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pp Coulomb Phase Shift for Different Proton Charge Distributions

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Abstract

The dependence of the pp Coulomb phase shift on the form of the proton charge distribution function is studied and discussed. The uniform, exponential and Gaussian distributions are used. The dependence on the form of the charge distribution for impact parameter values less than the radius of the proton is clear. The charge distribution effect leads to a decrease in the absolute value of the Coulomb phase shift.

1. Introduction

In the work of Franco and Varma (1975) the Gaussian form of the proton charge distribution function was used to obtain the high energy pp Coulomb phase shift by considering the finite size of the incident and target protons. The result for the point charge–charge distribution case is

$$\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) = 2n[\ln(b/2a) + \tfrac{1}{2}E_1(b^2/a_\mathrm{t}^2)]\,, \tag{1}$$

where the charge distribution of the target proton is taken as

$$\rho(\bar{\boldsymbol{r}}) = (1/\pi a_\mathrm{t}^2)^{3/2}\exp(-r^2/a_\mathrm{t}^2)\,, \tag{2}$$

$\bar{\boldsymbol{b}}$ is the two-dimensional impact parameter vector, a is the screen radius parameter of the Coulomb potential, $n = e^2/\hbar v$ with the relative velocity v of the incident proton and the charge e of proton, and $E_1(x)$ is the exponential integral. In the case where the charge distributions of the incident and target protons are taken into account, Franco and Varma (1975) obtained the result

$$\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) = 2n[\ln(b/2a) + \tfrac{1}{2}E_1\{b^2/(a_\mathrm{i}^2 + a_\mathrm{t}^2)\}]\,, \tag{3}$$

where a_i is the corresponding parameter for the incident proton with the same form of charge distribution (2).

For a uniform charge distribution of the target proton

$$\rho(\bar{\boldsymbol{r}}) = \begin{cases} \alpha, & r \le a_\mathrm{t} \quad (4\mathrm{a}) \\ 0, & r > a_\mathrm{t}\,, \quad (4\mathrm{b}) \end{cases}$$

and for a point charge incident proton, the pp Coulomb phase shift is (Fäldt and Hulthage 1978)

$$\begin{aligned}\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) &= 2n[\ln\{(a_{\mathrm{t}} + \sqrt{a_{\mathrm{t}}^2 - b^2})/2a\} - \tfrac{1}{3}(\sqrt{a_{\mathrm{t}}^2 - b^2}/a_{\mathrm{t}})^3 \\ &\quad - \sqrt{a_{\mathrm{t}}^2 - b^2}/a_{\mathrm{t}}]\,, \qquad b \leq a_{\mathrm{t}} \qquad (5\mathrm{a}) \\ &= 2n\ln(b/2a)\,, \qquad b > a_{\mathrm{t}}\,. \qquad (5\mathrm{b})\end{aligned}$$

The constant $\alpha = 3/4\pi a_{\mathrm{t}}^3$ is determined from the condition $\int \rho(\bar{\boldsymbol{r}})\, \mathrm{d}\bar{\boldsymbol{r}} = 1$.

In this work we try to obtain the pp Coulomb phase shift in the case of a uniform charge distribution of the incident and target protons, the generalisation of equation (5). Also, we consider an exponential charge distribution for the proton and try to obtain the corresponding forms of the Coulomb phase shift. Further, one purpose of this study is to examine the dependence of the pp Coulomb phase shift on the form of the proton charge distribution function. Of course, the results can be used in particle–nucleus and nucleus–nucleus scattering cases with the same form of charge distribution function, where $n = Z_{\mathrm{i}}\, Z_{\mathrm{t}}\, e^2/\hbar v$ and $Z_{\mathrm{i}}, Z_{\mathrm{t}}$ are the charge numbers of the incident and target particles.

