Aust. J. Phys., 1990, **43**, 65–75

Characteristics of the Solitary Waves in Multicomponent Plasmas

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Abstract

We have investigated ion–acoustic solitary waves in a multicomponent plasma. The study shows an interaction of the negatively charged particles with the solitary waves, due to which interesting physical behaviour of the solitary waves is observed. Moreover, the isothermality and the non-isothermality of the plasma exhibit different soliton-type solutions and a transition of the soliton's behaviour can be shown through the changes of the non-isothermality in the plasma.

1. Introduction

The study of ion–acoustic solitary waves in plasmas arising as a result of the Korteweg–de Vries (KdV) equation has lead to much interest in the last few decades. Washimi and Taniuti (1966) derived the KdV equation in a simple plasma with the assumption of a small amplitude approximation. Later, extensive theoretical studies revealed much of the new physics and facts on the existence and behaviour of the solitary waves in various plasma models by many authors (Kellog 1964; Jeffrey and Kakutani 1972), while some authors (Ikezi *et al.* 1970; Ikezi 1973) experimentally investigated the properties of ion–acoustic solitary waves in plasmas. With knowledge of the nature of solitary waves in simple plasmas, Watanabe (1984) later extended the plasma studies with several ionic species with a view to showing the interaction of solitons with multiple ions. A great number of investigations of solitary waves in plasmas has lead, over the years, to diverse applications of the KdV equation. Lonngren (1983) compiled works on the solitary wave behaviour existing in different plasmas obtained theoretically and experimentally, and discussed the essence of using the equation in a broad class of weakly nonlinear dispersive media. Gabl and Lonngren (1984) have investigated the similar properties of solitary waves in plasmas with multiple ions. However, earlier works dealing with multicomponent plasmas ignored completely the possible interaction of the solitary waves in plasmas having both kinds of ions and multiple electrons. Jones *et al.* (1975) were probably the first to consider the effect of multiple electrons in plasmas and derive the KdV equation to show that even a small fraction of multiple electrons might cause a detectable change in the properties of solitons. Goswami and Buti (1976) and Patraya and Chegelishvilli (1977)

0004-9506/90/010065$03.00

continued to emphasise the physics of solitary waves in plasmas with multiple electrons. The recent study by Das *et al.* (1986) has shown that multiple electrons in warm non-isothermal plasma have a significant effect on the characteristic features of the solitary waves. More recently Das and Karmakar (1988) have analysed the behaviour of the solitary wave in the presence of multicomponent and multiple non-isothermal electrons with negative ions in plasmas. They showed that the reduction of non-isothermality of the plasma leads to an interesting interaction of electrons, which casts doubt on the existence of solitons in plasmas. Although we do not claim priority on the procedure of studying solitary waves in plasmas, and the investigation is very much a sequel to earlier work (Das *et al.* 1986; Das and Karmakar 1988), we believe that our study of the development of the present problem will give some insight into the characteristics of solitary waves in plasmas. In the present analysis we observe the features of the solitons in the presence of multicomponent plasma and multiple non-isothermal electrons and negative ions. The study also shows that consideration of a generalised multicomponent plasma exhibits various fascinating characteristics of the solitons which differ from the features obtained in simple models. The characteristics of the solitons, as expected, are observed to be different due to the interaction of the solitons with negative ionic species.

2. Mathematical Formulation

We consider a plasma consisting of both kinds of ions having mass m_α moving with velocity v_α, temperature T_α and density n_α together with multiple electrons where $\alpha = i, j$; here i stands for positive ions and j stands for negative ions. Further, the plasma contains two types of electrons at high and low temperatures and with $T_{\mathrm{el}}, T_{\mathrm{eh}} \gg T_\alpha$.

Following Das *et al.* (1986), the basic equations describing plasma dynamics in a normalised one-dimensional form are

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial (n_\alpha v_\alpha)}{\partial x} = 0, \tag{1}$$

$$n_\alpha\left(\frac{\partial v_\alpha}{\partial t} + v_\alpha \frac{\partial v_\alpha}{\partial x}\right) + \frac{\theta_\alpha}{\mu_\alpha}\frac{\partial p_\alpha}{\partial x} = \frac{q_\alpha}{\mu_\alpha}\frac{\partial \phi}{\partial x}. \tag{2}$$

