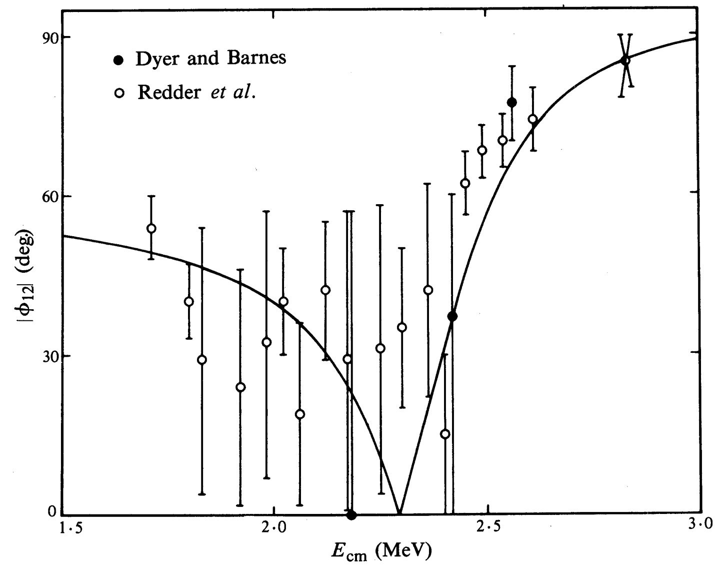

Fig. 6. Relative phase of $^{12}C(\alpha,\gamma)^{16}O$ E1 and E2 amplitudes as a function of c.m. energy. The experimental points are from Dyer and Barnes (1974) and Redder *et al.* (1985). The curve is the prediction of equation (8).

Table 2. Values of $S_{E1}(0\cdot3$ MeV) obtained by various methods

Method	Authors	$S_{E1}(0\cdot3$ MeV) (MeV b) DB[A]	K[B]
Hybrid *R*-matrix	Koonin *et al.* (1974)	$0\cdot08^{+0\cdot05}_{-0\cdot04}$	
	Tombrello *et al.* (1982)	0·09, 0·11	
	Langanke and Koonin (1983)	0·15	0·34
	Langanke and Koonin (1985)	0·16, 0·17	0·28, 0·29
R-matrix	Weisser *et al.* (1974)	0·17	
	Dyer and Barnes (1974)	$0\cdot14^{+0\cdot14}_{-0\cdot12}$	
K-matrix	Humblet *et al.* (1976)	$0\cdot08^{+0\cdot14}_{-0\cdot07}$	
'*S*-matrix'	Kettner *et al.* (1982)		0·25
Present work		$0\cdot14^{+0\cdot13}_{-0\cdot05}$	$0\cdot18^{+0\cdot16}_{-0\cdot10}$

[A] Fit to $^{12}C(\alpha,\gamma)^{16}O$ data of Dyer and Barnes (1974).
[B] Fit to $^{12}C(\alpha,\gamma)^{16}O$ data of Kettner *et al.* (1982).

It is seen that the value of $S_{E1}(0\cdot3$ MeV) depends on the $^{12}C(\alpha,\gamma)^{16}O$ data being fitted more sensitively in the hybrid *R*-matrix method than in the present approach; this is because the value of $\gamma_{11}^{(1)2}$ [or $\theta_\alpha^2(7\cdot12)$] is determined mainly by the $^{12}C(\alpha,\gamma)^{16}O$ data themselves in the hybrid model but by the ^{16}N α spectrum in the present case. The value of $S_{E1}(0\cdot3$ MeV) from the hybrid model is also sensitive to the description of the ^{16}O ground state and continuum wavefunctions, as is shown by the different results obtained by Koonin *et al.* (1974) and Langanke and Koonin (1983, 1985) from fits to the same data.

