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Nonlinear Instability of Ion–Cyclotron Whistlers in the Ionosphere

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Abstract

We provide a theoretical investigation on the nonlinear instability of ion–cyclotron whistlers in the ionosphere. The threshold power of the unstable electromagnetic wave, the total attenuation and the generated magnetic field are calculated for a proton whistler. Finally, the variation of different ionospheric parameters due to nonlinear effects in the medium are shown both numerically and graphically. Possible applications of this investigation to space plasmas are also pointed out.

1. Introduction

Linear propagation of ion–cyclotron whistlers in the ionosphere has been studied by many workers both theoretically and experimentally and it has been used as a diagnostic tool of ionospheric parameters (Gurnett *et al.* 1965; Gurnett and Brice 1966; Gurnett and Shawhan 1966; Singh *et al.* 1976; Das and Sur 1986; Das *et al.* 1987; and others). But, when the power of the wave is large, its nonlinear interaction with the background plasma can no longer be ignored; many interesting phenomena then occur. These have been investigated by Matsumoto *et al.* (1980), Brinca (1981), Das (1983), Serra (1984), Murtaza and Shukla (1984), Nunn (1984) and others. Recently, Chakraborty *et al.* (1986) theoretically studied the nonlinear propagation of ion–cyclotron whistlers in the ionosphere, and calculated the group travel time of the wave. Paul *et al.* (1987) derived the wave number shift of the ion–cyclotron whistler from the nonlinear dispersion relation. However, due to nonlinear effects, the whistler waves can feed energy into other eigenmodes present in the plasma causing parametric instability (Murtaza and Shukla 1984). Computer studies of the nonlinear whistler instability have been made elsewhere (Denavit and Sudan 1975; Matsumoto and Yasuda 1976; Kumagi *et al.* 1980; Vomvoridis and Denavit 1980; and others). The parametric instability of the lower-hybrid ion–cyclotron waves is also of great interest in space plasmas where high power VLF signals are artificially injected into the Earth's ionosphere (Shawhan 1979). Murtaza and Shukla (1984) have shown that a finite amplitude whistler can nonlinearly excite a lower-hybrid and a short wavelength ion–cyclotron wave. They pointed out that the electron Landau damping of the lower-hybrid wave can cause electron heating, whereas the ion–cyclotron waves can heat the ions preferentially, i.e. both plasma species could gain energy at the expense of the whistler wave.

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In the present paper, we derive the nonlinear change of the propagation vector of the ion–cyclotron whistler, following the dispersion relation by Chakraborty *et al.* (1986). Numerical estimations are made to find the minimum intensity of the electric field required for the instability of the ion–cyclotron whistler. The variation of the attenuation factor with ion density and power of the whistler wave are also investigated graphically. Application of our investigation to space plasmas is discussed.

2. Nonlinear Dispersion Relation of Whistlers

The ionospheric plasma is assumed to be cold, homogeneous, and collisionless, and both ions and electrons are assumed mobile. We further assume that the whistler is purely transverse and circularly polarised in the first order electric field $E_{1\pm} = a(e^{i\theta_\pm}+e^{i\theta_\mp})$, where $\theta_\pm = k_\pm z-\omega t$, a is the amplitude of the electromagnetic wave, ω the wave frequency and k the propagation vector. The upper and lower signs represent parameters for the LCP and RCP waves respectively.

Thus, using the basic system of equations governing the plasma dynamics (Chakraborty *et al.* 1986; Sur *et al.* 1987, present issue p. 665), the nonlinear dispersion relations for the LCP and RCP waves are obtained as (where only first harmonic parts correct up to the third order electric field are considered)

