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The Rate of Star Formation in Galactic Disks*

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Abstract

A simple prescription is presented for the rate of star formation in normal galactic disks, motivated by semi-phenomenological dynamical arguments. When energy input into the interstellar medium is included as a feedback mechanism to couple the star formation driven by disk instabilities with gaseous dissipation, one can obtain expressions for both disk surface density and the Tully–Fisher relation in good accord with observation. One plausibly expects variations at a level of order $\sim$10% in the Tully–Fisher zero-point due to age and/or star formation history that may be correlated over supercluster scales. I also show that there is a limiting rate of star formation, maintained by the porosity of the hot component of the multiphase interstellar medium, that can be interpreted as arising in starbursts that are triggered by galaxy mergers. The predicted star formation rate is found to be sensitive to the ambient pressure and results in enhanced star formation in galaxies that are undergoing first infall at cluster peripheries (out to $\sim 10h^{-1}$ Mpc from an Abell cluster).

1. Introduction

The microscopic physics of star formation is currently poorly understood, and likely to remain so despite the plethora of data from infrared and millimetre astronomy. The reason for this is that any theory of, for example, the initial stellar mass function (IMF) must take into account a great variety of physical processes. These include ionisation and heating by X-rays, cosmic rays and ultraviolet radiation, growth and destruction of dust grains, adsorption and photodesorption of molecules from grain surfaces, gas phase and surface molecular chemistry, cooling transitions by molecular and atomic species, molecular photodissociation, recombination of ions and electrons, radiative transfer of molecular lines important for cooling, heat input via internal shocks, turbulence dissipation, Alfven wave dissipation, internally and externally generated radiation fields, and feedback from protostars via bipolar flows, winds and flares. Nor does this list include dynamics of cloud contraction, collapse and fragmentation, or the role of magnetic fields in cloud support and ambipolar diffusion. The origin of the IMF seems intractable, although one might hope to gain a better understanding of characteristic mass scales, such as the average and minimum stellar masses that are produced.

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A global approach to star formation seems more promising. The principal aim is to infer the rate of star formation in the interstellar medium of a star-forming galaxy. The IMF is treated as a black box, to be inferred empirically. However, I will argue that the star formation rate (*SFR*) can be derived in a semi-phenomenological approach, and used to study star formation histories of galactic disks (Section 2). I apply these results to the Tully–Fisher relation (Section 3), and develop a simple formulation of the *SFR* in a starburst galaxy (Section 4) with applications to galaxy clustering and to galaxy formation theory (Section 5).

2. Star Formation Rate in Galactic Disks

A straightforward approach to global star formation has the following ingredients: a cold stellar disk containing HI and H_2 undergoing differential rotation is unstable to axisymmetric density perturbations and non-axisymmetric shear perturbations that concentrate the gas into dense clouds. The isothermal gas complexes become gravitationally unstable, and form stars. Feedback from the massive stars restricts the process to be inefficient, most of the gas being recirculated into the more diffuse HI/H_2 medium. Dissipation and cooling of the gas maintains the instability until the gas supply is depleted. To compare with detailed observations of disks, one needs to construct an empirical model. Input includes the stellar initial mass function, the gas density distribution, and stellar nucleosynthetic yields, as well as the theoretical prescription for the star formation rate. There are so many parameters in the empirical model that not all of them can be constrained. For example, the closed box model has difficulty in simultaneously obtaining the present gas fraction and metallicity in the disk, but allowance for infall of metal-poor gas or outflow of enriched gas means that chemical evolution no longer poses any difficulty. In the remainder of this discussion, I will accordingly focus on a derivation of the global star formation rate.

The Toomre parameter $Q_g \approx \kappa\sigma_g/(\pi G\mu_{gas}) \lesssim 1$ demarcates the onset of instability to local axisymmetric perturbations of a thin, differentially rotating, gaseous (Goldreich and Lynden-Bell 1965) or stellar (Toomre 1964, with π replaced by $3\cdot 36$, μ_{gas} by μ_{disk}, and σ_g by the stellar radial velocity dispersion) disk. A similar criterion also applies for the disk to be unstable to global non-axisymmetric perturbations, resulting in bar formation or swing amplification of spiral density waves (Toomre 1981). Here κ is the disk epicyclic frequency, equal to $2^{1/2}\Omega(2 + d\,ln\Omega/d\,lnr)$ for rotation rate $\Omega(r)$, σ_g is the gas velocity dispersion and μ_{gas} is the gas surface density. One can write $Q_g = \mu_{cr}/\mu_{gas}$, where $\kappa \equiv \kappa_\odot = 36\,\mathrm{km\ s^{-1}\,kpc^{-1}}$ and $\mu_{cr} \approx 2^{1/2}\Omega\sigma/\pi G = 8M_\odot\ \mathrm{pc}^{-2}$ at the solar circle. One observes locally $\mu_{HI+H_2} \equiv \mu_\odot \approx 13M_\odot\ \mathrm{pc}^{-2}$. This suggests that our galaxy, at least near the sun, is just unstable to star formation. In fact, Kennicutt (1989) has found in a study of nearby disk galaxies that the surface density of cold gas (HI+H_2) remains near the critical value where star formation is occurring, and drops below μ_{cr} (or $Q_g > 1$) outside the radius where HII regions are observed. This suggests that disk self-regulation, as originally discussed by Miller *et al.* (1970) and Quirk (1971), really does occur.