The radiation width of the $7\cdot12$ MeV level was taken to be 62 ± 5 meV in all cases in Table 2, except that Kettner *et al.* (1982) fitted 57 ± 5 meV and the present fit uses 55 ± 3 meV. As pointed out in I (see equation 23), these are values of the observed width, whereas in several of the fits (Weisser *et al.* 1974; Dyer and Barnes 1974; Kettner *et al.* 1982) the measured value was taken as a formal width. This turns out to be unimportant, however, because the formal and observed widths of this level differ by only about 5%, due to the small value of $\gamma_{11}^{(1)2}$. Also, in many-level R-matrix formulae (and in the hybrid R-matrix model), the same fit to data can be obtained for any choice of the boundary condition parameter B_1, although the parameter values giving this fit depend on the value of B_1 (Barker 1972). Fits made with the hybrid R-matrix model used $B_1 = 0$, but the formula (5) in Koonin *et al.* (1974) and (3·5) in Langanke and Koonin (1983) is true only for the particular choice $B_1 = S_1(7\cdot12\ \mathrm{MeV})$. By using this formula with $B_1 = 0$, Koonin *et al.* actually fitted an experimental value of the radiation width of 84 meV.

The hybrid R-matrix model requires rather detailed discussion because it has been claimed (Langanke and Koonin 1985) that this approach is superior to the usual R-matrix fits and because it is the results of this model that have been adopted in compilations for use in astrophysical calculations (Fowler *et al.* 1975; Harris *et al.* 1983; Caughlan *et al.* 1985). In this model only the $7\cdot12$ MeV level is included explicitly, and contributions from the $9\cdot59$ MeV and higher levels are described by a potential and effective dipole strength; the separation of this potential contribution into parts due to the $9\cdot59$ MeV level and a background is also given by Koonin *et al.* (1974). The parameters describing the $9\cdot59$ MeV level and the background are therefore related, since both come from the same potential and the same dipole strength, which is assumed to be independent of energy (except in Tombrello *et al.* 1982 where smooth energy dependences are investigated). As a result, the uncertainty in $S_{\mathrm{E1}}(0\cdot3\ \mathrm{MeV})$ is small. But this model implies that the states contributing to the background are dominantly $T = 0$ and have large α widths, and that they all have the same (small) $T = 1$ admixture as the $9\cdot59$ MeV level. It is not clear why this should be so, nor why one can neglect contributions from mainly $T = 1$ states with large ground-state E1 matrix elements and small $T = 0$ admixtures. In fact it is likely that contributions to the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ cross section from the two types of background state (mainly $T = 0$ and mainly $T = 1$) more or less cancel in the energy region under study. The claim (Koonin *et al.* 1974) that the hybrid model has 'accounted for the coherent background from the high-lying states in a physically plausible manner' does not seem to be justified.

Different treatments of the background contribution have been used in the different methods. In the R- and K-matrix methods, no restriction was placed on the radiation width of the background 'level', but in the present case it is assumed that this radiation width is zero, from the model-dependent arguments given in I. Kettner *et al.* (1982) neglected background contributions altogether, and in fact fitted the $^{12}\mathrm{C}+\alpha$ p-wave phase shift with a one-level R-matrix approximation (corresponding to the $9\cdot59$ MeV level).

Likewise, different choices of the channel radius a were made. Koonin *et al.* (1974) chose $a = 5\cdot35$ fm (although this was not the optimum value, as is seen from Table 3 of Tombrello *et al.* 1982), and their uncertainties allow for a change of $\pm0\cdot6$ fm in this value. Langanke and Koonin (1983, 1985) assumed $a = 5\cdot3$ fm, while Kettner *et al.* (1982) took $a = 5\cdot40$ fm. In the R-matrix fits, a particular value $a = 5 \cdot 5$ fm was used because changing this did not change appreciably the value of $S_{E1}(0 \cdot 3$ MeV), due to the background parameters being free to vary. In the present fit, the value of $S_{E1}(0 \cdot 3$ MeV) is well determined for a given value of a (see Fig. 2), and uncertainty in the choice of a is the main source of the uncertainty in the $S_{E1}(0 \cdot 3$ MeV) values given in equations (1). The K-matrix method has the apparent advantage of not introducing a channel radius; the fit is, however, essentially identical with an R-matrix fit with zero channel radius, which accounts for the small value of $S_{E1}(0 \cdot 3$ MeV).