$$
\begin{aligned}
\frac{k_\pm^2 c^2}{\omega^2} = 1 &- \frac{\omega_{pe}^2}{\omega(\omega\pm\Omega_e)} - \frac{\omega_{pi}^2}{\omega(\omega\mp\Omega_i)} \pm \frac{1}{4\omega^2-\omega_{pe}^2-\omega_{pi}^2} \\
&\times\left\{\frac{(\omega_{pe}^2-4\omega^2)\omega_{pi}^2 e^2 a^2 \Omega_i k_\mp}{4m_e^2\omega^4(\omega^2-\Omega_i^2)}\left(\frac{k_+}{\omega+\Omega_i}+\frac{k_-}{\omega-\Omega_i}\right)\right. \\
&-\frac{(\omega_{pi}^2-4\omega^2)\omega_{pe}^2 e^2 a^2 \Omega_e k_\mp}{4m_e^2\omega^4(\omega^2-\Omega_e^2)}\left(\frac{k_+}{\omega-\Omega_e}+\frac{k_-}{\omega+\Omega_e}\right) \\
&-\frac{\omega_{pi}^2\omega_{pe}^2 e^2 a^2 \Omega_e k_\mp}{4m_e m_i\omega^4(\omega^2-\Omega_e^2)}\left(\frac{k_+}{\omega+\Omega_i}+\frac{k_-}{\omega-\Omega_i}\right) \\
&\left.+\frac{\omega_{pi}^2\omega_{pe}^2 e^2 a^2 \Omega_i k_\mp}{4m_i m_e\omega^4(\omega^2-\Omega_i^2)}\left(\frac{k_+}{\omega-\Omega_e}+\frac{k_-}{\omega+\Omega_e}\right)\right\} \\
&+\frac{(k_++k_-)(\omega^2-k_\mp^2c^2)e^2a^2}{2(4\omega^2-\omega_{pe}^2-\omega_{pi}^2)\omega^3}\left\{\frac{1}{m_e^2}\left(\frac{k_+}{\omega-\Omega_e}+\frac{k_-}{\omega+\Omega_e}\right)\right. \\
&\left.-\frac{1}{m_i^2}\left(\frac{k_+}{\omega+\Omega_i}+\frac{k_-}{\omega-\Omega_i}\right)\right\},
\end{aligned}
\tag{1}
$$

where $\Omega_s = eH_0/m_s c$, the cyclotron frequency of the s-type particles, and $\omega_{ps} = (4\pi n_0 e^2/m_s)^{1/2}$, the plasma frequency of the s-type particles; s = i, e where i and e represent ions and electrons.

Now under the condition $\omega \approx \Omega_i$, the dispersion relations for the LCP and RCP waves are

$$\frac{c^2 k_+^2}{\omega^2} = \frac{\omega_{pi}^2}{\omega(\Omega_i-\omega)} + \alpha^2 c^2 \omega_{pi}^2 \left\{ \frac{\Omega_i k_-}{4\omega^2(\Omega_i^2-\omega^2)}\left(\frac{k_+}{\Omega_i+\omega} - \frac{k_-}{\Omega_i-\omega}\right) + \frac{k_-(k_+-k_-)}{4\omega^2\Omega_i^2} + \frac{k_-}{4\omega^2\Omega_i}\left(\frac{k_+}{\Omega_i+\omega} - \frac{k_-}{\Omega_i-\omega}\right) - \frac{k_-(k_+-k_-)}{4\omega^2(\Omega_i^2-\omega^2)}\right\} + \frac{(k_+^2-k_-^2)(\omega^2-k_-^2 c^2)\alpha^2 c^2}{2\omega_{pi}^2\omega\Omega_i} - \frac{(k_++k_-)(\omega^2-k_-^2 c^2)\alpha^2 c^2}{2\omega_{pe}^2\omega}\left(\frac{k_+}{\Omega_i+\omega} - \frac{k_-}{\Omega_i-\omega}\right), \quad (2)$$

$$\frac{c^2 k_-^2}{\omega^2} = \frac{\omega_{pi}^2}{\Omega_i(\Omega_i+\omega)} + \alpha^2 c^2 \omega_{pi}^2\left(\frac{k_+^2\omega}{4\Omega_i^2(\Omega_i-\omega)(\Omega_i+\omega)^2} + \frac{k_+ k_- \omega}{4\Omega_i^2(\Omega_i-\omega)^2(\Omega_i+\omega)}\right) + \alpha^2 c^4 k_+^2\left(\frac{k_+^2}{2\Omega_i\omega\omega_{pi}^2} + \frac{k_-^2}{2\omega_{pi}^2\omega\Omega_i} + \frac{k_+ k_-}{\omega_{pe}^2(\Omega_i+\omega)(\Omega_i-\omega)}\right), \quad (3)$$

where $\alpha = ea/m_i\omega c$, the dimensionless amplitude of the electromagnetic wave. In (2) and (3) it is seen that both equations contain k_+ and k_-, and so it would be difficult to study the LCP and RCP waves separately. However, using the linear dispersion relation in these equations, nonlinear dispersion relations exclusively for the LCP or RCP waves can be obtained as follows. The relations (2) and (3) in the linear approximation (i.e. $\alpha = 0$) become

$$k_+ = \frac{\omega_{pi}\omega^{\frac{1}{2}}}{c(\Omega_i-\omega)^{\frac{1}{2}}}, \qquad k_- = \frac{\omega_{pi}\omega^{\frac{1}{2}}}{c(\Omega_i+\omega)^{\frac{1}{2}}}, \quad (4, 5)$$