Global spiral density waves can be maintained by the competing effects of gas in destabilising as well as in stabilising spiral instabilities. This may be accomplished either by infall of cold gas (Sellwood and Carlberg 1984) or by gas dissipation (Bertin and Romeo 1988). The parameter Q is maintained to be of order unity, with disk instabilities forming OB stars, whose energy feedback heats the gas and tends to stabilise it. Once OB star formation is partially suppressed, however, the disk cools and destabilises. The detailed coupling is most likely a combination of expanding HII regions, stellar winds, and supernova remnants. This heats the gas, raises σ_g, increases the scale height, and reduces μ_{gas}, as long as massive stars are forming. Incorporation of the stellar component would lead to a more complicated expression (e.g. Jog and Solomon 1984) of the (asymptotically correct) form $1/Q \propto (\mu_{gas}/\sigma_g + \mu_*/\sigma_*)$: it should be evident that the cold gas will dominate the disk instabilities until the gas is depleted.

A semi-phenomenological expression that incorporates the relevant physics, at least globally, for the rate of star formation is given by

$$SFR = \epsilon \; \mu_{gas}/t_{instability}.$$

Here ϵ is an empirically determined efficiency for converting gas into stars and the growth rate for disk instabilities, $\sim\pi G\mu_{gas}/\sigma$ in linear theory, is written in the form

$$t^{-1}_{instability} \approx \kappa Q^{-1}(1-Q)\,, \qquad Q \leq 1\,.$$

Thus there is a simple, dynamically motivated prescription which accounts for the global present-day star formation rate in disks without appeal to speculative astrophysical scenarios, for example of gas infall, for which there is little current evidence. One may set

$$SFR = \epsilon\mu^2_{gas}(\pi G/\sigma_g)(1-Q), \tag{1}$$

motivated by the disk linear instability growth rate $t_{instability} \sim \pi G\mu_{gas}/\sigma_g$.

Gas dissipation is primarily due to energy loss in cloud–cloud collisions, which occur at a rate $t^{-1}_{coll} \sim (\mu_{gas}\mu_{cl})(\sigma_g/H)$ where μ_{cl} is the mean cloud surface density and the disk scale height is

$$H = \frac{\sigma_g}{\pi G}\left(\frac{\mu_{gas}}{\sigma_g} + \frac{\mu_*}{\sigma_*}\right)^{-1} \approx Q\sigma_g/\Omega\,.$$

In the interstellar medium of our Galaxy, molecular clouds satisfy (Solomon *et al.* 1987) $\mu_{cl} \approx 160 M_\odot\,\mathrm{pc}^{-2}$, with a dispersion of only about 10%. Locally, the disk surface density of about $60 M_\odot\,\mathrm{pc}^{-2}$ bounds μ_{gas} early in the evolution of the galactic disk.

Assume first that clouds form at a rate $\sim\Omega$, as expected if cloud growth is driven by coagulation of smaller gas clouds in a gravitationally unstable disk. It seems reasonable to expect that a pure gas disk will be sufficiently unstable that self-regulation will not be enforced until a substantial fraction of the gas has formed stars. In a steady state, with cloud motions being maintained by disk gravitational instabilities, $t^{-1}_{coll} \sim \Omega$ or $Q \approx \mu_{gas}/\mu_{cl}$, and I conclude that

$$SFR = (1-Q)\epsilon\mu_{cl}\Omega\,. \tag{2}$$

Hence the star formation rate is approximately constant, as indeed is observed for spiral galaxies, at least to within a factor of a few.

Next, consider what happens as the gas reservoir begins to deplete via star formation and the gas settles into the disk potential well. The following feedback mechanism should become operative. Once massive stars form and die, the energy input from HII regions, OB star winds and, especially, supernova remnants provides a source of momentum to the interstellar gas. I incorporate the supernova feedback by writing

$$SFR\ .\ p_{SN} = \mu_{gas}\sigma_g\Omega. \tag{3}$$

Here $p_{SN} = 2E_{SN}(v_c M_{SN})^{-1}$ is the final specific momentum deposited into the interstellar medium by a supernova remnant of initial kinetic energy $10^{51}E_{51}$ erg, and

