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Radio Emission Model of a 'Typical' Pulsar*

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Abstract

The generation of radio waves in the plasma of the pulsar magnetosphere is considered taking into account the inhomogeneity of the dipole magnetic field. It is shown that the growth rate of the instability of the electromagnetic waves calculated in the non-resonance case turns out to be of the order of $1/\tau_0$ (where τ_0 is the time of plasma escape from the light cylinder). However, the generation of electromagnetic waves from a new type Cherenkov resonance is possible, occurring when the particles have transverse velocities caused by the drift due to the inhomogeneity of the magnetic field. Estimates show that the development of this type of instability is possible only for pulsars with ages which exceed 10^4 yr. We make an attempt to explain some peculiarities of 'typical' pulsar emission on the basis of the model developed.

1. Introduction

There are many difficulties in the development of a successful pulsar magnetosphere model (see for example Arons 1981; Michel 1982; Beskin *et al.* 1983). Meanwhile, according to general energetic considerations and observational data, it seems possible to assume that a high density electron–positron plasma is created in polar regions of the magnetosphere. In such a plasma, the pulsar emission must be generated. The distribution function of the plasma appears to have a tail elongated in one direction (see Fig. 1). Besides, as a result of synchrotron radiation in the strong magnetic field B_0 of the pulsar, particles rapidly lose their transverse momenta near the stellar surface and the distribution function is one-dimensional. The magnetic field B_0 is assumed to be dipole and decreasing with an increase of the distance from the centre of the star, obeying the law $B_0 = B_{0s}(r_0/r)^3$, where $r_0 \approx 10^6$ cm is radius of a neutron star and $B_{0s} \approx 10^{12}$ G is the magnetic field strength at the stellar surface.

The pulsar magnetosphere contains an electron–positron plasma penetrated by a high energy, low density relativistic beam. There exist two types of waves in such a plasma (see for example Hardee and Rose 1976; Krasnosel'skikh *et al.* 1985). One of them is a mixed longitudinal–transversal wave, the other a purely electromagnetic t-wave (the dispersion curves of t-waves are represented in Fig. 2).

There are only two possibilties for wave excitation in the homogeneous relativistic electron–positron plasma: one being Cherenkov resonance and the other the result of

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an asymmetry of the distribution function. Estimates show that the growth rate of the beam–plasma instability turns to be $<1/\tau_0$ (Egorenkov *et al.* 1983), where τ_0 is the time of plasma escape from the light cylinder (the surface where the co-rotation velocity is the velocity of light).

The pulsar emission mechanisms provide information on wave excitation and propagation in the magnetosphere plasma, along with their escape outside the light cylinder. After leaving the magnetosphere the waves must propagate in the interstellar medium (vacuum) and reach us as pulsar emission. All this is most natural for purely transverse electromagnetic waves with a dispersion law close to that for waves propagating in vacuum ($\omega = kc$).

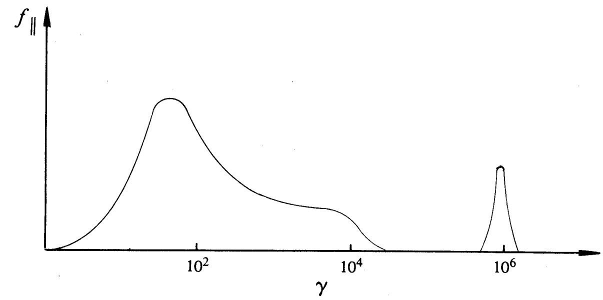

Fig. 1. Distribution function of particle energy.

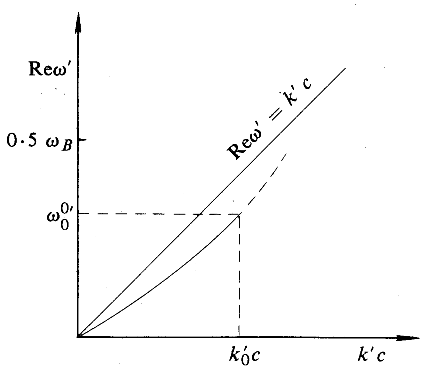

Fig. 2. Spectrum of t-waves in the plasma rest frame.

According to Machabeli and Usov (1979) and Lominadze *et al.* (1983), in the magnetospheres of the young pulsars PSR 0531+21 and PSR 0833−45, the distribution function appears to be unstable relative to the excitation of t-waves and the growth rate of this instability is $\Gamma \gg 1/\tau_0$. However, the rotation of the pulsars decelerates. This changes the plasma parameters and in 10^4 yr the excitation condition for t-waves is broken (Lominadze *et al.* 1983).