The basic equations are supplemented by the following equations of state and Poisson's equation:

$$\frac{\partial p_\alpha}{\partial t} + v_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial v_\alpha}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\mathrm{el}} + n_{\mathrm{eh}} + \sum_{j=1}^{m} n_j - \sum_{i=l}^{n} n_i, \tag{4}$$

where n_{el}, n_{eh} are the electron densities, $q_\alpha = 1$ for $\alpha = i$, $q_\alpha = -1$ for $\alpha = j$ and $\theta_\alpha = T_\alpha / T_{\mathrm{ef}}$, and where $T_{\mathrm{ef}} = T_{\mathrm{el}}\, T_{\mathrm{eh}} / (\nu T_{\mathrm{el}} + \mu T_{\mathrm{eh}})$, with $\mu_\alpha = m_\alpha / m_i$.

The boundary conditions appropriate to the model considered for studying the ion–acoustic waves at $|x| \to \infty$ are

$$
\begin{aligned}
&\text{(i) } n_{\mathrm{el}} \to \mu,\ n_{\mathrm{eh}} \to \nu, \text{ where } \mu \text{ and } \nu \text{ are initial densities of low and high temperature electrons;} \\
&\text{(ii) } n_\alpha = n_\alpha^{(0)}; \\
&\text{(iii) } v_\alpha^{(0)} \to 0; \\
&\text{(iv) moreover, the overall charge neutrality condition is always maintained in the plasma and takes the form } \mu+\nu = \textstyle\sum n v_i^{(0)} - \sum n_j^{(0)}.
\end{aligned}
\tag{4a}
$$

For further specification of the plasma model, we first consider the non-isothermality of plasma augmented through the electron densities n_{el} and n_{eh} as (Schamel 1973)

$$
\begin{aligned}
n_{\mathrm{el}} &= \mu\left\{1 + \frac{\phi}{\mu+\nu\beta} - \frac{4}{3}b_{\mathrm{l}}\left(\frac{\phi}{\mu+\nu\beta}\right)^{3/2} + \frac{1}{2}\left(\frac{\phi}{\mu+\nu\beta}\right)^{2} + \dots\right\}, \\
n_{\mathrm{eh}} &= \nu\left\{1 + \frac{\beta\phi}{\mu+\nu\beta} - \frac{4}{3}b_{\mathrm{h}}\left(\frac{\beta\phi}{\mu+\nu\beta}\right)^{3/2} + \frac{1}{2}\left(\frac{\beta\phi}{\mu+\nu\beta}\right)^{2} + \dots\right\},
\end{aligned}
\tag{5}
$$

where $b_{\mathrm{l,h}}$ are the arbitrary non-isothermal parameters defined through the electron temperatures as $b_{\mathrm{l,h}} = (1-\beta_{\mathrm{l,h}})/\pi^{1/2}$ and where $\beta_{\mathrm{l,h}} = T_{\mathrm{el,h}}/T_{\mathrm{ef}}$ and $\beta = T_{\mathrm{el}}/T_{\mathrm{eh}}$. In order to derive the desired KdV equation, we assume the following stretched coordinates:

$$
\xi = \epsilon^{1/4}(x-\lambda t), \qquad \psi = \epsilon^{3/4} t, \tag{6}
$$

and the plasma parameters are expanded as

$$
\begin{aligned}
n_\alpha &= n_\alpha^{(0)} + \epsilon n_\alpha^{(1)} + \epsilon^{3/2} n_\alpha^{(2)} + \epsilon^2 n_\alpha^{(3)} + \dots, \\
v_\alpha &= \epsilon v_\alpha^{(1)} + \epsilon^{3/2} v_\alpha^{(2)} + \epsilon^2 v_\alpha^{(3)} + \dots, \\
\phi &= \epsilon \phi^{(1)} + \epsilon^{3/2}\phi^{(2)} + \epsilon^2 \phi^{(3)} + \dots, \\
p_\alpha &= 1 + \epsilon p_\alpha^{(1)} + \epsilon^{3/2} p_\alpha^{(2)} + \epsilon^2 p_\alpha^{(3)} + \dots,
\end{aligned}
\tag{7}
$$

where λ is an unknown phase velocity to be determined later.