$$v_c = 208E_{51}^{1/14}Z^{-3/14}n^{1/7}\ \mathrm{km\ s^{-1}}$$

is the cooling velocity at which a supernova remnant expanding into a uniform medium of density n and metallicity Z (relative to solar) first enters the approximately momentum-conserving phase (Cioffi *et al.* 1989). The mean mass in stars that form for each supernova is M_{SN}, with a typical numerical value being $M_{SN} \equiv 300M_{300}M_\odot$ with $M_{300} \approx 1$ for a present-day star formation rate of $\sim 6M_\odot\,\mathrm{yr^{-1}}$ and disk supernova rate of $\sim 0{\cdot}02\,\mathrm{yr^{-1}}$. One obtains $p_{SN} = 600E_{51}^{13/14}n^{-1/7}M_{300}^{-1}$ km s^{-1}. The star formation rate now decreases in proportion to μ_{gas}: the e-folding time-scale for gas depletion is

$$\sim(p_{SN}/\sigma_g)\Omega^{-1} \sim 100\sigma_{10}\Omega^{-1},$$

where $\sigma_g \equiv 10\sigma_{10}$ km s^{-1}.

Finally, I assume that self-regulation of the star formation rate occurs and is maintained with $Q \approx 1$. I suppose, for simplicity, that gas self-gravity dominates and define $Q_g = \sigma_g\Omega/\pi G\mu_{gas}$. Combination of the feedback condition with the previous expression for the star formation rate as a function of Q then yields, for a flat rotation curve,

$$SFR\ = Q_g\pi G\mu_{gas}^2/p_{SN}\,. \tag{4}$$

Self-regulation (Q_g = constant $\lesssim 1$) implies

$$\epsilon = \frac{Q_g}{1-Q_g}\frac{\sigma_g}{p_{SN}} = 0{\cdot}01\sigma_{10}\frac{n^{1/7}M_{300}}{E_{51}^{13/14}}\,.$$

If Q_0 is specified by the initial conditions that gave rise to the disk, the only freedom remaining is in the mass fraction in cold clouds. Gas infall or outflow, as well as radial gas flows, are complicating factors that I will not consider here. The cold gas mass fraction f_m is relevant for gas self-regulation via disk instability and star formation. I write $\mu_{gas} = f_m\mu_0$, $Q = Q_0/f_m$, where μ_0 is the total gas surface density as a function of galactic radius. One can then write the quadratic law (1) in terms of f_m as

$$SFR = (SFR)_0\ f_m(f_m/Q_0 - 1)\,, \tag{5}$$

where

$$(SFR)_0 = \epsilon\ \mu_0\ \kappa = 6 \times 10^{-9} \left(\frac{\epsilon}{0{\cdot}01}\right) \left(\frac{\mu_0}{\mu_\odot}\right) \left(\frac{\kappa}{\kappa_\odot}\right) M_\odot\ \mathrm{pc}^{-2}\ \mathrm{yr}^{-1},$$

$$Q_0 = \frac{\mu_{cr}}{\mu_0}\,, \quad \mu_{cr} = 13 \left(\frac{\kappa}{\kappa_\odot}\ \sigma_{10}\right)\ M_\odot\ \mathrm{pc}^{-2}\,,$$

and for spiral galaxies similar to the Milky Way, $f_m \approx 1$. Hence the self-regulation hypothesis specifies ϵ and *requires* that $f_m \lesssim Q_0$, where Q_0 increases when μ_0 drops below μ_{cr}, as expected for an exponential disk outside several scale lengths. The resulting stabilisation should produce a sharp outer edge in the distribution of newly formed stars. However, stellar feedback may well couple into small-scale gas processes and allow the Q parameter to fall below unity at least for the gas component, once a substantial star component has formed.

Kennicutt (1989) indeed observed that the inner disks at the present epoch satisfy a star formation law that is intermediate between a linear and a quadratic dependence on μ_{gas}, and consistent with $Q \approx$ constant < 1, but steepening rapidly beyond a Holmberg radius. Wyse and Silk (1989) demonstrated earlier that a star formation rate proportional to $\Omega(r)\mu_{gas}(r)$ satisfactorily reproduces colour and metallicity gradients in galaxy disks. This was motivated by a cloud coagulation model, with giant molecular cloud complexes forming on a dynamical time-scale. The inner galaxy undergoes more rapid star formation and gas depletion than the region outside the corotation radius, where Ω decreases approximately as r^{-1}.

