# A Simple Microstrip Phase Shifter 

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#### Abstract

A novel, inexpensive, series-connected loaded-line (SCOLL) phase shifter has been developed for the Molonglo Observatory Synthesis Telescope. The phase shifter, which operates at 0.843 GHz , consists of a $\sim 3 \lambda / 8$ length of microstrip with series connected varactors at each end. Although designed as a binary phase shifter, the insertion phase varies almost linearly with varactor reactance, with very little gain variation for phase ranges up to $90^{\circ}$. Thus, driven by a digital-to-analogue (D/A) converter, a SCOLL becomes a programmable phase shifter of any desired resolution. The same technique would be suitable for narrow-band low power applications in the frequency range $0 \cdot 1$ to 10 GHz . This paper gives the basic theory of SCOLL phase shifters, a design chart, construction details and performance data.


## 1. Introduction

The device described in this paper originated in the need to develop a $60^{\circ}$ binary phase shifter to be included in the 0.843 GHz signal path of the Molonglo Observatory Synthesis Telescope, or MOST (Mills 1981). This instrument is a phased-array radio telescope, and the phase shifters were needed as part of an upgrading to improve the quality of the synthesised images (Amy and Large 1990). The requirement was for a total of 176 inexpensive phase shifters to be distributed along the 1.6 km length of the antenna, located in existing amplifier and IF conversion boxes. The rms scatter of phase and gain was specified to be not more than $2^{\circ}, 0.2 \mathrm{~dB}$ over the range of temperature $\left(-10^{\circ}<T<+40^{\circ}\right)$ and relative humidity ( $15 \%<R H<80 \%$ ) experienced in the boxes, in order to preserve the beamshape of the telescope. Power handling capacity and switching speed were not important considerations.

There are four circuit techniques for introducing phase directly into a signal path: switched line, reflection, loaded line and highpass/lowpass (Sharma 1989). Our requirement for a large number of phase shifters to be fitted into limited space persuaded us that the most appropriate type of device was a loaded line phase shifter using UHF varactors and microstrip technology (Nguyen 1989). Other systems would appear to use more components and occupy more space on a circuit board. The most usual form of loaded-line phase shifter consists of a $\lambda / 4$ length of transmission line with variable or switched reactances connected in parallel at each end (Garver 1972). The series-connected loaded-line (SCOLL)


Fig. 1. A SCOLL phase shifter consists of a section of transmission line of phase length $\beta l$ and characteristic impedance $Z_{p}$ with series-connected adjustable reactances $X$ as shown. The impedances looking towards the load at points A, B and C are plotted on a Smith impedance chart (see Fig. 2) for the two states in which the phase shifter is matched ( $Z_{i n}=Z_{0}$ ) and the gain is unity. The phase difference between the two states is given by equation (4).


Fig. 2. Impedance-coordinate Smith chart illustrating the operation of a SCOLL phase shifter. The chart is normalised to the characteristic impedance $Z_{p}$ of the transmission line. The points labelled $a, b$ and $c$ on the chart represent the impedances looking towards the load at the positions labelled A, B and C in Fig. 1. The first varactor takes the impedance to $a$ (or $a^{\prime}$ ). The transmission line then transforms the impedance to $b$ (or $b^{\prime}$ ) and the second varactor matches the impedance to $c$ (i.e. $Z_{0}$ ).
phase shifter described in this paper differs from this in two ways. Firstly, the varactors are connected in series with the transmission line because the reactances required (at 0.843 GHz ) are more readily obtained with available varactors in this configuration. Secondly, the length of transmission line and its characteristic impedance are chosen so that the phase shifter is exactly matched to the external circuit at both of its operating points.

## 2. Description

Fig. 1 is a schematic circuit diagram of an idealised SCOLL, assuming lossless components and no parasitic (stray) reactances. A section of transmission line of phase length $\beta l$ and characteristic impedance $Z_{p}$ has series-connected variable reactances $X$ at each end. The source and load impedances $Z_{0}$ are taken to be equal and purely resistive.