The system of equations (1)–(4) together with (5)–(7) leads to a solution for λ in biquadratic form as

$$
\begin{aligned}
&\lambda^4 - \left\{\left(n_i^{(0)} + \frac{n_j^{(0)}}{\mu_j}\right) + 3\left(\frac{\theta_i}{n_i^{(0)}} + \frac{\theta_j}{\mu_j n_j^{(0)}}\right)\right\}\lambda^2 \\
&\quad + \frac{3}{\mu_j n_i^{(0)} n_j^{(0)}}\left\{3\theta_i\theta_j + \theta_j n_i^{(0)2} + \theta_i n_j^{(0)2}\right\} = 0.
\end{aligned}
\tag{8}
$$

The inclusion of thermal effects of the ionic species in the plasma gives rise to multiple ion–acoustic modes propagating with different phase velocities and, consequently, there will be multiple ion–acoustic solitary waves. Following the usual procedure the modified KdV equation describing the ion–acoustic wave is obtained in the form

$$R\frac{\partial \phi^{(1)}}{\partial \psi} + Q(\phi^{(1)})^{1/2}\frac{\partial \phi^{(1)}}{\partial \xi} + P\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{9}$$

where

$$\begin{aligned} P &= \lambda/2, \\ Q &= \lambda(b_1\mu + b_h\nu\beta^{3/2})/(\mu+\nu\beta)^{3/2}, \\ R &= 1 + \frac{3\theta_i}{(\lambda^2 - 3\theta_i/n_i^{(0)})^2} + \frac{3\theta_j}{\mu_j^2(\lambda^2 - 3\theta_j/\mu_j n_j^{(0)})^2}, \end{aligned} \tag{10}$$

and the solitary wave solution takes the form

$$\phi^{(1)} = \phi^{(0)}\mathrm{sech}^4(\chi/2\delta_1), \tag{11}$$

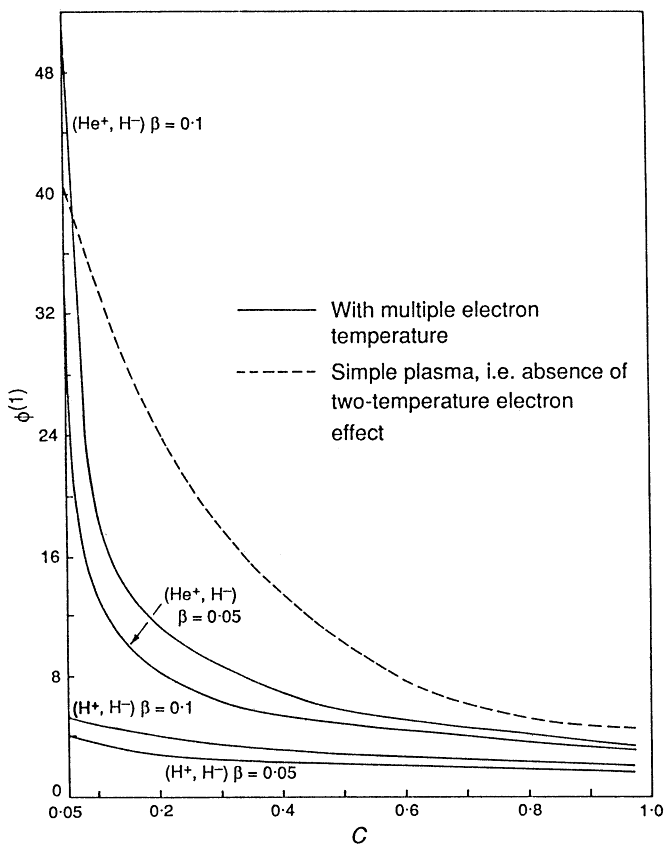

Fig. 1. Solitary wave amplitude $\phi^{(1)}$ against concentration of negative ions C for two values of the electron temperature ratio β and for the ionic species (He^+, H^-) and (H^+, H^-).

where $\phi^{(0)} = 225R^2U^2/64Q^2$ is the amplitude of the solitary wave and the corresponding width is $\delta_1 = 2\ (P/RU)^{1/2}$.