In fact, these star formation laws are empirically indistinguishable at the present epoch. An unresolved issue is the question of whether the current gas fraction and gas distribution can be explained. The ratio of gas surface density to star formation rate at radius r yields a time-scale $t_* \equiv \mu_{gas}(r)/SFR \sim [\epsilon(1-R)\Omega(r)]^{-1}Q(1-Q)^{-1}$, where R is the gas fraction returned by massive stars to the interstellar medium. One obtains a satisfactory present epoch star formation rate and gas density within the solar circle by adopting a linear star formation law with $Q \approx$ constant and $\epsilon \simeq 0{\cdot}01$, an efficiency that is consistent with the feedback hypothesis (3) as well as with direct observation of molecular cloud complexes. At the solar radius, for example, $\Omega^{-1} \approx 3 \times 10^7$ yr, so that with $R = 0{\cdot}4$, one has $t_* = 5 \times 10^9$ yr. With the observed gas surface density $\mu_0 = 13 M_\odot\,\mathrm{pc}^{-2}$, we infer $SFR = 3 M_\odot\,\mathrm{pc}^{-2}\,\mathrm{Gyr}^{-1}$.

A critical test of such a model arises with predictions of enrichment (Wang and Silk 1992). The most important hurdle is the G-dwarf problem: the paucity of low metallicity stars. This is most easily resolved by recognising that thin disk formation began with finite metallicity, perhaps $Z = 0{\cdot}2 Z_\odot$, resulting from heavy element synthesis and ejection by massive stars that formed in the bulge, halo and thick disk components prior to thin disk formation. Other solutions have been advocated over the years that are *ad hoc* and possess varying degrees of implausibility, including infall of primordial gas, metal-enhanced star formation, and bimodal star formation.

Effective dissipation is an important prerequisite for self-regulation to occur. As disk instabilities operate and drive massive star formation, one expects gas to be heated and ejected out of the disk. The ratio of cooling time to dynamical time for interstellar gas clouds moving at the virial velocity, equivalent to the escape velocity from the disk, is

$$\frac{t_{cool}}{t_{dynamical}} = \left(\frac{M}{10^{11} M_\odot}\right) \frac{\mu_{disk}}{\mu_{gas}} \frac{1}{\lambda(T)},$$

where $\lambda(T) \equiv T^{1/2}\Lambda(T)$, with $\Lambda(T)$ equal to the cooling function, is a slowly varying function of T (or M/R) over the temperature range of interest, $10^5 \lesssim T \lesssim 10^7$ K. Evidently, efficient dissipation, crucial to maintaining $Q \sim 1$, occurs primarily for low mass galaxies. At the same time, the low mass galaxies suffer mass loss via supernova heating of the interstellar medium that results in a galactic wind. Mass loss driven by supernova energy input, when most of the initial gas reservoir is ejected, occurs if the following energetic requirement is satisfied, based on generalising the momentum balance condition (3):

$$p_{SN}\dot{\mu}_* > \mu_{gas} v_{rot}/t_{dis}, \tag{6}$$

where t_{dis} is the momentum dissipation time-scale. In addition, one requires, in order for a stable wind to develop, the cooling time to exceed the flow time at the sonic radius. For the present purpose, however, it will suffice to assume that the supernova input to the interstellar gas, and any other energy input that is proportional to the star formation or death rate, balances the gas dissipation rate when the gas disk self-regulates.

3. Application to Tully–Fisher Relation

The global star formation rate, utilising the linear theory-inspired formulation of disk instability with $Q \approx$ constant, integrates over the disk to give

$$\dot{M}_* \approx \frac{k_m \epsilon}{G\sigma_g} v_{rot}^4 \left\langle \left(\frac{\mu_{gas}}{\mu_{disk}}\right)^2 \right\rangle, \tag{7}$$

where $k_m \approx 0{\cdot}3$ and I have assumed that μ_{gas} and μ_{disk} have similar radial profiles, with

$$\mu_{disk} = \frac{\alpha^2}{2\pi} M_{disk} e^{-\alpha r}. \tag{8}$$

This omits the contribution of the spheroid to the rotation velocity. This should be unimportant within one or two disk scale lengths. The disk mass is M_d; any halo mass contribution within a disk scale length α^{-1} will be neglected. Empirically, one measures a maximum rotation velocity attained near $\alpha r \approx 2$ (Freeman 1970) that satisfies

$$v_{rot} = k_d (\alpha G M_{disk})^{1/2}, \qquad k_d \approx 0{\cdot}8. \tag{9}$$

The current star formation rate evidently satisfies a Tully–Fisher-like law, as also does the integrated luminosity. For example, from the above disk equations (8) and (9),

$$L_d = \beta v_{rot}^4 \ \mu_{disk}^{-1} (M_{disk}/L_{disk})^{-1} G^{-2}, \tag{10}$$

where $\beta = (2\pi k_d^4)^{-1} \approx 0{\cdot}09$.