Thus, a beam–plasma instability cannot be a source of wave generation in the magnetosphere, while for the second mechanism the excitation of t-waves is possible only for fast rotating pulsars (such as the Crab and Vela). Hence, it is a very important challenge to explain the origin of emission from relatively old pulsars.

This paper deals with the problem of t-wave excitation taking into account the specific motion of the particles in the dipole magnetic field of the pulsar. In the papers by Blandford (1975) and Melrose (1978) the possibility of wave excitation due to the magnetodrift mechanism was discussed. However, in both papers the particle drift across the magnetic field was neglected. It was shown by Zheleznyakov and Shaposhnikov (1979), Ochelkov and Usov (1980) and Chugunov and Shaposhnikov (1987) that, if the drift motion of the particles is taken into account, wave amplification is possible. [Further, Ochelkov and Usov (1980) found the conditions for wave amplification.] The amplification of Langmuir waves due to the curvature of magnetic field lines has been discussed by Shaposhnikov (1981). In the papers mentioned above, the non-resonant mechanism of wave excitation based on summation of the magnetodrift emission of each individual plasma particle moving along the curved lines of the pulsar magnetic field is considered. It is noteworthy that in these papers dispersion and linear polarisation of t-waves are not taken into account and the calculation methods applied do not allow a quantitative estimation of the growth rate of the instability. Here we shall consider two possibilities for t-wave generation: non-resonant and on the Cherenkov resonance.

2. Generation of t-waves in Pulsar Magnetospheres

The magnetic field weakens with the distance from the star and the contribution of the inhomogeneity of the magnetic field in the transverse motion of the particles becomes significant (Machabeli and Usov 1979). Neglecting terms of the order of $p_\perp/p_z \ll 1$, we obtain the smoothed equation for the charged particles in the dipole magnetic field that describes the motion of the guiding centre (Sivukhin 1963):

$$\dot{\boldsymbol{R}} = v_z\,\boldsymbol{h} + \frac{v_z\,c p_z}{R_B\,e B_0}\,\boldsymbol{j}. \tag{1}$$

Here p_z and $p_\perp$ are the particle momentum components along and transverse to the magnetic field, R_B is the curvature radius of the magnetic field line, the z-axis is chosen in the initial moment of time along the magnetic field, and $\boldsymbol{h}$ and $\boldsymbol{j}$ are unit vectors. The vector $\boldsymbol{h}$ is directed along the tangent to the field line and $\boldsymbol{j}$ is perpendicular to the $\boldsymbol{h}$ direction and the plane where the curved field line lies. In the cylindrical system the angular coordinate ϕ is cyclic (p_ϕ = const.). From this and the energy conservation law it follows that, besides drift motion, particles rotate around a Larmor circle, the radius of which is determined by the drift velocity:

$$r_{\mathrm{L}} \approx u_\perp/\tilde{\omega}_B\,, \qquad u_\perp = v_z\,c p_z(e R_B\,B_0)^{-1}\,,$$

$$\tilde{\omega}_B = \frac{\omega_B}{\gamma}\,, \qquad \omega_B = \frac{eB_0}{mc}\,,$$

where $\gamma = \{1 + (p_z^2 + p_\perp^2)/m^2c^2\}^{1/2}$, and c and m are the electron charge and rest mass respectively. Consequently, we obtain

$$\boldsymbol{k}.\boldsymbol{R} = \boldsymbol{k}.z + v_z \int_0^t \boldsymbol{k}.\boldsymbol{h}\,\mathrm{d}t' + k_\perp\,u_\perp\,t$$

$$+ \frac{k_\perp\,u_\perp}{\tilde{\omega}_B}\{\sin(\tilde{\omega}_B\,t - \beta) - \sin\beta\}\,, \tag{2}$$

where β is the initial phase. Equation (2) relates the coordinates $\boldsymbol{R}$ at the present time with the coordinates r at the initial time. At the chosen initial moment of time, the vector $\boldsymbol{h}$ forms an arbitrary angle θ with the z-axis. At time t the z component of the $\boldsymbol{h}$ vector is expressed by

$$h_z = \cos(\tilde{\omega}^* t + \theta), \tag{3}$$

where $\tilde{\omega}^* = \omega^*/\gamma$ and $\omega^* = p_z/mR_B$. All calculations, except where specified otherwise, are made in the observer's frame of reference.