3. Discussion

The expressions for amplitude and width of the solitary wave show that the negative ions and the multiple electrons change the existence properties of the solitons, compared with solitons observed in a simple plasma. The changes in the features of the solitary waves have been calculated numerically and exhibit a significant interaction caused by the negative charged particles. Fig. 1 shows the variation of solitary wave amplitude with negative ion concentration C. We have considered the plasma with multiple ions (He^+, H^-) and (H^+, H^-). The amplitude decreases with an increase in the relative concentrations of the negative ions. With an increase of negative ion concentration the amplitude decreases and ultimately becomes independent of the variation of negative ion concentration. Similar behaviour is observed with varying ionic masses. Again, Fig. 1 shows that the presence of a small percentage of negative ion concentration gives a larger variation of the solition amplitude compared with a high concentration of negative ions. Thus, in order to get the salient features of the solitons arising due to negative ions in a laboratory plasma, one has to

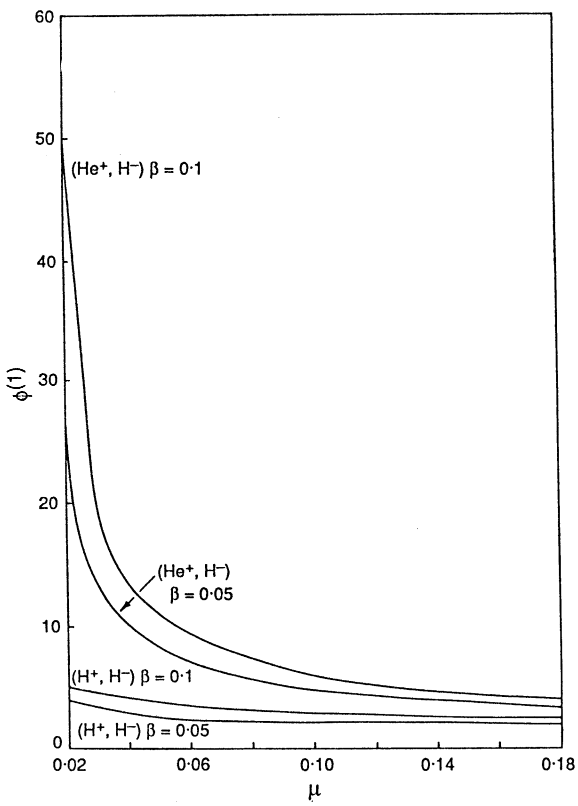

Fig. 2. Solitary wave amplitude $\phi^{(1)}$ against the initial low-temperature electron density μ.

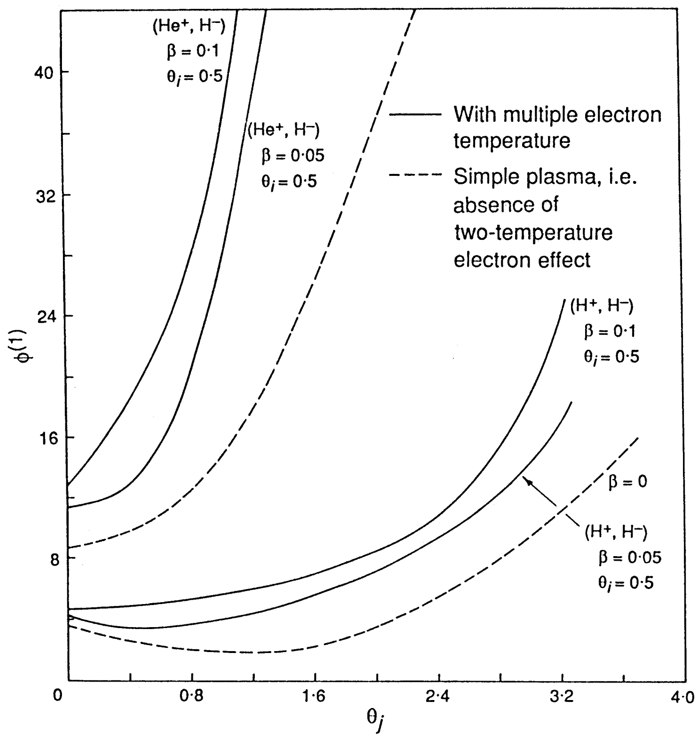

Fig. 3. Solitary wave amplitude $\phi^{(1)}$ against negative ion temperature θ_j for fixed positive ion temperature.