The predicted dispersion is dominated by the dispersion either in $(\mu_{gas}/\mu_{disk})^2$ or in $\mu_{disk}^{-1}(M_{disk}/L_{disk})^{-1}$. Surface densities, both of star and gas, are constant for 'normal' spirals: the central surface brightnesses satisfy Freeman's law ($21{\cdot}65$ B mag arcsec^{-2}, with a dispersion of $0{\cdot}3$ mag) over a 5 magnitude range in M_B (Freeman 1970), and the gas surface densities show no systematic trend with galaxy luminosity (Giovanelli and Haynes 1984). The decrease in infrared surface brightness with decreasing 21 cm linewidth ΔV for galaxies with $\Delta V \lesssim 300$ km s^{-1} (Mould *et al.* 1989) is attributable to the decreasing bulge contribution with later Hubble type. While the dispersion in central blue surface brightness is about $0{\cdot}3$ mag, and the ratio of HI mass to blue diameter is about 70%, one may regard the ratio of gas to stellar mass to be a prime demarcator of Hubble type and therefore likely to at least possess a low dispersion within a given Hubble type.

However, the principal problem with these interpretations is that the low observed Tully–Fisher dispersion is attributable to a low dispersion in disk surface brightness for which no explanation is provided. This constitutes a notorious weakness in all previous Tully–Fisher relation interpretations. I propose here a simple self-regulation hypothesis, based on the self-regulation and feedback arguments in the previous section, that yields a Tully–Fisher relation with intrinsically low dispersion.

I first note that the postulated disk star formation law yields the gas evolution, since $(1-R)\mu_* + \mu_{gas} = \mu_{disk}$. From (2), the stellar surface density can be inferred to be

$$\mu_{disk} \approx \mu_* \approx \epsilon \mu_{cl} \Omega t_d\,,$$

where t_d is the duration of the early gas-rich phase of disk formation. Once feedback is important, the gas depletion time-scale can be identified as $\sim(p_{SN}/\sigma_g)\Omega^{-1}$. This enables one to infer that

$$\mu_* \approx \epsilon \mu_{cl} p_{SN}/\sigma_g\,.$$

This is the principal result: the disk surface *brightness* depends on parameters that seem reasonably robust, or at least are plausibly determined by local star formation physics. For whatever reason [e.g. photodissociation (McKee 1989), magnetic support (Myers and Goodman 1988), gravitational stability (Chieze 1987) have all been proposed], μ_{cl} is observed to have a remarkably low dispersion in the nearby interstellar medium.

Next, I substitute this expression for disk surface brightness into equation (10) to obtain

$$L_d = v_{rot}^4 \frac{\beta \sigma_g}{G \epsilon p_{SN} \mu_{cl} (M_d/L_d)} \,. \tag{11}$$

The self-regulation limit results in a quadratic star formation law and a gas surface density in the late-time limit of the solution to equation (4)

$$\mu_{gas} = \frac{p_{SN}}{Gt} \,.$$

Thus the gas-to-star surface density ratio is

$$\frac{\mu_{gas}}{\mu_*} = \frac{\sigma_g}{Gt\epsilon\mu_{cl}} \,.$$

Inserting this into expression (7) for the star formation rate results in

$$\dot{M}_* = v_{rot}^4 \left(\frac{k_m \epsilon}{G\sigma_g}\right) \left(\frac{\sigma_g}{Gt\epsilon\mu_{cl}}\right)^2 \,.$$

Incorporating appropriate numerical normalisations, one has, provided $\mu_{gas} > \mu_{cr}$, the following expressions:

$$M_d = 1 \times 10^{11} \left(\frac{n^{1/7} \sigma_{10} M_{300}}{\epsilon_{-2} E_{51}^{13/14} \mu_{160}}\right) v_{250}^4 M_\odot \,, \tag{12}$$

$$\dot{M}_* = 5{\cdot}4 \left(\frac{\sigma_{10} v_{250}^4}{\epsilon_{-2} t_{10}^2 \mu_{160}^2}\right) M_\odot \, \mathrm{yr}^{-1} \,,$$

$$\mu_* = 96 \left(\epsilon_{-2} \frac{E_{51}^{13/14} \mu_{160}}{M_{300} \sigma_{10} n^{1/7}}\right) M_\odot \, \mathrm{pc}^{-2} \,,$$

$$\frac{\mu_{gas}}{\mu_*} = 0{\cdot}14 \left(\frac{\sigma_{10}}{\epsilon_{-2} t_{10} \mu_{160}}\right) ,$$

where $v_{rot} \equiv 250 v_{250}$ km s^{-1}, the disk age $t \equiv 10^{10} t_{10}$ yr, and $\epsilon \equiv 0{\cdot}01\epsilon_{-2}$. This yields a robust prediction for disk surface brightness: the observed blue central surface brightness is (Freeman 1970; Boroson 1981)

$$\mu_d^B = 145 L_\odot \, \mathrm{pc}^{-2} \,,$$

in good agreement, at say one disk scale length, with the predicted value of μ_{disk} for a canonical IMF, disk age and mass-to-luminosity ratio. The derived value for μ_{disk} is insensitive to disk parameters other than age, and therefore, in effect, star formation history. However the M/L ratio need not be constant and independent of disk properties, although observations restrict it to be only weakly varying. For example, one finds from the fundamental plane analyses for ellipticals that $M/L \propto L^{1/6}$.