Let us choose the x-axis along $k_\perp$ ($k^2 = k_\perp^2 + k_z^2$). Using expressions (2) and (3) and the method of integration along the particle trajectory, we obtain the dispersion relation for t-waves propagating in the dipole magnetic field of a pulsar

$$N^2 = \mathrm{Re}\,\epsilon_{22} + \mathrm{i}\,\mathrm{Im}\,\epsilon_{22}; \qquad N^2 \equiv k^2c^2/\omega^2. \tag{4}$$

For the real part of the dielectric constant ϵ, the curvature has some influence, so that

$$\mathrm{Re}\,\epsilon_{22} = 1 - \mathrm{Re}\sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2} \int \frac{(\omega - k_z v_z) f_{\alpha\|}\, \mathrm{d}p_z}{\gamma(\omega - k_z v_z - \tilde{\omega}_{B\alpha})}, \tag{5}$$

but for the imaginary part we get

$$\mathrm{Im}\,\epsilon_{22} = \mathrm{Im}\sum_\alpha \frac{2\omega_{\mathrm{p}\alpha}^2}{\omega^2} \sum_{s=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \int \mathrm{d}p_z \frac{J_l^2(k_z R_B)}{\gamma(\nu_s - l)} \frac{\omega_{B\alpha}}{\omega^*}$$

$$\times \left(\{ sJ_s'^2(b) + sJ_s'(b)\, J_s''(b) k_\perp c p_{\perp 0}/\omega_{B\alpha} \} f_{\alpha\|} \right.$$

$$\left. - \frac{k_z c}{\omega_{B\alpha}} p_{\perp 0} J_s'^2(b) \frac{\partial f_{\alpha\|}}{\partial p_z} \right). \tag{6}$$

Here $J_s(b)$, $J_l(k_z R_B)$ and $J_s'(b)$ are Bessel functions and their derivatives with respect to the argument, while $b \equiv k_\perp p_{\perp 0}/m\omega_B$, $p_{\perp 0} = m\gamma u_1$, $\nu_s = (\omega - s\tilde{\omega}_B)/\tilde{\omega}^*$ and $\omega_{p\alpha}^2 = 4\pi e_\alpha^2 n_\alpha/m$. The sum over α is over particle types, n_α is the particle density and e_α the particle charge. The distribution function is chosen as

$$f_\alpha = (1/p_{\perp 0})\delta(p_\perp - p_{\perp 0}) f_{\alpha\|}; \qquad \int f_\alpha\, p_\perp\, \mathrm{d}p_\perp = f_{\alpha\|}.$$

Let us rewrite ω in the form $\omega = \omega_0 + \mathrm{i}\,\Gamma$, where ω_0 and Γ are the real and imaginary parts of ω respectively. The value of $\Gamma/\tilde{\omega}^* \equiv \Delta$ can be very large i.e. $\Gamma/\tilde{\omega}^* \geqslant 1$. In this case it is impossible to limit oneself with one term in the sum taken over l; this means that the resonance is smeared. Let us estimate the infinite sum over l in (6). For the sum over s we take only the terms $s = 0, \pm 1$, because the argument of the Bessel function $J_s(b)$ is small ($b \ll 1$). We use the well-known formula

$$\sum_{l=-\infty}^{+\infty} \frac{J_l^2(k_z R_B)}{\nu - l} = \frac{\pi}{\sin \nu\pi} J_\nu(k_z R_B)\, J_\nu(k_z R_B), \tag{7}$$

and the Shlefli representation of Bessel functions

$$J_{\pm\nu}(x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \exp(\pm \mathrm{i}\,\nu\phi - \mathrm{i}\,x\sin\phi)\,\mathrm{d}\phi$$
$$\mp \frac{\sin\nu\pi}{\pi}\int_{0}^{\infty} \exp(\pm\nu\phi - x\sinh\phi)\,\mathrm{d}\phi\,. \tag{8}$$