and, using (5) and (4) in the right-hand sides of (2) and (3) respectively, we get

$$P_1 k_+^2 - Q_1 k_+ + R_1 = 0, \quad (6)$$

$$P_2 k_-^2 - Q_2 k_- + R_2 = 0, \quad (7)$$

where

$$P_1 = \frac{c^2}{\omega^2} + \frac{\alpha^2 c^2}{2\Omega_i(\Omega_i+\omega)},$$

$$Q_1 = \frac{\alpha^2 c\omega_{pi}^3(2\Omega_i^3 - 2\Omega_i\omega^2 - \omega^3)}{4\omega^{\frac{3}{2}}\Omega_i^2(\Omega_i+\omega)^{\frac{5}{2}}(\Omega_i-\omega)},$$

$$R_1 = \frac{\alpha^2\omega_{pi}^4(2\Omega_i^3 - 2\Omega_i\omega^2 + \omega^3)}{4\omega\Omega_i^2(\Omega_i-\omega)^2(\Omega_i+\omega)^2} - \frac{\alpha^2\omega\omega_{pi}^2}{2\Omega_i(\Omega_i+\omega)^2} - \frac{\omega_{pi}^2}{\omega(\Omega_i-\omega)},$$

$$P_2 = \frac{c^2}{\omega^2} - \frac{\alpha^2 c^2}{2\Omega_i(\Omega_i - \omega)},$$

$$Q_2 = \frac{\alpha^2 c \omega_{pi}^3 \omega^{\frac{3}{2}}(1 + 4\Omega_i^2/\omega_{pe}^2)}{4\Omega_i^2(\Omega_i - \omega)^{\frac{5}{2}}(\Omega_i + \omega)},$$

$$R_2 = -\frac{\omega_{pi}^2}{\Omega_i(\Omega_i + \omega)} - \frac{\alpha^2 \omega_{pi}^4 \omega^2}{4\Omega_i^2(\Omega_i - \omega)^2(\Omega_i + \omega)^2} - \frac{\alpha^2 \omega_{pi}^2 \omega}{2\Omega_i(\Omega_i - \omega)^2}.$$

3. Nonlinear Instability, Total Attenuation and the Generated Magnetic Field

To investigate the nonlinear instability of the LCP wave in the frequency limit $\omega \lesssim \Omega_i$, we assume that the LCP wave is unstable, i.e. k_+ is complex. Therefore, writing $k_+ = k_{\gamma+} + \mathrm{i}\, k_{i+}$ in (6) and then evaluating the real and imaginary parts, we obtain

$$k_{\gamma+} = \left(\frac{c^2}{\omega^2} + \frac{\alpha^2 c^2}{2\Omega_i(\Omega_i + \omega)}\right)^{-1} \times \left(\frac{\alpha^2 c \omega_{pi}^3 (2\Omega_i^3 - 2\Omega_i \omega^2 - \omega^3)}{8\omega^{\frac{3}{2}} \Omega_i^2 \Delta\omega (\Omega_i + \omega)^{\frac{5}{2}}}\right), \tag{8}$$

$$k_{i+} = \pm\left(\frac{c^2}{\omega^2} + \frac{\alpha^2 c^2}{2\Omega_i(\Omega_i + \omega)}\right)^{-1} \left\{\left(\frac{c^2}{\omega^2} + \frac{\alpha^2 c^2}{2\Omega_i(\Omega_i + \omega)}\right) \times \left(\frac{\alpha^2 \omega_{pi}^4 (2\Omega_i^3 - 2\Omega_i \omega^2 + \omega^3)}{4\omega \Omega_i^2 (\Delta\omega)^2 (\Omega_i + \omega)^2} - \frac{\alpha^2 \omega \omega_{pi}^2}{2\Omega_i(\Omega_i + \omega)^2} - \frac{\alpha^2 \omega_{pi}^2}{\Delta\omega\, \omega}\right) - \frac{\alpha^2 c^2 \omega_{pi}^6 (2\Omega_i^3 - 2\Omega_i \omega^2 - \omega^3)^2}{64\omega^3 \Omega_i^4 (\Omega_i + \omega)^5 (\Delta\omega)^2}\right\}^{\frac{1}{2}}, \tag{9}$$

where $\Delta\omega = \Omega_i - \omega$. Similarly, to study the instability of the RCP wave, we assume that $k_- = k_{\gamma-} + \mathrm{i} k_{i-}$. Therefore, from (7) we get