A further comment needs to be made on the formula for the $^{12}C(\alpha,\gamma)^{16}O$ S factor used by Kettner *et al.* (1982), in which a one-level R-matrix approximation was assumed for each of the $7 \cdot 12$ and $9 \cdot 59$ MeV levels, and an S-matrix amplitude was formed by adding coherently these two amplitudes with a phase difference given by the difference of their resonance phase shifts (for this reason we have called this an S-matrix method in Table 2). This phase difference is not what one expects from a rigorously-based reaction theory such as R-matrix theory (Lane and Thomas 1958) or complex eigenvalue theory (Humblet and Rosenfeld 1961).

Tombrello *et al.* (1982) criticised the R- and K-matrix expansions, saying that appreciable truncation error may be involved due to an inadequate description of the $9 \cdot 59$ MeV level. Their argument, however, was based on the assumption that the $9 \cdot 59$ MeV level is more accurately described by a potential model than by a single term in an R-matrix expansion, and they gave no evidence for this. In fact the quality of fits to data obtained here with an R-matrix expansion is equally as good as that obtained in the hybrid model.

Values of $\theta_{\alpha}^{2}(7 \cdot 12)$ have also been obtained from α-transfer reactions and from nuclear models of the ^{16}O states. Some such values have been summarised by Kettner *et al.* (1982), Tombrello *et al.* (1982) and Descouvement *et al.* (1984). These values have a considerable spread, particularly those from α-transfer reactions (see also Shyam *et al.* 1985). There is no unique relation between the values of $\theta_{\alpha}^{2}(7 \cdot 12)$ and $S_{E1}(0 \cdot 3$ MeV), in part because different definitions have been used for $\theta_{\alpha}^{2}(7 \cdot 12)$. In a recent calculation, Descouvement *et al.* (1984) gave $\theta_{\alpha}^{2}(7 \cdot 12) = 0 \cdot 09$ and $S_{E1}(0 \cdot 3 \text{ MeV}) = 0 \cdot 30$ MeV b; in contrast, the values in the first row of Table 1, for example, give $\theta_{\alpha}^{2}(7 \cdot 12) = 0 \cdot 094$ and $S_{E1}(0 \cdot 3 \text{ MeV}) = 0 \cdot 152$ MeV b. But the $\theta_{\alpha}^{2}(7 \cdot 12)$ of Descouvement *et al.* was derived from an effective reduced width, which takes account of background contributions due to other 1^- levels, and the 'single-level' reduced width was larger by a factor of $3 \cdot 5$. Also Descouvement *et al.* neglected the radiation width of the $9 \cdot 59$ MeV level; inclusion of this would increase the value of $S_{E1}(0 \cdot 3$ MeV), due to constructive interference in the region between the levels.

In summary we suggest that the best value of $S_{E1}(0 \cdot 3$ MeV) at present available is $0 \cdot 15^{+0 \cdot 14}_{-0 \cdot 06}$ MeV b. This omits from consideration values obtained from the hybrid R-matrix model and from K- and S-matrix fits, for the reasons given above, and attributes more weight to the fits using the Dyer and Barnes data because they are more consistent with the ^{16}N α-spectrum data.

(b) The $^{12}C(\alpha,\gamma)^{16}O$ E2 Cross Section

Dyer and Barnes (1974) pointed out that the bound $6 \cdot 92$ MeV 2^+ level of ^{16}O might provide a significant contribution to the $^{12}C(\alpha,\gamma)^{16}O$ cross section at stellar energies, and estimated the ratio of the E2 to the E1 contributions to be $0 \cdot 1$–$0 \cdot 3$.