The operation of a SCOLL as a binary phase shifter depends on the existence of two states, corresponding to two different particular values of $X$, for which the device is matched to the source and load (i.e. $Z_{i n}=Z_{0}$ ). How this useful property comes about can be understood by reference to a Smith impedance chart (Fig. 2) on which the impedances at positions A, B and C (Fig. 1) are plotted. The impedance looking towards the load at A is $Z_{0}+j X$. For just two values of $X$, the effect of the length of transmission line is to transform this impedance to $Z_{0}-j X$ at B . The addition of the second reactance $X$ makes the impedance at C equal to the source impedance $Z_{0}$. Thus for these two values of $X$ the device is matched and the gain is unity. In the example shown $\beta l=136^{\circ}, Z_{p} / Z_{0}=0 \cdot 8$, both values of $X$ are capacitative (negative) and the phase difference between the states is $60^{\circ}$.

## 3. Theory

We have analysed the performance of SCOLLs by setting up the transfer matrix and hence evaluating the complex gain $G$ (or scattering parameter $S_{21}$ ). For a review of these techniques see, for example, Ramo et al. (1984). The result is most simply expressed in terms of $1 / G$ :

$$
\begin{align*}
1 / G= & \cos \beta l-\left(X / Z_{p}\right) \sin \beta l+j\left(X / Z_{0}\right) \cos \beta l \\
& +(j / 2)\left(Z_{p} / Z_{0}+Z_{0} / Z_{p}-X^{2} / Z_{p} Z_{0}\right) \sin \beta l \tag{1}
\end{align*}
$$

This expression gives the gain and phase of a SCOLL as a continuous function of the series reactance $X$. The two states of a binary phase shifter then correspond to the points at which the gain is unity, i.e.

$$
\begin{equation*}
|G|=1 \tag{2}
\end{equation*}
$$

Analysis of equation (1) confirms that this condition is satisfied for two values of $X\left(X_{r}, r=1,2\right)$ given by

$$
\begin{equation*}
X_{r}=Z_{p} \cot \beta l+(-1)^{r}\left[\left(Z_{p} \cot \beta l\right)^{2}+\left(Z_{p}^{2}-Z_{0}^{2}\right)\right]^{\frac{1}{2}} \tag{3}
\end{equation*}
$$

The phase difference $\Delta \phi$ between these two states is given by

$$
\begin{equation*}
\cos (\Delta \phi / 2)=\left(Z_{0} / Z_{p}\right) \sin \beta l . \tag{4}
\end{equation*}
$$

The simplicity of a SCOLL phase shifter arises from the fact that the reactances $X_{1}$ and $X_{2}$ can both be negative, and hence realised by varactors without the need for any other reactive components. Inspection of equation (3) shows that
this condition is satisfied if the line length lies in the range $180^{\circ}>\beta l>90^{\circ}$ and the line impedance $Z_{p}$ is less than $Z_{0}$.

By re-arranging equations (3) and (4) useful expressions for the device characteristics can be obtained in terms of $X_{1}$ and $X_{2}$; the transmission line characteristic impedance is given by

$$
\begin{equation*}
Z_{p}=\left(Z_{0}^{2}-X_{1} X_{2}\right)^{\frac{1}{2}} \tag{5}
\end{equation*}
$$

and its phase length is given by

$$
\begin{equation*}
\beta l=\operatorname{arccot}\left[\left(X_{1}+X_{2}\right) / 2 Z_{p}\right] . \tag{6}
\end{equation*}
$$

The phase interval between the two states is then

$$
\begin{equation*}
\Delta \phi=2 \arctan \left[\left(X_{1}-X_{2}\right) / 2 Z_{0}\right] \tag{7}
\end{equation*}
$$



Fig. 3. Design chart for SCOLL binary phase-shifters using capacitative reactances. The full curves are isoreactances of $X_{1}\left(X_{1} / Z_{0}=-0 \cdot 5,-0 \cdot 67,-1,-1 \cdot 5,-2,-4\right.$ and -8$)$ and the dashed curves are isoreactances of $X_{2}\left(X_{2} / Z_{0}=-0 \cdot 67,-0 \cdot 5,-0 \cdot 25\right.$ and $\left.-0 \cdot 125\right)$. The use of the chart is explained in Section 4.