Note that $\mathrm{Re}\,\nu \gg 1$ and $x = k_z R_B \gg 1$. We see that the first integral in (8) contains a rapidly oscillating function and is different from zero (and of the order of unity) only at the point $\phi = 0$. There is a multiplier $\exp(-x\sinh\phi)$ in the second integral. For this reason the value of the integral is significant only for small values of ϕ, and the absolute value of the integral is of order unity. Supposing $\nu = q + \mathrm{i}\,\Delta$ (where q is a large integer) and putting the real and imaginary parts of (4) equal to zero, each taken separately after some transformations, we obtain

$$\Delta = (-1)^{q+1} A \sinh(\pi\Delta)\,. \tag{9}$$

We are interested only in the order of magnitude of Δ and for simplicity we assume that $f_{\alpha\|} = \delta(p_z - p^{\alpha}_{z0})$, where p^{α}_{z0} is the average momentum of particles of type $\alpha = \mathrm{p,b}$. Then, we get

$$A = \omega_0^{-2}\left(\frac{\omega_{\mathrm{b}}^2\,\omega_B\,R_B}{2\gamma_{\mathrm{b}}^2\,c}\{\cos(\omega_B R_B/\gamma_{\mathrm{b}}\,c) + \sin(\omega_B R_B/\gamma_{\mathrm{b}}\,c)\}\right.$$
$$\left. -\,\omega_{\mathrm{p}}^2(k_z R_B)(u_\perp^{\mathrm{p}}/c)^2\{\cos(\omega_B R_B/\gamma_{\mathrm{p}}\,c) + \sin(\omega_B R_B/\gamma_{\mathrm{p}}\,c)\}\right),$$

where $\omega_{\mathrm{b}}^2 = 4\pi e^2 n_{\mathrm{b}}/m$ and $\omega_{\mathrm{p}}^2 = 8\pi e^2 n_{\mathrm{p}}/m$, while $n_{\mathrm{b}}, \gamma_{\mathrm{b}}$ and $n_{\mathrm{p}}, \gamma_{\mathrm{p}}$ are the density and average Lorentz factors of beam and plasma particles respectively, and where $n_{\mathrm{b}}\gamma_{\mathrm{b}} \approx n_{\mathrm{p}}\gamma_{\mathrm{p}}$ and $u_\perp^{\mathrm{p}} = \gamma_{\mathrm{p}}\,c^2/R_B\,\omega_B$. If $u_\perp^{\mathrm{p}} \to 0$ in the expression for A, the contribution of plasma electrons is fully compensated for by positrons, as it is proportional to the odd power of electron charge, in particular $\sim e_\alpha^3$. The beam consists only of primary electrons detached from the stellar surface and, therefore, there is no compensation for its contribution. It follows from (9) that for radio frequencies the value Δ is of the order of several units. By definition we have

$$\Gamma = \tilde{\omega}^* \Delta \equiv (c/R_B)\Delta\,. \tag{10}$$

In the wave generation regions we have $R_B > r$, where r is the distance from the centre of the pulsar to the place of generation. Thus, we have $\Gamma < (c/r)\Delta \approx \Delta/\tau_0$, even if the growth rate Γ surpasses $1/\tau_0$only insignificantly. Hence, we conclude that the magnetodrift non-resonant mechanism cannot be a source of wave generation.

Let us consider the resonant case. In the dispersion relation (6) the argument of the Bessel function $J_l(k_z R_B)$ is large ($k_z R_B \gg 1$) and the value of $J_l(k_z R_B)$ peaks when $l_0 \approx k_z R_B$ [in this case the asymptotic expansion of the Bessel function for large arguments and indices can be used, i.e. $J_{l_0} \sim (1/l_0)^{1/3}$]. Comparing this value with asymptotic expansions of $J_l(k_z R_B)$ when $l \to \infty$ and $k_z R_B \to \infty$ [valid if $|l - k_z R_B| \gg (k_z R_B)^{1/3}$], it is easy to show that the value of the Bessel function is significantly smaller than $l_0^{-1/3}$ (Watson 1958). Hence, the contribution of the corresponding terms in the sum over l is negligibly small. For the main terms of the sum we have $|l - k_z R_B| \lesssim (k_z R_B)^{1/3}$ and, as $k_z R_B \approx \nu_0$, provided that $\Delta \gg l_0^{1/3}$ in the denominator of this term we change $\nu_0 - l_0 + \mathrm{i}\,\Delta$ to $\mathrm{i}\,\Delta$ (the resonant case). Thus, when the condition $\Delta \gg \nu_0^{1/3}$ is met in the dispersion relation, the curved motion of the particles along the magnetic field is negligible, but the drift across B_0 must be considered. Hence, from the dispersion relation (4) it follows that