One may define the local star formation efficiency as the fraction of gas consumed at the present star formation rate in a dynamical time. This gives

$$SFE = \frac{\dot{\mu}_*}{\mu_{gas}}\, t_{dyn} = \epsilon \mu_{gas} \frac{\pi G}{\sigma_g}\, t_{dyn} = \epsilon \pi \frac{t_{dyn}}{t} \frac{p_{SN}}{\sigma_g} \approx 0{\cdot}03\,.$$

One can also evaluate the ratio of present to average past star formation rates:

$$b = (1-R)\dot{M}_* t / M_{disk} \approx (1-R)\pi\beta \frac{\mu_{gas}}{\mu_*} \approx 0{\cdot}5(1-R)\left(\frac{\sigma_{10}}{\epsilon_{-2} t_{10} \mu_{160}}\right).$$

Since the model star formation rate remains nearly constant, only declining relatively recently as $\dot{M}_* \propto t^{-2}$, it is not surprising that $b \sim 0{\cdot}3$. The self-regulation condition adjusts the gas velocity dispersion to maintain marginal instability to fragmentation, and this condition is approximately satisfied at present: for a flat rotation curve, one has

$$Q_g \equiv \frac{\kappa \sigma_g}{\pi G \mu_{gas}} = \left(\frac{\Omega \sqrt{2} \sigma_g}{\pi p_{SN}}\right) \approx 1\,.$$

In summary, the proposed law for the global rate of star formation in disk galaxies fits the usual constraints from chemical evolution, gradients in gas and abundances. Current star formation makes a significant contribution to the luminosity of disk galaxies. From Charlot and Bruzual (1991), one can see that for a constant star formation rate, about 70% of the B-band light is from main sequence stars. In the near infrared, recent spectral evolution models (Bruzual and Charlot 1992) show that AGB star and red supergiants, from massive star precursors, dominate the light of star-forming galaxies. In a starburst, most of the light after a few Gyr have elapsed is due to red giants. While a typical disk may have undergone past starbursts, it is presently described by a constant or slowly declining star formation rate. The Tully–Fisher relation, between blue or near-infrared luminosity and maximum rotational velocity, is best described by evaluating the *current* and *recent* star formation rates rather than, say, the total mass that has ever undergone star formation, although the history of star formation also plays a major role. Consequently, the past, as well as present, environments and the history of star formation all play important roles in determining the normalisation of the predicted Tully–Fisher relation, since gas outflows (and infall) will modify the present star formation rate.

The derived dependence of $\dot{M}_*$ on μ_{gas}/μ_{disk} guarantees that the Tully–Fisher relation steepens for later, more gas-rich Hubble types, and in particular, for galaxies with 21 cm linewidth below about 300 km s^{-1}, as observed. When μ_{gas} approaches μ_{cr}, star formation becomes effectively quenched as the disk stabilises. Dwarf galaxies have rising specific star formation rates, since $\mu_{cr} \propto \Omega$ increases towards lower rotational velocities: $\Omega = G\mu_{disk}/v_{rot} \propto v_{rot}^{-1}$ implies that $\mu_{cr}/\mu_{disk} \propto v_{rot}^{-1}$ and the approximate proportionality $\dot{M}_* \propto (1-\mu_{cr}/\mu_{disk})$ therefore steepens the Tully–Fisher slope. Luminous galaxies have central surface brightnesses and a Tully–Fisher relation with little intrinsic dispersion.

4. Starbursts

The preceding derivation of the formation rate describes star formation in normal disks where the gas is predominantly cold, $f_m \approx 1$. Imagine a starburst situation in which a strong tidal interaction triggers intense star formation, reducing f_m below unity. Restriction of f_m will quench the instability and star formation, leading to a new mode of self-regulation, the starburst mode. To model the star formation rate in a starburst, I proceed as follows.

Let supernova remnants overlap and produce a hot coronal medium that pressurises interstellar clouds. When the hot medium becomes sufficiently porous and dominates the volume, the cold gas fraction will diminish. Define the porosity of the hot interstellar medium by (McKee 1990)

$$P = \dot{\mu}_* \nu_{SN} / 4 H M_{SN} ,$$

where H is the disk scale height and ν_{SN} is the 4–volume occupied by an old supernova remnant halted by the pressure of the ambient medium. A factor of 2 reduction is included to allow for only about half of all supernovae being in the disk. A spherically symmetric remnant expanding into a uniform medium of pressure $p \equiv 10^4 p_4 k$ cm^{-3} K and density n fills a 4–volume

$$\nu_{SN} = 7{\cdot}82 \times 10^{12} p_4^{-1{\cdot}36} n^{-0{\cdot}11} E_{51}^{1{\cdot}26} Z^{-0{\cdot}204} \ \mathrm{pc}^3 \ \mathrm{yr} ,$$

where Z (in units of $Z_\odot$) is the metallicity.