focus attention on observing soliton behaviour in the low range of negative ion concentration. However, in the case of a plasma with many more positive ions (but without negative ions) the amplitude variation can never be large. Fig. 1 shows the effect of the variation of β; the electron temperature does not produce any marked feature in the solitary waves. Consequently one concludes that the multiple electrons have little effect if the low and high temperature ratio of electrons is very small (e.g. $\beta \approx 0 \cdot 05$). The variation of β does not show a large effect on the amplitude variation and it leads to a similar effect on the amplitudes as obtained in the case of a plasma with negative ions in isolation. The increase of the negative ion concentration C for the typical values $\beta = 0 \cdot 05$ and $0 \cdot 1$ produces more effective variation of the amplitude especially in the low ranges of negative ion concentration. But with the increase of negative ion concentration, the amplitude variation slowly becomes independent of negative ion concentration even though the variation of β and the ionic masses are added. Moreover, with the variation of β, the schematic variation of the amplitude is qualitatively similar to that earlier. Such salient features are observed in the plasma whenever the ions are of different masses. However, when the soliton's amplitude is large, the reductive perturbation technique is not applicable in our derivation of the soliton, thus showing a contradiction in using the reductive perturbation technique here. However, Poisson's equation shows that the amplitude cannot be large as the ion–acoustic wave is reflected from a barrier induced by the positive ions before it attains a large magnitude. Das (1975) has already shown in detail that the same reductive perturbation technique is applicable in the plasma even with the negative ions.

For the particular model of a plasma consisting of ionic species (He^+, H^-) and (H^+, H^-), we show the amplitude variation of the solitary waves with the initial density of low-temperature electrons in Fig. 2. In this case, the electron temperature ratio β and concentration C of the negative ion species are taken to be constant for the plasma model considered. Fig. 2 shows that in the range of the lower temperature electron density, the amplitude of the soliton increases with the addition of heavier positive ions. Otherwise in the case of higher μ the amplitude variation does not depend on the masses of the ions as well as on the μ. As in Fig. 1 here a large amplitude solitary wave is also obtained, and similar arguments could be given for the use of perturbation technique and should be overcome with the analysis mentioned by Das (1975).

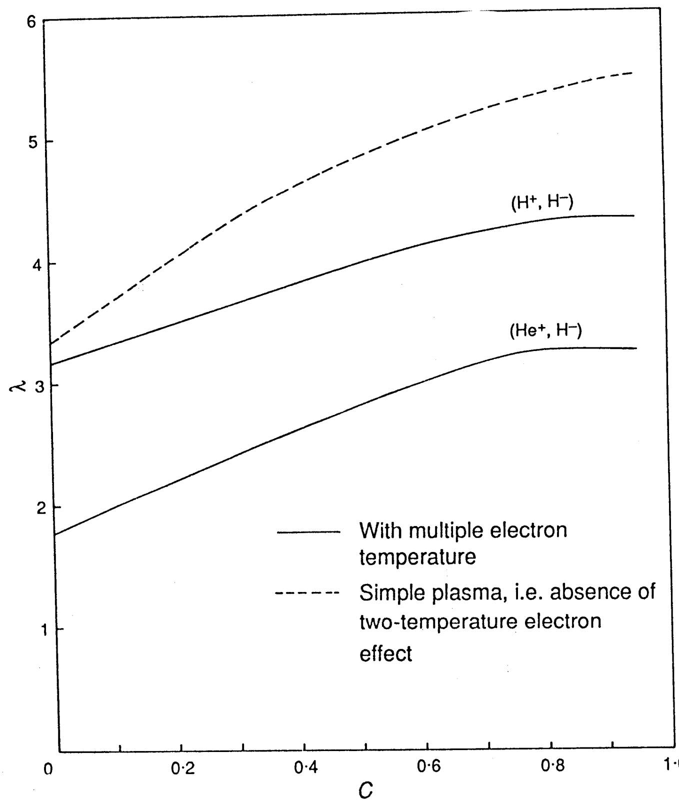

Fig. 4. Solitary wave phase velocity λ against concentration of negative ions C.

Fig. 3 shows the variation of soliton amplitude with negative ion temperature. It is observed that the amplitude increases with negative ion temperature and can be very large at certain temperatures, showing again similar characteristics to what we have already discussed in Figs 1 and 2. From Figs 1 and 3 we conclude that an increase of negative ion concentration lowers the magnitude of the amplitude while a temperature increase increases the amplitude. Finally Fig. 3 shows the variation of the amplitude in a multicomponent plasma compared with that obtained in a simple plasma (dashed curves).

Fig. 4 shows the effects of negative ion concentration C on the wave phase velocity λ. It is seen that λ increases with C, but in the range of higher concentrations becomes almost independent C. It is also clear that λ depends on the presence of two-temperature electrons and, in the absence of two-temperature electrons, we get the same behaviour in λ as discussed in Das (1975).