$$\Gamma = \operatorname{Im}\omega \approx -\operatorname{Im}\frac{\omega_{\mathrm{p}}^2}{\omega_0}\int \mathrm{d}p_z\Bigg\{\sum_{s=-\infty}^{+\infty}\left(\frac{s^2}{b^2}-1\right)\{sJ_s^2(b) - J_s(b)\,J_{s+1}(b)\,b\}$$

$$\times\frac{s\omega_B}{\gamma^2}\left(\frac{f_{\|}}{\omega_0 - k_z v_z - k_\perp u_\perp + s\tilde{\omega}_B} - \frac{f_{\|}}{\omega_0 - k_z v_z - k_\perp u_\perp - s\tilde{\omega}_B}\right)$$

$$+(1/\gamma^2)\frac{{p_{\perp 0}}^2}{m^2c^2}J_0'^2(b)\frac{k_z v_z}{\omega_0 - k_z v_z - k_\perp u_\perp}\frac{\partial f_{\|}}{\partial p_z}\Bigg\}, \tag{11}$$

$$\omega_0 \approx kc\left(1 - 2\frac{\omega_{\mathrm{p}}^2}{\omega_B^2}\bar{\gamma}\right); \qquad \bar{\gamma} = \int_{-\infty}^{+\infty}\gamma f_{\|}\,\mathrm{d}p_z. \tag{12}$$

From the first two terms in the second line of (11), it follows that for obtaining wave generation it is necessary to have more particles satisfying the resonance condition $\omega_0 - \boldsymbol{k}.\boldsymbol{v} + \tilde{\omega}_B = 0$ than particles satisfying the condition $\omega_0 - \boldsymbol{k}.\boldsymbol{v} - \tilde{\omega}_B = 0$. In the reference frame where the main mass of plasma is at rest the resonance condition $\omega - \boldsymbol{k}.\boldsymbol{v} + \tilde{\omega}_B = 0$ is fulfilled only for particles with positive momenta (moving from the pulsar to the light cylinder), and damping occurs for particles with negative momenta. According to Machbeli and Usov (1979), wave generation occurs at certain conditions which are only fulfilled for young pulsars. For the pulsars considerd by us, these two terms can only give damping. In the low frequency (radio) range the cyclotron resonance condition is not satisfied because there are no particles with negative momenta of corresponding values. Thus, the term in the third line of equation (11) has maximum value. It should be noted that the beam part of the distribution function has a sharp maximum and its derivative has a large value. It is clear that the result depends on the type of the distribution function. If we assume $f_{\|}$ to be a delta function (hydrodynamic instability), the growth rate is

$$\frac{\Gamma}{\omega_0} \approx \frac{\omega_{\mathrm{b}}}{\omega_B}\left(\frac{u_\perp}{c}\right)^2\frac{k_\perp}{k},$$

and for the characteristic parameters of a plasma in a typical pulsar magnetosphere this has a negligibly small value. Calculating the growth rate of a hydrodynamic instability we assumed that the beam is monoenergetic. However, the electron beam is formed by the particles that are detached from the pulsar surface by the electric field parallel to B_0. This field accelerates particles to relativistic velocities. Thus, it can be assumed that the beam distribution function has a Maxwellian spread in momenta. Presumably the spread is defined by the energy expended by the electrons leaving the stellar surface, and is equal to $T \sim 1$ keV (Flowers *et al.* 1977). Indeed, the surface of a neutron star cannot contain particles with energies higher than T, so that $0 < \frac{1}{2}mv^2 < T$. The distribution function of beam particles $f_{\|}$ in the observer's frame may be given by

$$f_{\|} = \frac{1}{\pi^{\frac{1}{2}}}\eta \exp\left\{-\left(\frac{\eta^{\frac{1}{2}}}{mc}(p-p_{\mathrm{b}})\right)^2\right\}; \qquad \eta = \frac{mc^2}{T} \approx 10^2 .$$

Note that according to Gurevich and Istomin (1985), the monoenergetic beam after leaving the stellar surface undergoes a spread in momenta due to scattering and η has the above value. From (11), in the case $\partial f_{\|}/\partial p_z > 0$, we get

$$\frac{\Gamma}{\omega_0} \approx \left(\frac{k_{\perp}}{k}\right)^2\left(\frac{\omega_{\mathrm{p}}}{\omega_B}\right)^2 \frac{n_{\mathrm{b}}}{n_{\mathrm{p}}}\gamma_{\mathrm{b}}^{-2}\eta^2\left(\frac{p_{\perp 0}}{mc}\right)^4 . \tag{13}$$

We note that the growth rate (13) is obtained in the approximation $\Gamma/\omega_0 \ll 1$. Substituting the numerical values of the typical pulsar parameters into (13), we get $k_{\perp}/k \ll 1$ and $k \approx k_z + \frac{1}{2}(k_{\perp}/k_z)k_{\perp}$.