$$k_{\gamma-} = \left(\frac{c^2}{\omega^2} - \frac{\alpha^2 c^2}{2\Omega_i \Delta\omega}\right)^{-1} \left(\frac{\alpha^2 c \omega_{pi}^3 \omega^{\frac{3}{2}}(1 + 4\Omega_i^2/\omega_{pe}^2)}{8\Omega_i^2 (\Delta\omega)^{\frac{5}{2}} (\Omega_i + \omega)}\right), \tag{10}$$

$$k_{i-} = \pm i\left(\frac{c^2}{\omega^2} - \frac{\alpha^2 c^2}{2\Omega_i\,\Delta\omega}\right)^{-1}\left\{\left(\frac{c^2}{\omega^2} - \frac{\alpha^2 c^2}{2\Omega_i\,\Delta\omega}\right)\right.$$
$$\times\left(\frac{\alpha^2\omega_{pi}^2}{\Omega_i(\Omega_i+\omega)} + \frac{\alpha^2\omega_{pi}^4\,\omega^2}{4\Omega_i^2(\Delta\omega)^2(\Omega_i+\omega)^2} + \frac{\alpha^2\omega_{pi}^2\,\omega}{2\Omega_i(\Delta\omega)^2}\right)$$
$$\left. + \frac{\alpha^4 c^2\omega_{pi}^6\,\omega^3(1+4\Omega_i^2/\omega_{pe}^2)}{64\Omega_i^4(\Delta\omega)^5(\Omega_i+\omega)^2}\right\}^{\frac{1}{2}}. \tag{11}$$

It is seen from (9) and (11) that, for $\omega \lesssim \Omega_i$, k_{i+} is real and k_{i-} is imaginary. Therefore k_+ is complex and k_- is real, which indicates that the LCP wave is unstable and the RCP wave is stable in the frequency range $\omega \lesssim \Omega_i$ due to nonlinear effects in the whistler. It is important to note that k_{y+} in (8) and k_{y-} in (10) exist only due to nonlinear effects. If nonlinear effects are not taken into accounts, i.e. for $\alpha = 0$, $k_{y+} = 0 = k_{y-}$, but k_{i+} and k_{i-} become imaginary, i.e. k_+ and k_- are real. These results also follow from the linear dispersion relation.

In order to obtain the total attenuation due to the damping of the ion–cyclotron whistlers, we integrate the imaginary part k_{i+} over the propagation path h, following the work of Das and Sur (1986), and obtain the attenuation

$$\beta = 2\int_0^h k_{i+}\,\mathrm{d}h$$
$$= \frac{\omega_{pi}(0)}{4c\Omega_i'(0)}\int_{\Delta\omega(0)}^{\Delta\omega(h)} \frac{\{\alpha^2\omega_{pi}^2(0)-16\Omega_i(0)\,\Delta\omega\}^{\frac{1}{2}}\,\mathrm{d}(\Delta\omega)}{\Delta\omega}$$
$$- \frac{\alpha^2\omega_{pi}^5(0)}{512\,c\Omega_i'(0)\,\Omega_i^2(0)}\int_{\Delta\omega(0)}^{\Delta\omega(h)} \frac{\mathrm{d}(\Delta\omega)}{\Delta\omega\{\alpha^2\omega_{pi}^2(0)-16\Omega_i(0)\,\Delta\omega\}}, \tag{12}$$

where $\Omega_i(h) = \Omega_i(0)+h\Omega_i'(0)$, and where $\Omega_i'(0)$ is the gradient of the ion–cyclotron frequency and $\Omega_i(0)$ the ion–cyclotron frequency at the satellite.

Integrating (12) and then substituting $\Delta\omega(h) = \eta\,\Delta\omega(0)$, the total attentuation β is obtained approximately as

$$\beta = \frac{\omega_{pi}(0)}{c\Omega_i'(0)}[\{\alpha^2\omega_{pi}^2 - 16\eta\Omega_i\,\Delta\omega(0)\}^{\frac{1}{2}} - \{\alpha^2\omega_{pi}^2 - 16\Omega_i\,\Delta\omega(0)\}^{\frac{1}{2}}]$$
$$+ \frac{\alpha\omega_{pi}^2(0)\{\alpha^2\omega_{pi}^2(0)-256\Omega_i^2(0)\}}{512c\Omega_i'(0)\,\Omega_i^2(0)}$$
$$\times \log\frac{1}{\eta}\left(\frac{\alpha\omega_{pi}(0)+\{\alpha^2\omega_{pi}^2(0)-16\eta\Omega_i(0)\,\Delta\omega(0)\}^{\frac{1}{2}}}{\alpha\omega_{pi}(0)+\{\alpha^2\omega_{pi}^2(0)-16\Omega_i(0)\,\Delta\omega(0)\}^{\frac{1}{2}}}\right)^2, \tag{13}$$

where $\eta \gg 1$, since $\Delta\omega(h) \gg \Delta\omega(0)$ (Gurnett and Shawhan 1966).