To get an insight into the physics of ion–acoustic solitary waves we now introduce a small percentage of non-isothermality into the plasma by considering the parameters $b_{\mathrm{l,h}}$ to be of small order $\epsilon^{1/2}$ given by $b_{\mathrm{l,h}} = \epsilon^{1/2} b'_{\mathrm{l,h}}$, where $b'_{\mathrm{l,h}} > 0$. In the plasma model the scheme of perturbation is considered in the following form:

$$\begin{aligned} n_a &= n_\alpha^{(0)} + \epsilon n_\alpha^{(1)} + \epsilon^2 n_\alpha^{(2)} + \dots, \\ v_\alpha &= \epsilon v_\alpha^{(1)} + \epsilon^2 v_\alpha^{(2)} + \epsilon^3 v_\alpha^{(3)} + \dots, \\ p_\alpha &= 1 + \epsilon p_\alpha^{(1)} + \epsilon^2 p_\alpha^{(2)} + \epsilon^3 p_\alpha^{(3)} + \dots, \\ \phi &= \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \dots, \end{aligned} \tag{12}$$

along with the stretched coordinates ξ and ψ as

$$\xi = \epsilon^{1/2}(x - \lambda t), \qquad \psi = \epsilon^{3/2} t. \tag{13}$$

Following the usual procedure, the expressions (12), (10) and (5) are used in the basic system (1)–(4) and the lowest order terms of ϵ give the expression for the phase velocity similar to equation (8). The next higher order terms lead to the modified KdV equation

$$R \frac{\partial \phi^{(1)}}{\partial \psi} + \left\{ T\phi^{(1)} + S(\phi^{(1)})^{1/2} \right\} \frac{\partial \phi^{(1)}}{\partial \xi} + P \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{14}$$

where

$$\begin{aligned} T &= \frac{\lambda}{2}\left\{ -\frac{\mu + \nu\beta^2}{(\mu+\nu\beta)^2} + 3\left(\frac{\sum n_i^{(0)}}{(\lambda^2 - 3\theta_i/n_i^{(0)})^2} - \frac{\sum n_j^{(0)}}{(\mu_j \lambda^2 - 3\theta_j/n_j^{(0)})^2} \right) \right. \\ &\quad \left. + 12\left(\frac{\theta_i}{(\lambda^2 - 3\theta_i/n_i^{(0)})^3} - \frac{\theta_j}{(\mu_j\lambda^2 - 3\theta_j/n_j^{(0)})^3} \right) \right\}, \end{aligned} \tag{15}$$

$$S = \frac{\lambda(b'_{\mathrm{l}}\mu + b'_{\mathrm{h}}\nu\beta^{3/2})}{(\mu+\nu\beta)^{3/2}},$$

and where R and P are the same as obtained in (10), with a slight modification in changing $b_{\mathrm{l,h}}$ to $b'_{\mathrm{l,h}}$.

It is seen that as in the earlier case, the phase velocity of the solitary wave does not depend on the non-isothermality of the simple plasma. But, in the case of a multicomponent plasma, the phase velocity depends on the non-isothermality, as well as on multiple electron temperatures in the plasma. To obtain a rigorous analysis for the solitary wave solution of (14), we introduce a travelling frame of reference $\chi = \xi - U\psi$, with the boundary conditions given by (4a). Following Das and Karmakar (1988) the solitary wave solution is

$$\phi^{(1)} = \left\{ \frac{4S}{15UR} + \left(\frac{16S^2}{225U^2R^2} + \frac{T}{3UR} \right)^{1/2} \cosh(\chi/\delta_1) \right\}^{-2}, \tag{16}$$

where δ_1 is the width of the solitary wave defined earlier.