Similarly Tombrello *et al.* (1982) obtained a ratio of about 0·5. More recent estimates of $S_{E2}(0\cdot3 \text{ MeV})$ are listed in Table 3.

Table 3. Values of $S_{E2}(0\cdot3$ MeV) obtained by various methods

Method	Authors	$S_{E2}(0\cdot3$ MeV) (MeV b)
'*S*-matrix'	Kettner *et al.* (1982)	0·18
Generator coordinate	Descouvement *et al.* (1984)	0·09
Potential	Langanke and Koonin (1985)	0·07
Coupled channels	Funck *et al.* (1985)	0·10
Present work		$0\cdot03^{+0\cdot05}_{-0\cdot03}$

Kettner *et al.* (1982) fitted the energy dependence of their $^{12}C(\alpha,\gamma)^{16}O$ total cross section with incoherent E1 and E2 contributions. Since the E2 contribution is small at laboratory energies, it is not well determined by this method and the resulting value of $S_{E2}(0\cdot3 \text{ MeV})$ must have large uncertainties. In spite of this, Kettner *et al.* placed rather small uncertainties on the combined E1 and E2 contributions, $S(0\cdot3 \text{ MeV}) = 0\cdot42^{+0\cdot16}_{-0\cdot12}$ MeV b. As in the E1 case, Kettner *et al.* treated the measured radiation width of the 6·92 MeV level as a formal width, and they assumed a phase difference between the resonant and direct-capture contributions to the E2 *S*-matrix amplitude that is not what one expects from *R*-matrix or complex-eigenvalue theory.

Using the generator coordinate method, with ^{16}O levels assumed to have an $\alpha+{}^{12}C_{gs}$ cluster structure and with a parameter in the nucleon–nucleon force adjusted to fit each level energy exactly, Descouvement *et al.* (1984) calculated $S_{E2}(0\cdot3 \text{ MeV}) \approx 0\cdot024$ MeV b. This calculation, however, did not fit the observed radiation width of the 6·92 MeV level, so they renormalised the bound state contribution to the E2 matrix element by a factor of 1·9, leading to the much increased value of $S_{E2}(0\cdot3 \text{ MeV})$ given in Table 3.

Earlier, Langanke and Koonin (1983) had calculated the E2 cross section using a microscopically founded potential model, based on $\alpha+{}^{12}C_{gs}$ structure of the ^{16}O states, in which the potential parameters were chosen to fit the $^{12}C+\alpha$ d-wave phase shift and properties of the 6·92 MeV level. They found $S_{E2}(0\cdot3 \text{ MeV}) = 0\cdot0054$ MeV b. Langanke and Koonin (1985) pointed out that this calculation was in error, and that correction led to an increase in $S_{E2}(0\cdot3 \text{ MeV})$ by a factor of about 6. They also 'improved' the calculation by adjusting the potential parameters to fit properties of the 10·35 MeV 4^+ level of ^{16}O, assumed to belong to the same $\alpha+{}^{12}C$ molecular band as the 6·92 MeV 2^+ level, but apparently they no longer required a fit to the $^{12}C+\alpha$ d-wave phase shift; this led to a further increase in $S_{E2}(0\cdot3 \text{ MeV})$ by a factor of about 2, giving the final value as in Table 3. Funck *et al.* (1985) extended the potential model of Langanke and Koonin to a coupled channels calculation, by including coupling to cluster states with $\alpha+{}^{12}C^*$ structure, where $^{12}C^*$ is the 2^+ first excited state of ^{12}C. The difference between their value and that of Langanke and Koonin (1985) is due to different constraints applied in determining the potentials, and not to the coupling to the excited-state channel. Thus values of $S_{E2}(0\cdot3 \text{ MeV})$ from about 0·03 to 0·10 MeV b have been obtained from the Langanke and Koonin potential model by requiring the potential parameters to fit different measured properties of the ^{16}O levels.