Let us consider now how the resonance conditions necessary for the development of kinetic instability

$$\omega_0 - k_z v_z - k_{\perp} u_{\perp} = 0 \tag{14}$$

are fulfilled. Using expression (12) we obtain

$$\left(\frac{k_{\perp}}{k}\right)^2 \approx 4\left(\frac{\omega_{\mathrm{p}}}{\omega_B}\right)^2 \bar{\gamma} . \tag{15}$$

From (15) it follows that for a typical pulsar $k_{\perp}/k_z \sim 10^{-2}$–$10^{-3}$ and $(\Gamma/\omega_0) \sim 10^{-2}$. For the parameters of young pulsars, particularly for PSR 0531+21, the growth rate appears to be rather small and the instability has no time to develop.

3. Pulsar Emission Model

Let us determine the region of the magnetosphere where emission can be generated. For this purpose we recall the relation between the curvature radius R_B and the distance to the magnetic axis of the pulsar (Manchester and Taylor 1977):

$$R_B \approx 4r^2/3a , \qquad 0 \leqslant a \leqslant (\Omega r/c)^{\frac{1}{2}} .$$

Using $B_0 = B_{0\mathrm{s}}(r_0/r)^3$ and $n = n_0(r_0/r)^3$ we get

$$\Gamma \approx 10^5\left(\frac{r}{r_{\mathrm{c}}}\right)^{10}\left(\frac{a}{r_{\mathrm{c}}}\right)^4 \omega_0 , \tag{16}$$

where $r_{\mathrm{c}} \equiv c/\Omega \approx 5\times10^9$ cm. The emission frequency ω_0 in the observer's frame is connected to the frequency ω_0' in moving frame by

$$\omega_0 = \frac{\omega_0'(1-v^2/c^2)^{\frac{1}{2}}}{1-(v/c)\cos\theta} ,$$

where $\theta \approx k_\perp/k \ll 1$ is the angle in observer's frame and is determined by expression (15). Changing $\cos\theta \approx 1-\frac{1}{2}\theta^2$ and $v/c = 1-1/2\gamma_p^2$ and taking the maximum value of the frequency in the plasma rest frame $\omega_0^{0\prime} \approx \alpha\omega_B = \alpha\omega_{B_0}(r_0/r)^3$ ($\alpha < 1$) (see Fig. 2), we obtain

$$\omega_0^0 = 10^7 (r/r_c)^{-6}. \tag{17}$$

Substituting (17) into (16) we get

$$\Gamma \approx 10^{12}\left(\frac{r}{r_c}\right)^4 \left(\frac{a}{r_c}\right)^4. \tag{18}$$

For the existence of kinetic instability, fulfillment of the condition $\Gamma/\omega_0 \ll 1$ is necessary. It gives

$$\Gamma \ll 10^7 (r/r_c)^{-6}. \tag{19}$$

On the other hand, for the development of the instability the angle L/R ($L \gg c/\Gamma$) between the field line and original direction (beginning from the moment of wave generation) of wave vector $\boldsymbol{k}$ must not surpass the angle $\theta \approx 2(\omega_p/\omega_B)\bar{\gamma}^{1/2}$ [in the reverse case the resonance condition (15) is broken]. Hence, we have

$$\Gamma \gg c/R_B\theta = 10\,\frac{a}{r_c}\left(\frac{r}{r_c}\right)^{-\frac{7}{2}}. \tag{20}$$

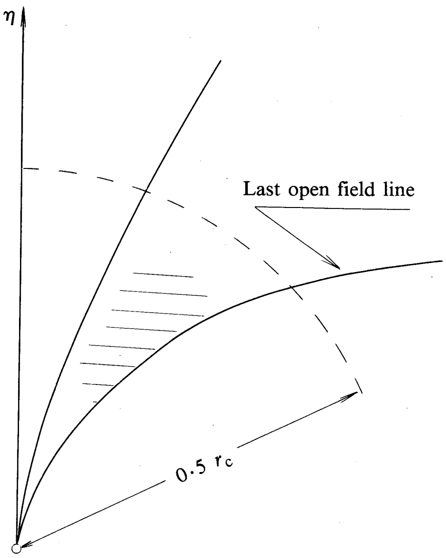

Fig. 3. Wave generation region in the magnetosphere (dashed region). The curves denote magnetic field lines and η is the magnetic dipole axis.