From relation (16) it is seen that the characteristic variation of the solitary wave depends on the expressions T, R, S and P, which are functions of the multiple electron temperatures and multiple ionic species in the plasma. It is to be noted that the values of R, S and P are always positive whereas the value of T can be either positive or negative. Thus the existence of the solitary wave depends on the expression within the square root. This is required to be positive otherwise there is the possibility of a complex solution, one prone to exhibit a non-propagating solitary wave in the plasma. The soliton solution shows that multiple electrons as well as negative ions, together with the thermal effect, do have an important role in determing the existence and behaviour of solitons in plasma. In order to show the characteristic features of solitary waves in the plasma model considered, we look at the following degenerate cases. These cases, as will be seen, give an insight into the physics of solitary waves. First of all, we consider the case where the nonlinear effect introduced by T is much smaller than that introduced by S. In this case the solution (16) reduces to

$$\phi^{(1)} = \frac{225U^2R^2}{64S^2} \left(1 - \frac{75TUR}{32S^2} \cosh(\chi/\delta_1)\,\mathrm{sech}^2(\chi/2\delta_1) \right) \mathrm{sech}^4(\chi/2\delta_1), \tag{17}$$

where terms in S^{-6} are neglected.

The soliton solution shows that the singular behaviour observed in the plasma with multiple electrons and negative ions will not be seen in the modified soliton solution. As a special case, when T is sufficiently small, the solution (17) takes the form

$$\phi^{(1)} = \phi_N^{(0)}\,\mathrm{sech}^4(\chi/2\delta_1), \tag{18}$$

where $\phi_N^{(0)} = 225U^2R^2/64S^2$. This solution resembles an ion–acoustic solitary wave solution in the plasma with non-isothermal electrons. The solution does not depend on the value of T, and thus the question of non-existence of solitons does not arise here. Here $\phi_N^{(0)}$ depends on the values of R and S which are always positive for all plasma parameters. The amplitude $\phi_N^{(0)}$ is different from the expression obtained by Das (1975). Also, R appears as a factor for amplifying the amplitude of the ion–acoustic solitary wave. Then, the solution (17) takes the form

$$\phi^{(1)} = \frac{3UR}{T}\left(1 - \frac{8Q^2}{5(3TR)^{1/2}}\operatorname{sech}(\chi/\delta_1) - \frac{16Q^2}{75TUR}(1-3\operatorname{sech}^2(\chi/\delta_1))\right)\operatorname{sech}^2(\chi/\delta_1). \tag{19}$$

The values T and R are the main factors in the formation of the ion–acoustic solitary waves. The value of R is always positive and thus the existence of the solitary wave wholly depends on T which, of course, appears under the square root. The value of T is required always to be positive, otherwise it would be complex which indicates non-propagation of the wave. However, if Q is negligibly small then the solution (19) is

$$\phi^{(1)} = \frac{3UR}{T}\operatorname{sech}^2(\chi/\delta_1).$$

This is similar to the solution of a KdV equation derived in a plasma with isothermal electrons. Then the amplitude depends on the values of R and T. When both R and T are small or large the ratio R/T has a finite order of magnitude and in this case, the soliton's behaviour is schematically very similar as shown in the figures. Otherwise R/T is either small or large showing that the amplitude of the soliton will be either small or large respectively. As a result when the amplitude is large, then the width of the soliton will be small and there is a possibility of breaking the soliton into many more solitons. Again, when the amplitude is small the width of the solitary wave is wide and in this case the wave will be dispersive dominant.

4. Conclusions

We have presented the characteristics of the existence and behaviour of solitary waves in a generalised multicomponent plasma studied as a result of the solitary wave solution of the KdV equation. The main emphasis has focussed on the variation of amplitude and the phase velocity of the solitons with negative ion concentration, electron temperature and negative ion temperature. The study shows that the ion–acoustic wave can achieve a large amplitude arising from the presence of a small percentage of negative ions, but the amplitude decreases with increasing C. Such solutions are prone to show the non-applicability of the reductive perturbation technique to study solitons in plasma. The large amplitude soliton produces the possibility of breaking the soliton into many more solitons. Such a situation for the solitons arises in the isothermal plasma, whereas the non-isothermal plasma does not produce such a role for the ion–acoustic wave. The reduction of the non-isothermality of the plasma gives the solitary waves different properties from the case of the non-isothermal plasma. Moreover, we would like to point out that the nature of the nonlinear coefficient gives different types of solitons in plasmas. When it is positive the usual solitary wave solution showing a compressional soliton propagates in the plasma. However, when it is negative, in general, the soliton shows rarefactive behaviour in the plasmas instead of compressional behaviour. Both compressional and rarefactive solitons could be observed in the laboratory. Such solitons in plasma will be studied in the near future.

Acknowledgment

We wish to thank a referee for various suggestions and helpful comments in bringing the manuscript to the present form.

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Manuscript received 25 May, accepted 24 October 1989