To find β in terms of the group travel time $t(\omega)$, we use (4) and (5) in the right-hand side of (2) and then evaluate the group velocity U_g of the ion–cyclotron whistler. Thus, equation (2) yields for the LCP wave

$$U_g = 2c\left\{\frac{\omega_{pi}^2}{\Omega_i\,\Delta\omega} + \alpha^2\omega_{pi}^2\left(\frac{3\omega_{pi}^2}{16\,2^{\frac{1}{2}}\,\Omega_i^{\frac{5}{2}}(\Delta\omega)^{\frac{3}{2}}} - \frac{\omega_{pi}^2}{8\Omega_i(\Delta\omega)^3} + \frac{1}{4\Omega_i\,\Delta\omega}\right)\right\}^{\frac{1}{2}}$$

$$\times\left\{\frac{\omega_{pi}^2(2\Omega_i-\omega)}{\Omega_i(\Delta\omega)^2}\,\alpha^2\omega_{pi}^2\left(\frac{3\omega_{pi}^2(4\Omega_i-\omega)}{32\,2^{\frac{1}{2}}\,\Omega_i^{\frac{5}{2}}(\Delta\omega)^{\frac{5}{2}}} - \frac{\omega_{pi}^2(2\Omega_i+\omega)}{8\Omega_i(\Delta\omega)^4} + \frac{2\Omega_i-\omega}{4\Omega_i(\Delta\omega)^2}\right)\right\}^{-1}. \tag{14}$$

The group travel time is then given by

$$t(\omega) = \int_0^h \frac{\mathrm{d}h}{U_g}$$

$$= \int_0^h \frac{\omega_{pi}(2\Omega_i-\omega)}{2c\Omega_i^{\frac{1}{2}}(\Delta\omega)^{\frac{3}{2}}}\left\{1+\alpha^2\left(\frac{3\omega_{pi}^2}{6\,2^{\frac{1}{2}}\,\Omega_i^{\frac{3}{2}}(\Delta\omega)^{\frac{1}{2}}} - \frac{\omega_{pi}^2}{8(\Delta\omega)^2} + \tfrac{1}{4}\right)\right\}^{-\frac{1}{2}}\mathrm{d}h$$

$$+\int_0^h \alpha^2\left(\frac{3\omega_{pi}^3(4\Omega_i-\omega)}{64\,2^{\frac{1}{2}}\,c\Omega_i^2(\Delta\omega)^2} - \frac{\omega_{pi}^3(2\Omega_i+\omega)}{16c\Omega_i^{\frac{1}{2}}(\Delta\omega)^{\frac{7}{2}}} + \frac{\omega_{pi}(2\Omega_i-\omega)}{8c\,\Omega_i^{\frac{1}{2}}(\Delta\omega)^{\frac{3}{2}}}\right)$$

$$\times\left\{1+\alpha^2\left(\frac{3\omega_{pi}^2}{16\,2^{\frac{1}{2}}\,\Omega_i^{\frac{3}{2}}(\Delta\omega)^{\frac{1}{2}}} - \frac{\omega_{pi}^2}{8(\Delta\omega)^2} + \tfrac{1}{4}\right)\right\}^{-\frac{1}{2}}\mathrm{d}h. \tag{15}$$

However, it may be easily seen that for the proton whistler

$$\left|\,\alpha^2\left(\frac{3\omega_{pi}^2}{16\,2^{\frac{1}{2}}\,\Omega_i^{\frac{3}{2}}(\Delta\omega)^{\frac{1}{2}}} - \frac{\omega_{pi}^2}{8(\Delta\omega)^2} + \tfrac{1}{4}\right)\right| < 1.$$

Therefore, after some simple algebra, equation (15) gives the following value where the higher order terms of α^2 have been neglected:

$$t(\omega) = \lambda t'(\omega), \tag{16}$$

where

$$\lambda = 1 + \frac{3\alpha^2\omega_{pi}^2(0)}{32\,2^{\frac{1}{2}}\,\Omega_i^{\frac{3}{2}}(0)\{\Delta\omega(0)\}^{\frac{1}{2}}} - \frac{\alpha^2\omega_{pi}^2(0)}{16\{\Delta\omega(0)\}^2} + \frac{\alpha^2}{8},$$

$$t'(\omega) = \frac{\omega_{pi}(0)\,\Omega_i^{\frac{1}{2}}(0)}{c\Omega_i'(0)\{\Delta\omega(0)\}^{\frac{1}{2}}},$$

$$\Delta\omega(0) \ll \Delta\omega(h).$$

We note that the value of $t'(\omega)$ was obtained by Gurnett and Shawhan (1966) for the linearised approximation.