The volume filling factor of the cold medium is $f_v = e^{-P}$, and the mass fraction is

$$f_m = \min \left[1, \left\langle \frac{\rho_{cl}}{\rho_{gas}} \right\rangle e^{-P} \right] ,$$

where $\langle \rho_{cl}/\rho_{gas} \rangle$ is the ratio of characteristic cloud density to average density of the cold disk gas. Star formation occurs if the disk is unstable to local axisymmetric perturbations, a condition that requires

$$Q \equiv \mu_{cr} / \mu_0 f_m \lesssim 1 .$$

This occurs if the porosity is not too large:

$$P \lesssim ln \left[\langle \rho_{cl}/\rho_{gas} \rangle \frac{1}{Q_0} \right] .$$

On the other hand, the triggering of star formation drives high porosity and ensuing gas outflows via a supernova-driven wind if $f_m < 1$; that is if

$$P \gtrsim ln(\langle \rho_{cl}/\rho_{gas} \rangle),$$

one can self-regulate star formation by disrupting the gas supply. A typical value for $\langle \rho_{cl}/\rho_{gas} \rangle$ is 10, and $\langle Q_0 \rangle$ is about $0{\cdot}5$. I conclude that self-regulation during a starburst restricts the porosity to $P \equiv P_0 \approx 2$–3.

I can now deduce the star formation rate during a starburst:

$$\dot{\mu}_* = 4HP_0M_{SN}\nu_{SN}^{-1} = 2{\cdot}3\times10^{-8}(H_{150}\ M_{150}\ p_4^{1\cdot36}\ n^{0\cdot11})M_\odot\,\mathrm{pc}^{-2}\,\mathrm{yr}^{-1}$$

for scale height $H = 150H_{150}$ pc, which integrates to give

$$\dot{M}_* = 20\ (\alpha_4^{-2}H_{150}\ M_{150}\ p_4^{1\cdot36}\ n^{0\cdot11})M_\odot\ \mathrm{yr}^{-1}$$

for the global star formation rate with $\alpha^{-1}\equiv 4\alpha_4^{-1}$ kpc. I conclude that in a starburst

$$\dot{M}_* \propto p^{1\cdot36}n^{0\cdot1}\alpha^{-2}H.$$

The specific star formation time-scale $t_* \equiv M_*/\dot{M}_* \propto M_*\sigma_g v_{rot}^{-6\cdot8}$.

The analogous equation for a spherical system rather than a disk results in

$$\dot{M}_* \propto p^{1\cdot4}n^{0\cdot1}r^3 \propto v^{5\cdot8}(M_{gas}/M_*)^{3/2}.$$

The star formation time-scale is $t_* = M_*^{-0\cdot9}\rho_{gas}^{-1}(\rho_*/\rho_{gas})^{1/2}$, suggesting that low mass systems may remain gas-rich until a late epoch. A porosity larger than unity results in a wind if the star formation rate is larger than this value. However, the star formation rate cannot exceed the gas supply available in spherically symmetric free-fall, namely v^3/G, as inferred from the collapse rate of an isothermal sphere, where v is the velocity dispersion appropriate to stars in circular orbits. It follows that $v \lesssim 50(M_*/M_{gas})^{1/2}$ km s^{-1}, a condition that is necessary for a wind-driven outflow to occur and restricts such a phenomenon to gas-rich dwarf galaxies undergoing a starburst. For a wind to actually be generated, one also requires $t_{cool} > t_{dyn}$, generally satisfied for dwarfs.

5. Implications

Star formation is enhanced in a high pressure environment. This includes gas interactions in a merger that utilises tidal interactions to drive non-circular orbital velocities of gas clouds (Barnes and Hernquist 1991). The resulting inelastic encounters help concentrate gas within the central kiloparsec of the merged galaxy, and the gas pressure consequently rises by an order of magnitude or more relative to the fiducial local interstellar medium value $p/k = 3600$ cm^{-3} K (Jura 1975).