## 4. A Design Chart

We have used equations (5) and (7) to construct a design chart (Fig. 3) which illustrates the range of possible SCOLL binary phase shifters using capacitative reactances. In this chart the axes are the phase difference $\Delta \phi$ and the normalised line impedance $Z_{p} / Z_{0}$. The corresponding reactances are plotted as lines of constant $X_{1} / Z_{0}$ (full curves) and $X_{2} / Z_{0}$ (dashed curves). To design a SCOLL of a particular phase interval $\Delta \phi$, choose a convenient line impedance $Z_{p}$ and read
off $X_{1}$ and $X_{2}$. A lower ratio of $X_{1} / X_{2}$ (i.e. lower ratio of varactor capacitance) is obtained by choosing a lower line impedance. The line length $\beta l$ and precise values of $X_{1}$ and $X_{2}$ may be found using equations (3) and (4).

Although a SCOLL phase shifter using capacitative reactances (varactors) is particularly simple, equations (3)-(7) are valid for both capacitative and inductive reactances. If both reactances are inductive, the design chart (Fig. 3) may be used by changing the sign of both $X_{1}$ and $X_{2}$. The phase length of the line is still given by equation (6) and now lies in the range $0^{\circ}<\beta l<90^{\circ}$. The phase interval is (trivially) changed in sign.

If one of the reactances is capacitative and the other is inductive then solutions exist only for $1<Z_{p} / Z_{0}<\sec (\Delta \phi / 2)$. These solutions are not included in Fig. 3, do not seem to be very useful in designing SCOLLs and are not considered further in this paper. Study of Fig. 3 together with equations (3)-(7) gives a thorough understanding of the scope and limitations of SCOLLs as binary phase switches. An example of the use of the design chart is given in the following section.

Table 1. Three possible designs for a $0.843 \mathrm{GHz}, 60^{\circ}$ phase shifter

| $Z_{p}(\Omega)$ | $C_{1}(\mathrm{pF})$ | $C_{2}(\mathrm{pF})$ | $\beta l$ |
| :---: | :---: | :---: | :---: |
| 45 | 26 | $2 \cdot 9$ | $129^{\circ}$ |
| 40 | 15 | $2 \cdot 7$ | $136^{\circ}$ |
| 35 | 11 | $2 \cdot 5$ | $143^{\circ}$ |

## 5. Construction and Tests

For the purposes of upgrading our radio telescope we required $60^{\circ}$ binary phase shifters operating at 0.843 GHz . In designing the SCOLL for this purpose we were thus able to chose one parameter, corresponding to the choice of a point on the $\Delta \phi=60^{\circ}$ ordinate of the design chart. Examples of three possible designs are given in Table 1, where the reactances $X_{r}$ are converted into capacitances $(C=-1 / \omega X)$ and $Z_{0}=50 \Omega$. The values in the second example were adopted for the MOST phasers, as the necessary capacitances were readily attained with available UHF varactors.

Fig. 4 shows the completed printed circuit board incorporating SCOLLs and other components. It is produced on double-sided 1.6 mm fibreglass board, using surface mounted components. The ground areas on the circuit side are connected to a continuous ground plane on the other side with plated-through holes at $\sim \lambda / 10$ spacing. Each printed circuit board includes two identical phase shifters, input and output attenuator pads, UHF chip amplifiers and diode logic circuits for decoding the switching signal.

The 8 dB input attenuator maintains the impedance presented to the input of the phase shifter at close to $50 \Omega$, regardless of the voltage standing-wave ratio (VSWR) of the signal source (which is any one of 176 UHF amplifiers distributed along the 1.6 km length of the antenna). The two matched varactors are connected with their cathodes to the microstrip, and are biassed by a positive potential applied to the microstrip via a UHF choke. The other phase shifter on the board is connected with the varactor anodes to the microstrip, and is adjusted by a negative potential applied to the microstrip. On the MOST, the phase shifters are controlled in pairs by a


Fig. 4. One of the 100 phasing units built for the MOST project. The components are surface-mounted, and numerous plated-through holes make connection to the ground plane. Each board comprises two $60^{\circ}$ SCOLL binary phase shifters and associated circuits. One of these phase shifters is labelled as follows: (1) circuit for generating the varactor bias (applied to the varactors via UHF chokes and the microstrip). (2) $50 \Omega$ input point. (3) 8 dB input attenuator. (4) varactor. (5) $40 \Omega, 136^{\circ}$ microstrip. (6) varactor. (7) 6 dB output attenuator. (8) UHF IC amplifier (the DC supply is carried by a UHF choke). (9) $50 \Omega$ output. Each board is $72 \times 176 \mathrm{~mm}^{2}$.
steady potential on a single wire. A control voltage of zero ('fail safe') sets the relative phase of each phase shifter to zero. A control voltage of +30 V switches one phase shifter to $+60^{\circ}$ whereas a voltage of -30 V switches the other to $+60^{\circ}$.