Now, to find the variation of $t(\omega)$ due to nonlinear effects on the whistler propagation, we use data on proton whistlers (see Section 4). Moreover, we make the following approximation:

$$t(\omega) \approx \frac{\omega_{\rm pi}(0)\,\Omega_{\rm i}^{\frac{1}{2}}(0)}{c\Omega_{\rm i}'(0)\{\Delta\omega(0)\}^{\frac{1}{2}}} - \frac{\alpha^2\,\Omega_{\rm i}^{\frac{1}{2}}(0)\,\omega_{\rm pi}^3(0)}{16c\Omega_{\rm i}'(0)\{\Delta\omega(0)\}^{\frac{5}{2}}}\,. \tag{17}$$

The second term in (17) is the contribution due to nonlinear effects in the plasma. We define here the fractional contribution to the group travel time as

$$R = \frac{\alpha^2\,\Omega_{\rm i}^{\frac{1}{2}}(0)\,\omega_{\rm pi}(0)}{16c\Omega_{\rm i}'(0)\{\Delta\omega(0)\}^{\frac{5}{2}}}\,\frac{1}{t}\,, \tag{18}$$

so that $R\;(=t_{\rm NL}/t)$ is then the rate of decrease of $t(\omega)$ due to nonlinear effects. The variation of R could be obtained for ion–cyclotron whistlers in the ionosphere.

Next we substitute the value of $\Delta\omega(0)$ from (18) into (13) and obtain the total attenuation in terms of $t(\omega)$:

$$\begin{aligned}\beta &= A\{(\alpha^2\omega_{\rm pi}^2 - B\eta t^{-\frac{2}{5}})^{\frac{1}{2}} - (\alpha^2\omega_{\rm pi}^2 - Bt^{-\frac{2}{5}})^{\frac{1}{2}}\}\\ &\quad - D\log\frac{1}{\eta}\left(\frac{\alpha\omega_{\rm pi} + (\alpha^2\omega_{\rm pi}^2 - B\eta t^{-\frac{2}{5}})^{\frac{1}{2}}}{\alpha\omega_{\rm pi} + (\alpha^2\omega_{\rm pi}^2 - Bt^{-\frac{2}{5}})^{\frac{1}{2}}}\right)^2,\end{aligned} \tag{19}$$

where

$$A = \frac{\omega_{\rm pi}(0)}{c\Omega_{\rm i}'(0)}\,,$$

$$B = 16\Omega_{\rm i}(0)\left(\frac{\alpha^2\omega_{\rm pi}^3(0)\,\Omega_{\rm i}^{\frac{1}{2}}(0)}{16cR\Omega_{\rm i}'(0)}\right)^{\frac{2}{5}},$$

$$D = \frac{\alpha\omega_{\rm pi}^2\{256\Omega_{\rm i}^2(0) - \alpha^2\omega_{\rm pi}^2\}}{512\,\Omega_{\rm i}'(0)c\Omega_{\rm i}^2(0)}\,.$$

Following Das and Sur (1986), the generated magnetic field can then be obtained as a function of $t(\omega)$:

$$\begin{aligned}B_1 \sim \frac{1}{t^{\frac{1}{5}}}\exp\Bigg\{&-A\{(\alpha^2\omega_{\rm pi}^2 - \beta\eta t^{-\frac{2}{5}})^{\frac{1}{2}} - (\alpha^2\omega_{\rm pi}^2 - Bt^{-\frac{2}{5}})^{\frac{1}{2}}\} + D\log\frac{1}{\eta}\\ &\times\left(\frac{\alpha\omega_{\rm pi} + (\alpha^2\omega_{\rm pi}^2 - B\eta t^{-\frac{2}{5}})^{\frac{1}{2}}}{\alpha\omega_{\rm pi} + (\alpha^2\omega_{\rm pi}^2 - Bt^{-\frac{2}{5}})^{\frac{1}{2}}}\right)^2\Bigg\}.\end{aligned} \tag{20}$$