First infall of gas-rich galaxies into cluster peripheries also results in enhanced ram pressure acting on the interstellar medium within the disks. Strictly speaking, since both galaxies and gas are locally subject to the same gravitational field, one might not expect there to be any ram pressure if the infall is coherent. However, the gas initially in the vicinity of a galaxy has properties determined by the local nonlinear substructure, i.e. on galaxy group scales, and should have lower pressure than interstellar gas within the galaxy. The peculiar velocities are large ($\sim$1000 km s^{-1}), the intergalactic gas is cold, and gravitational tidal fields result in non-spherically symmetric peculiar motions. Hence shocks should be inevitable that result in ram pressure driven by the galaxy peculiar velocity acting on the interstellar gas. I can crudely estimate this pressure, defined as $\rho_{ambient}\, v^2_{peculiar}$, as follows. The run of galaxy overdensity and peculiar velocity with radial distance from an Abell cluster may be calculated from the galaxy-cluster correlation function. I find that there is a factor $2\langle\rho_{gas}/\rho_{gal}\rangle\Omega^{1\cdot 2}b^{-2}$ enhancement relative to the local interstellar medium pressure at $8h^{-1}$ Mpc from the cluster centre. Here ρ_{gas} is the local intercluster gas density, ρ_{gal} is the smoothed density in galaxies, and b is the bias factor that measures the ratio of mass to galaxy density fluctuations on a scale $8h^{-1}$ Mpc.

Some support for these implications comes from the following observations. An enhancement of post-starburst galaxies is found in high redshift clusters, with a spatial distribution that avoids the centre and enhanced velocities appropriate to infall, as opposed to virialised, motion (Dressler and Gunn 1991). A high proportion of cluster galaxies have low surface brightness compared to field samples, suggestive of a past history of enhanced star formation and gas loss (Impey *et al.* 1988). Luminous spirals, undergoing star formation at a higher rate than less luminous systems, are more strongly clustered, as evidenced by enhanced spatial correlations that extend out to a scale of $\sim 8h^{-1}$ Mpc (Hamilton 1988). The QDOT redshift survey of IRAS selected galaxies reveals enhanced counts in $40h^{-1}$Mpc cells (Efstathiou *et al.* 1990) that may however be due to 'hot spots' attributable to superclusters, especially if the proposed luminosity enhancement associated with first infall is important. Elsewhere (Silk 1992), I have predicted that luminous IRAS galaxies should display stronger spatial correlations than intrinsically lower luminosity galaxies since these are preferentially selected on the basis of star formation rate. I noted that enhanced luminosity of galaxies in a region results in a spurious positive radial velocity if the Tully–Fisher relation is applied and normalised to local calibrators. The present model can lead to either inferred positive or negative radial velocities, corresponding to enhancement of luminosity during the starburst ($\dot{M}_* \propto p^{1\cdot 4}$) or fading following the starburst. A similar effect has been recognised for ellipticals (Gregg 1992) that have apparently undergone a merger within the past few Gyr.

The cosmological implications for galaxy formation are also worthy of note. Protogalaxies of a specified mass experience more efficient star formation at earlier epochs. In a simple top hat model, the density of a newly formed system (before any dissipation occurs) scales as $(1+z)^3$, so that $v \propto (1+z)^{1/2}$ and t_* decreases rapidly with increasing redshift for a given mass scale. This results in early, efficient star formation (short t_*) for systems collapsing early, and results in formation and merging of dense stellar subsystems. These systems are efficient at transferring angular momentum, to result in slowly rotating elliptical galaxies (Zurek *et al.* 1988).

Whether this process of early efficient star formation actually predominates depends on how the various mass scales evolve with redshift. For example, newly formed clouds that are generated by hierarchical collapse of gravitationally unstable primordial density fluctuations in the early universe satisfy simple scaling relations, prior to occurrence of any dissipation, namely

$$p \propto (1+z)^{4(n+2)/(n+3)},$$

$$v^2 \propto (1+z)^{(n-1)/(n+3)},$$

$$M \propto (1+z)^{-6/(n+3)}.$$

These relations assume primordial gaussian density fluctuations that undergo linear evolution in an Einstein–de Sitter universe described by evolving the power spectrum of the density field, with Fourier amplitude δ_k, as a function of wavenumber k and redshift z according to $\mid \delta_k \mid^2 \propto k^n(1+z)^{-2}$. One may infer the following. Since $t_* \propto (1+z)^{-(2\cdot 9n-3\cdot 3)/(n+3)}$, then with $0 \gtrsim n \gtrsim -1$, as expected for primordial isocurvature perturbations in a baryon-dominated universe, one finds that there is more efficient star formation at high redshift and also in systems of lower mass. This allows the possibility of producing baryonic dark matter at high redshift in compact, tightly bound primordial clusters of subgalactic mass.

On the other hand, in a cold dark-matter dominated universe seeded by primordial adiabatic density fluctuations, one has $n \approx -1\cdot 5$ on galactic scales, and $n \lesssim -2$ on subgalactic scales. Hence t_* is an increasing function of redshift and also of decreasing mass, and I conclude that early star formation is inefficient in low mass systems. This means that the most common objects of galactic scale, disk galaxies, can form from gas-rich clouds embedded within dark halos at a relatively recent epoch. The dissipative collapse of gas-rich matter tends to conserve angular momentum and results in formation of rotationally supported stellar galactic disks.

One may conclude that, whatever one's prejudice about galaxy formation theory, an understanding of the past star formation rate is central to understanding the present morphology of galaxies.

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