The 6 dB attenuator at the output of the microstrip ensures a reasonable match to the line, and to the input of the UHF amplifier chip. The gain of this amplifier is $\sim 15 \mathrm{~dB}$, so that the net insertion loss of the device is close to zero.

We have produced 100 phaser boards (i.e. 88 for the telescope plus spares) using a matched set of Philips type BB215 UHF varactors. There were minor variations in the phase, mainly associated with the quoted maximum $3 \%$ difference between any two varactors in the matched set. By individually presetting the bias voltages on the varactors (by choice of fixed resistor values in the biassing networks) we readily set each phase shifter so that the phase step ( $\Delta \phi$ ) was the required $60^{\circ}$ and the gain step $(\Delta G)$ was nominally 0 dB . Bench tests showed that the rms scatter over the 200 phase shifters was $1^{\circ}$ in $\Delta \phi$, and 0.05 dB in $\Delta G$. The measured temperature coefficients of $\Delta \phi$ and $\Delta G$ were $-0^{\circ} .04 \mathrm{~K}^{-1}$ and $0.005 \mathrm{~dB} \mathrm{~K}^{-1}$ respectively. Over the 3 MHz bandpass of the MOST, $\Delta \phi$ and $\Delta G$ varied by less than $1^{\circ}$ and $0 \cdot 1 \mathrm{~dB}$. There was some increase in the scatter when the phasers were installed in situ on the antenna. However, the end result has been the reduction of grating lobe responses as predicted, with a corresponding very striking improvement in the quality of image produced by the telescope (Large and Whiteoak 1992).

## 6. Continuous Phase Shifting

## (a) Experimental Result

We have shown theoretically, and confirmed in practice, that with suitable choice of line length and impedance, a SCOLL forms the basis of a simple,


Fig. 5. Measured dependence of phase (full curve) and gain (dashed curve) on varactor bias for one of the SCOLL phase shifters built for the MOST. The zeros of phase and gain are arbitrary. The reactances $X_{1}$ and $X_{2}$ correspond to bias voltages of 4.0 and 20.6 V . According to equation (1) the gain falls by 0.09 dB between these points, but in this sample, as in many others, the gain dip was not seen. Its absence is attributed to parasitic reactances, slight mismatches, etc. Although designed as a $60^{\circ}$ binary phase shifter, the device has potential as a continuous phase shifter over a phase range somewhat greater than $60^{\circ}$.
practical, microwave binary phase shifter. The potential of SCOLLs as continuous microwave phase shifters was not recognised until we started making measurements on the MOST prototypes. Fig. 5 shows the performance of a sample MOST phase shifter at band centre. The gain and phase are plotted as a function of the varactor voltage over the range 0 to 26 V . The phase increases monotonically and almost linearly with varactor voltage over a range in excess of $60^{\circ}$ while the gain varies in a narrow range. Such devices could have applications in many branches of microwave engineering, so in this section we outline their characteristics.


Fig. 6. Phase and gain of a typical SCOLL plotted as a function of varactor reactance using equation (1). In this plot, $\Delta \phi=75^{\circ}, Z_{p}=30 \Omega, Z_{0}=50 \Omega$ and $\beta l=151^{\circ} \cdot 6$. Broadly similar results are obtained for a wide range of parameter values.

## (b) Choice of Parameters

We have explored the potential of SCOLLs as continuous phase shifters by using equation (1) to examine how the complex gain $G$ varies with the reactance $X$ for a wide range of the phase interval $\Delta \phi$ and line impedance $Z_{p}$, the phase length $\beta l$ being determined by equation (4). The result plotted in Fig. 6 is typical. From studies of similar plots we are able to make the following general observations:
(1) The phase is a surprisingly linear function of $X$, with a slope which depends on $\Delta \phi$, but is almost independent of $Z_{p}$.
(2) The broad dip in gain between the two operating points increases with $\Delta \phi$ but is independent of $Z_{p}$. The gain midway between the operating points [i.e. at $X=\left(Z_{1}+Z_{2}\right) / 2$ ] is

$$
\begin{equation*}
\left|G_{\mathrm{mid}}\right|=\frac{2}{\cos (\Delta \phi / 2)+\sec (\Delta \phi / 2)} \tag{8}
\end{equation*}
$$

(3) For any given phase interval $\Delta \phi$, the ratio of reactance required to cover the phase range is less for lower values of $Z_{p}$. This point is important in designing SCOLL phase shifters, as varactors have a limited capacitance ratio.