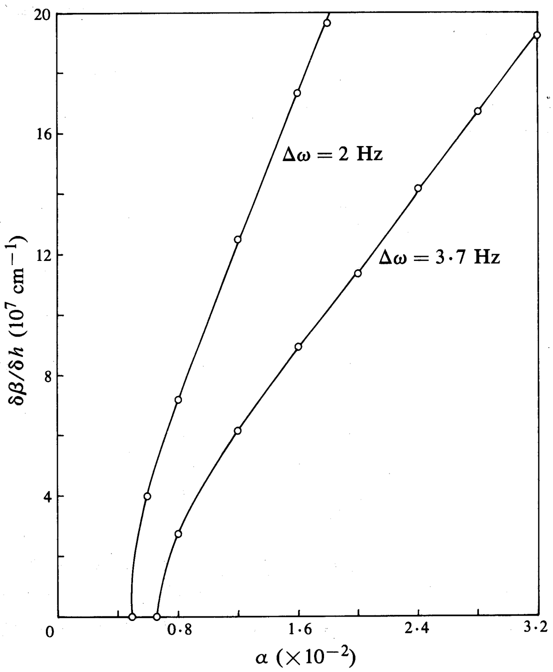

Fig. 1. Variation of the rate of attenuation with electric field intensity for the two values of $\Delta\omega$ indicated.

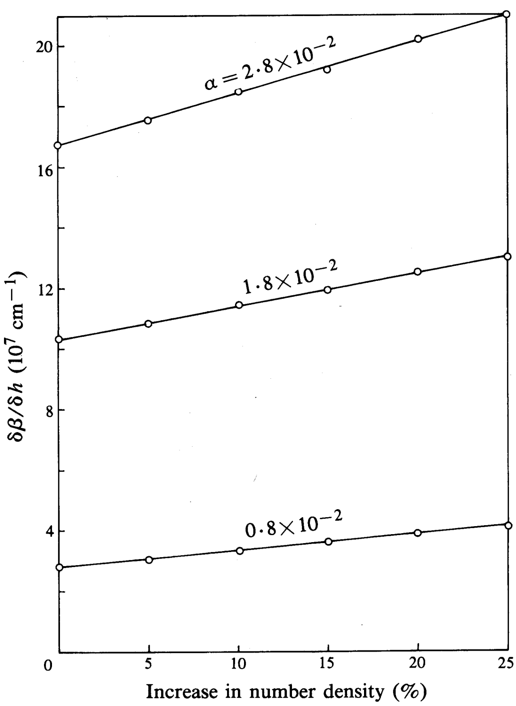

Fig. 2. Variation of the rate of attenuation with number density for the three values of α indicated.

4. Results and Discussion

To make quantitative estimations of the dependence of k_{i+}, β and B_1 on α, $t(\omega)$ and other parameters, we consider the experimental data on proton whistlers recorded on January 11, 1963 by the Injun-3 satellite (Gurnett and Brice 1966): $\Omega_i(0) = 410 \cdot 3$ Hz, $\Delta\omega(0) = 3 \cdot 7$ Hz and $n(\mathrm{H}^+) = 12 \cdot 5 \times 10^3\ \mathrm{cm}^{-3}$. The satellite observations were at 1308 km altitude where $\Omega_i'(0)$, the gradient of the proton gyrofrequency, is obtained from the geometry of the geomagnetic field variation as $1 \cdot 663 \times 10^{-6}\ \mathrm{Hz\,cm}^{-1}$ (Gurnett and Brice 1966).

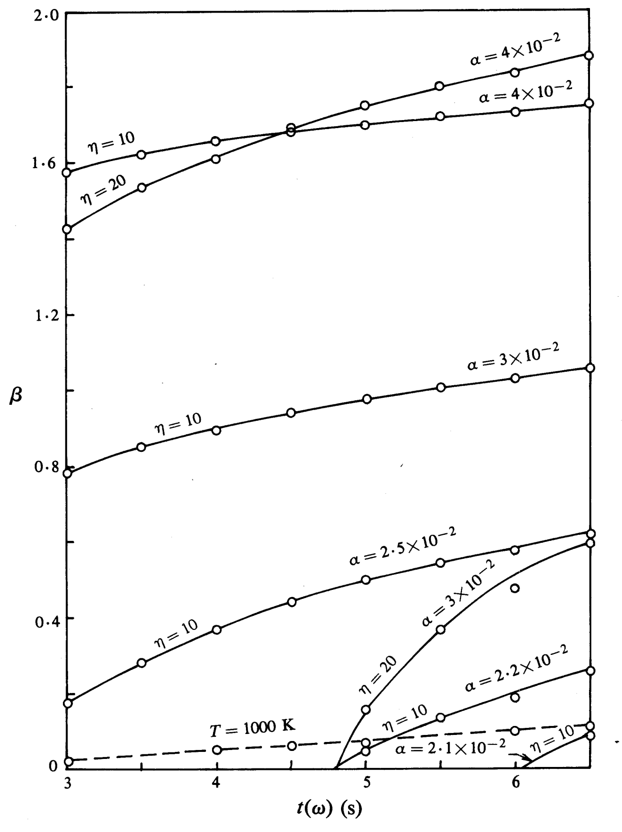

Fig. 3. Variation of the total attenuation with group travel time for the values of η and α indicated. The dashed line is from Singh *et al.* (1976) (see text).