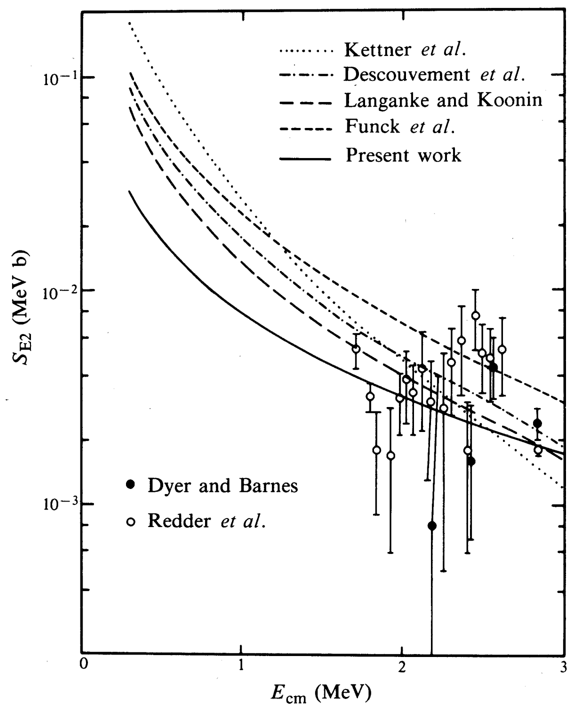

Fig. 7. The $^{12}C(\alpha,\gamma)^{16}O$ E2 S factor as a function of c.m. energy. The experimental points are as in Fig. 5. The curves are fits and calculated values from Kettner *et al.* (1982), Descouvement *et al.* (1984), Langanke and Koonin (1985), Funck *et al.* (1985), and the present work.

From such values of σ_{E2} obtained by calculations and by fits to data, Redder *et al.* (1985) derived values of σ_{E2}/σ_{E1}, which they compared with their measured values. This is not the best way of comparing σ_{E2} values, because most of the energy dependence is due to the peak in σ_{E1}. A direct comparison of calculated and fitted E2 cross sections (as S factors), showing their extrapolations to stellar energies, is made in Fig. 7, which also shows experimental values based on the σ_{E1} values of Dyer and Barnes (1974).

In view of the problems with a fully-microscopic calculation (Descouvement *et al.* 1984) and the flexibility of a potential model calculation (Langanke and Koonin 1983, 1985; Funck *et al.* 1985), both based on the assumption of $\alpha + {}^{12}C_{gs}$ structure for the ^{16}O levels, we prefer the results obtained from the present R-matrix fit to experimental data, even though the uncertainties are large, and recommend $S_{E2}(0\cdot3\ \mathrm{MeV}) = 0\cdot03^{+0\cdot05}_{-0\cdot03}$ MeV b.

5. Summary

From the preceding section, the recommended best values of the $^{12}C(\alpha,\gamma)^{16}O$ S factors at stellar energies are $S_{E1}(0\cdot3\ \mathrm{MeV}) = 0\cdot15^{+0\cdot14}_{-0\cdot06}$ MeV b and $S_{E2}(0\cdot3\ \mathrm{MeV}) = 0\cdot03^{+0\cdot05}_{-0\cdot03}$ MeV b. Then the total S factor is $S(0\cdot3\ \mathrm{MeV}) = 0\cdot18^{+0\cdot15}_{-0\cdot07}$ MeV b. This is just over twice the value of 0·08 MeV b that was accepted (Fowler *et al.* 1975) until 1983, but does not support factors as large as 3–5 that have recently been suggested (Kettner *et al.* 1982; Descouvement *et al.* 1984; Langanke and Koonin

1985), adopted (Caughlan *et al.* 1985), and used in astrophysical calculations (Arnett and Thielemann 1985; Thielemann and Arnett 1985; Woosley and Weaver 1986). The assigned uncertainties in $S(0\cdot 3\ \mathrm{MeV})$ are still much larger than the 20% suggested as desirable by Woosley (1985).

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