## (c) Phase/Voltage Linearity

We have shown that, over a wide range of parameters, the phase of a SCOLL is expected to vary almost linearly with the varactor reactance. Varactors are manufactured for use in the VHF to microwave regions of the spectrum with junction profiles tailored to produce a wide variety of reactance/bias-voltage characteristics $X(V)$ for various applications. (The data are usually presented in terms of varactor capacitance, but it is the reactance which is relevant in the present context.) When used as the variable reactance element in SCOLLs, the phase/bias-voltage characteristic $\phi(V)$ is essentially determined by $X(V)$. The SCOLL developed for the MOST has a linear $\phi(V)$ characteristic because, fortuitously, the varactors used have a nearly linear $X(V)$ characteristic over the range of bias voltage used.


Fig. 7. A $360^{\circ}$ programmable phase shifter comprising two binary phase switches which select the phase quadrant and a SCOLL which sets the phase over a $90^{\circ}$ range with a resolution determined by the precision of the D/A converter.

## (d) Digital Control

A widely used type of phase shifter (Sharma 1989) consists of a set of $N$ digitally controlled phase steps of $180^{\circ}, 90^{\circ} \ldots 360^{\circ} \times 2^{-N}$. The phase resolution is then determined by the number of steps (for example if $N=7$ the resolution is $2^{\circ} \cdot 8$ ). A single SCOLL can replace the lower order steps to produce a programmable phase shifter of any desired phase resolution. In the example shown in Fig. 7, $180^{\circ}$ and $90^{\circ}$ phase switches select the phase quadrant, and a SCOLL driven by a $\mathrm{D} / \mathrm{A}$ converter sets the phase within a $90^{\circ}$ range. A suitable SCOLL for this purpose would be the one characterised in Fig. 6. Over the $90^{\circ}$ phase range, the gain variation for this example is only 0.23 dB . If greater gain precision is necessary, binary steps can be used to select the octant, together with a SCOLL designed for a phase range of only $45^{\circ}$. The gain variation over the phase range could then, ideally, be less than 0.03 dB .

## 7. Limitations

The characteristics given by equation (1) and illustrated in Fig. 6 refer to an ideal SCOLL phase shifter. In practice, any parasitic reactances or mismatches of the source or load will affect the performance. We found such effects unimportant for the MOST devices. For example, arbitrary mismatches of source and load with the VSWR $\leq 1.2$ typically produced relative gain and phase errors at the operating points of $\sim 0.1 \mathrm{~dB}$ and $1^{\circ}$. For many purposes only the gain
discrepancy matters, as the phase is readily corrected by an adjustment of the varactor bias.

We have experience with SCOLL phase shifters only at 0.843 GHz . Looking to higher frequencies, it seems probable that the same techniques would be satisfactory up to about 10 GHz , although parasitic reactances would become increasingly troublesome. The lowest frequency of operation for a SCOLL phase shifter would probably be 100 MHz , determined by the availability of suitable high-capacitance varactors. The hyper-abrupt junctions used to produce such varactors would give rise to very nonlinear phase/bias-voltage characteristics.

SCOLL phase shifters are limited to low powers such that the signal voltage does not appreciably modulate the varactor capacitance. They are also essentially dispersive elements, and hence unsuited to wide-band applications.

## 8. Conclusions

Series-connected loaded-line (SCOLL) phase shifters with varactors as the sole reactive phasing element are particularly attractive for low power applications at frequencies $\sim 1 \mathrm{GHz}$. They form ideal binary phase switches and excellent continuous phase shifters for phases up to $90^{\circ}$. They can be used to replace the lower order bits in programmable phase shifters, providing a digitally controlled phase range of $360^{\circ}$ with any desired resolution.

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