These data are used in equation (9) to study the α dependence of k_{i+} for the proton whistler in the ionosphere; the relationship is shown in Fig. 1. It is found that the threshold value of the instability is for $\alpha = 0 \cdot 664 \times 10^{-2}$, and that it varies significantly with the intensity of the electric field. If we change the value of $\Delta\omega(0)$ from $3 \cdot 7$ to 2 Hz, the threshold value is for $\alpha = 0 \cdot 486 \times 10^{-2}$.

The effect of the number density variation of H^+ on the instability is estimated from (9). A plot of k_{i+} against $n(\mathrm{H}^+)$ in Fig. 2 for three values of α shows clearly the nature of the effect.

The variation of the total attenuation β from equation (19) as the proton whistler passes through is shown in Fig. 3. Here two values of η are considered for different values of α, and it can be seen that due to nonlinear effects the proton whistler is highly damped for high electric field intensity and that it also depends on $\Delta\omega(h)$, i.e. on the wave frequency of the whistler at the source point. The dashed line is from Singh *et al.* (1976) for the same proton whistler data at the temperature 1000 K, where the damping of the whistler was assumed to be due to thermal effects. This estimate shows that the nonlinear effect has a significant role in the damping of the whistler.

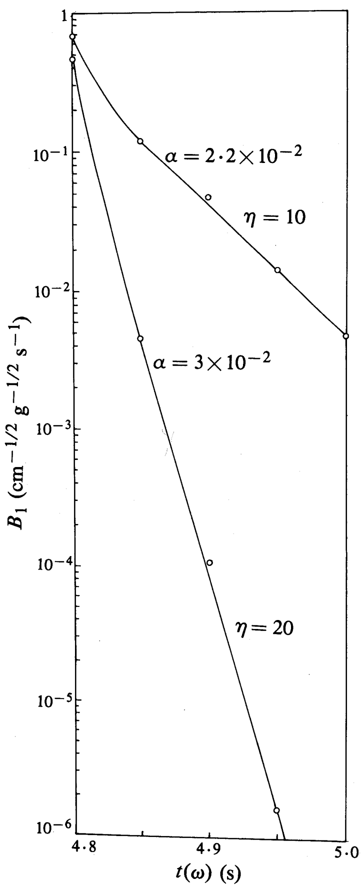

Fig. 4. Variation of B_1 with $t(\omega)$ for the values of η and α indicated.

From (20) we observe that for small $t(\omega)$ large values of the magnetic field are generated due to the nonlinear interaction. This is shown in Fig. 4 for the group travel time 4·8–5·0 s.

5. Concluding Remarks

Our work shows that ion–cyclotron whistlers may be used to estimate the generated magnetic field in the ionosphere through an estimation of damping phenomena. The damping rate due to nonlinear effects is sensitive to the variation of α, $\Delta\omega(h)$, $\Delta\omega(0)$ and the number density of the ions. The nonlinearity in the plasma, in comparison with other effects, is found to have a measurable contribution to the damping of ion–cyclotron whistlers. In the present mechanism, the damping of the ion–cyclotron whistlers can heat ions in the plasma (i.e. the plasma species can gain energy at the expense of the ion–cyclotron whistler). The following extensions to our work are proposed:

(i) Consideration of the thermal effect on the propagation of whistlers in the nonlinear plasma should give interesting results.

(ii) Stream velocities of the plasma species and newly formed ion effects should be taken into account (Das and Sur 1986) in further study of whistler waves.

(iii) Since the effect of negative ions on the propagation of whistlers (Smith 1965; Das *et al.* 1987; Sur *et al.* 1987) is not negligible, the nonlinear instability of whistlers in the presence of negative ions should be studied.

(iv) Slow variation of the number density of the plasma species has an effect on whistler propagation. Since this has not been considered here in the dispersion relation, the problem of nonlinear modification of whistlers in inhomogeneous plasmas remains open to investigation.

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