Using (18), the inequalities (19) and (20), and bearing in mind that

$$0 \leqslant a/r_c \leqslant (r/r_c)^{\frac{3}{2}},$$

the emission generation region in the pulsar magnetosphere can be found: it occurs at distances of the order of 10^9 cm (see Fig. 3). As shown, the generation region is very narrow compared with the magnetosphere dimensions. Before the waves reach the observer they must propagate through the pulsar magnetosphere. So, let us consider wave propagation through the magnetosphere plasma.

The spectrum of t-waves in the electron–positron plasma is given in Fig. 2 [equation (12) is valid in the case $\omega \ll \omega_B$]. It can be seen that above the point $k = k_0$ the curve $\omega = \omega(k)$ moves upwards and for $k > k_0$ the group velocity is greater than the velocity of light. Hence, waves with $k > k_0$ will not propagate (Lominadze *et al.* 1986). Calculation of the dispersion relation shows that the frequency ω_0^0 corresponding to the wave vector $k = k_0$ appears to be $\omega_0^0 \approx \alpha\omega_{B_0}(r_0/r)^3$ ($\alpha < 1$). For example, if we have a power-law distribution function with index $-3/2$, then $\alpha \approx 0\cdot3$. The waves excited in the inner regions of the magnetosphere on the way from the pulsar to the light cylinder reach regions with lower values of magnetic field and density. In these regions the frequency of the waves is higher than the maximum possible value of the t-waves and reflection takes place (see Fig. 4). However, if the waves reach, without reflection, the magnetosphere boundary where the density abruptly falls and the magnetic field remains practically unchanged, the ratio $(\omega_p^2/\omega_B^2)\bar{\gamma}$ goes to zero and the t-wave spectrum [i.e. $\omega \approx kc\{1-2(\omega_p^2/\omega_B^2)\bar{\gamma}\}$] goes close to the spectrum of vacuum waves. So, the waves can propagate in the interstellar medium.

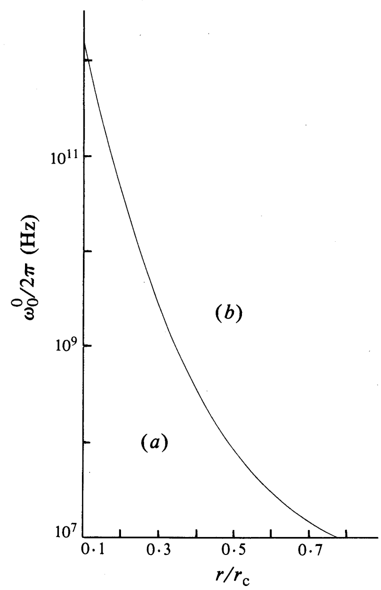

Fig. 4. Dependence of ω_0^0 on the distance from the stellar surface for (*a*) the transparency region and (*b*) the non-transparency region.

According to (19) and (20) wave generation is possible only at distances of the order of 10^9 cm. It is obvious that for the observer the generated frequencies (equation 17) will be in the radio range. It follows from Fig. 4 that if wave generation occurs at distances $r = 10^9$ cm, high frequencies of order 10^{10}–10^{11} Hz must be cut off. Indeed, according to recent papers (see for example Izvekova *et al.* 1985), for typical pulsars a high frequency cutoff is observed.

Thus, the proposed theory of wave generation in pulsar magnetosphere brings us to the following emission model: at distances $r \approx 10^9$ cm, in rather narrow regions, wave excitation takes place. The wave vectors form a hollow cone with the spread angle $(k_1/k) \sim 10^{-2}$–10^{-3} in the observer's frame (see Fig. 5). With this cone our model is similar to the model by Ruderman and Sutherland (1975); however, this is the only resemblance.

The model represented in Fig. 5 can be considered satisfactory only if it explains the basic laws of pulsar radio emission. However, such an explanation is not the purpose of the present paper, but rather as an illustration of the possibility of a model. Let us present an interpretation of some observational data for the pulsars with two subpulses.

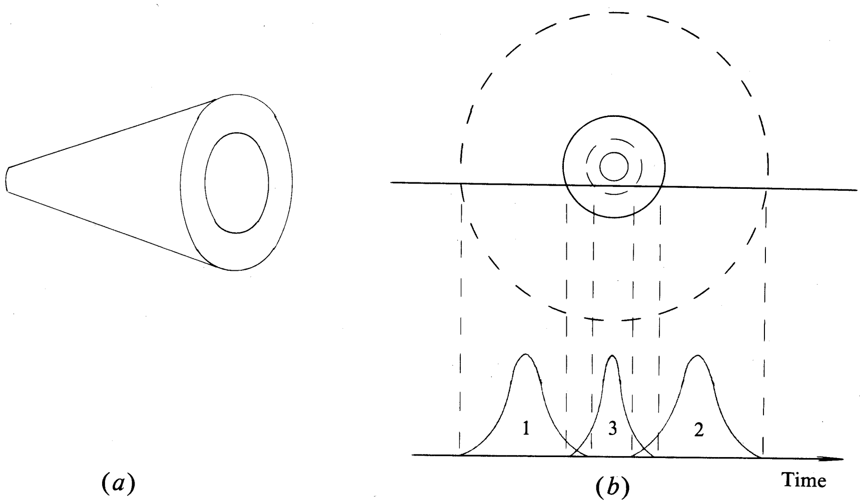

Fig. 5. (*a*) The emission cone. (*b*) The cross section of the cone. The continuous circle designates the high frequency radio emission region, and the dashed circle designates the low frequency region. The straight line corresponds to the observer's path, while 1 and 2 denote the integrated pulse profile in the low frequency range and 3 the high frequency range.

It is clear from Fig. 4 that low frequencies are generated over greater distances than for high frequencies. So, the spread angle for low frequencies is wider than for high frequencies. In Fig. 5*b* the emission cone cross section is shown, where the continuous circle designates the region in which high frequency emission is generated, and the dashed circle designates the region of low frequency emission. The straight line corresponds to the observer's path (the direction to the observer) as the pulsar rotates. It is obvious that we have two subpulses at low frequencies (segments 1 and 2) and one at high frequencies (segment 3).

Using the expressions for Γ and ω_0^0 we get

$$\Gamma/\omega_0^0 \propto (r/r_0)^{10}.$$

So, from the condition $\Gamma/\omega_0 \ll 1$ we conclude that waves are excited only in the height interval from $\sim 10^9$ to $\sim 1{\cdot}3\times 10^9$ cm. Consequently, the longitudinal dimensions of the wave generation region can be estimated as $L_0 \sim 3\times 10^8$ cm, and the subpulse width as $\delta \sim L_0/R_B \approx 0{\cdot}3$ rad $\approx 10°$. The latter is in good accord with observational data (see for example Manchester and Taylor 1977; Smith 1977; Kuzmin and Solovev 1986).

Note that t-waves in the electron–positron plasma have strong linear polarisation. The vector $\boldsymbol{E}$ is perpendicular to the plane containing the vectors $\boldsymbol{k}$ and $\boldsymbol{B}$. In the narrow region of wave generation the field lines of $\boldsymbol{B}$ are almost parallel, whereas the $\boldsymbol{k}$ vectors form a hollow cone. Thus, in going around the circle of the cross section of the hollow cone, the position angle must vary from 0° to 180° in the opposite points. The probability that the observer's path will intersect the cone cross section along the diameter is small. But as shown in Fig. 5*b*, the position angle will change significantly from one subpulse to the other, a result confirmed by observations.

4. Conclusions

In the framework of this theory we obtain the following results:

(1) The existence of subpulses is easily explained.

(2) The subpulse width estimated from the theory is in good agreement with observational data.

(3) It is evident from Fig. 5*b* why at low frequencies we obtained a double-humped pulse envelope, whereas there is a single-humped pulse at high frequencies.

(4) When a double-humped pulse at high frequencies is observed, if the line of intersection of the observer's path with the emission cone cross section is slightly changed, it is easy to see why the width between subpulses at low frequencies is greater than at high frequencies.

(5) Reaching the observer as radio emission, t-waves determine the observed high linear polarisation of subpulses.

(6) The observed variation of position angle through the pulse is explained.

(7) It is clear from Fig. 4 that pulsar radio emission must have a high frequency cutoff.

The present paper does not claim to explain all properties of pulsars. However, a more sophisticated version of the idealised picture suggested by the present model can serve as a foundation for the description and further investigation of other pulsar emission properties.

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