2. General Formalism

The Coulomb phase shift in the high energy approximation is given by (Glauber 1959)

$$\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) = -(1/\hbar v)\int_{-\infty}^{\infty} V(\bar{\boldsymbol{r}})\, \mathrm{d}z\,, \qquad (6)$$

where $\bar{\boldsymbol{r}} = \bar{\boldsymbol{b}} + \hat{\boldsymbol{k}}z$ is the position vector of the incident particle relative to the target particle, $\hat{\boldsymbol{k}}$ is the unit vector in the direction of the z axis (taken in the incident direction) and $V(\bar{\boldsymbol{r}})$ is the Coulomb potential energy. This integral diverges at both extremes of the integration over z. To avoid this problem, a screened Coulomb potential is used (Glauber 1959). In the point charge case, the screened Coulomb potential energy for the proton–proton interaction is given by

$$V(\bar{\boldsymbol{r}}) = \frac{e^2}{r}\, F(\boldsymbol{r})\,,$$

where $F(\boldsymbol{r})$ is a screen function which tends to zero as $\boldsymbol{r}$ tends to infinity. Therefore, for the two charge distributions $\rho_{\mathrm{t}}(\bar{\boldsymbol{r}}_1)$ and $\rho_{\mathrm{i}}(\bar{\boldsymbol{r}}_2)$ of the target and incident protons, the screened potential energy of the two charge elements $e\rho_{\mathrm{t}}(\bar{\boldsymbol{r}}_1)\, \mathrm{d}\bar{\boldsymbol{r}}_1$ and $e\rho_{\mathrm{i}}(\bar{\boldsymbol{r}}_2)\, \mathrm{d}\bar{\boldsymbol{r}}_2$ can be written as

$$\frac{e^2\rho_{\mathrm{t}}(\bar{\boldsymbol{r}}_1)\, \rho_{\mathrm{i}}(\bar{\boldsymbol{r}}_2)\, \mathrm{d}\bar{\boldsymbol{r}}_1\, \mathrm{d}\bar{\boldsymbol{r}}_2}{|\bar{\boldsymbol{r}} + \bar{\boldsymbol{r}}_2 - \bar{\boldsymbol{r}}_1|}\, F(|\bar{\boldsymbol{r}} + \bar{\boldsymbol{r}}_2 - \bar{\boldsymbol{r}}_1|)\,,$$

where $\bar{\boldsymbol{r}}_1$ and $\bar{\boldsymbol{r}}_2$ are taken relative to the centre of mass of the particle in each case and $\bar{\boldsymbol{r}}$ is defined as above. Note that $\int \mathrm{d}\bar{\boldsymbol{r}}_1\, \rho_{\mathrm{t}}(\bar{\boldsymbol{r}}_1) = \int \mathrm{d}\bar{\boldsymbol{r}}_2\, \rho_{\mathrm{i}}(\bar{\boldsymbol{r}}_2) = 1$. Thus, in general, the screened Coulomb potential energy can be written in the form

$$V(\bar{r}) = e^2 \iint \mathrm{d}\bar{r}_1\, \mathrm{d}\bar{r}_2\, \frac{\rho_{\mathrm{t}}(\bar{r}_1)\, \rho_{\mathrm{i}}(\bar{r}_2)}{|\bar{r}+\bar{r}_2-\bar{r}_1|}\, F(|\bar{r}+\bar{r}_2-\bar{r}_1|)\,. \tag{7}$$

Making use of equations (6) and (7) we get

$$\begin{aligned}\chi^{(\mathrm{C})}(\bar{b}) \;=\; & \iint \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\bar{r}_2\, \rho_{\mathrm{t}}(\bar{r}_1)\, \rho_{\mathrm{i}}(\bar{r}_2) \\ & \times \left(-n \int_{-\infty}^{\infty} \mathrm{d}z\, F(|\bar{r}+\bar{r}_2-\bar{r}_1|)/|\bar{r}+\bar{r}_2-\bar{r}_1| \right),\end{aligned} \tag{8}$$

but

$$\chi^{(\mathrm{C})}_{\mathrm{pt}}(\bar{b},\, \bar{s}_1,\, \bar{s}_2) = -n \int_{-\infty}^{\infty} \mathrm{d}z\, \frac{F(|\bar{r}+\bar{r}_2-\bar{r}_1|)}{|\bar{r}+\bar{r}_2-\bar{r}_1|} \tag{9}$$

is considered as a point charge phase shift, where $\bar{s}_1$ and $\bar{s}_2$ are the projections of $\bar{r}_1$ and $\bar{r}_2$ on the impact plane of $\bar{b}$, so that

$$\chi^{(\mathrm{C})}(\bar{b}) = \iint d\bar{r}_1\, d\bar{r}_2\, \rho_{\mathrm{t}}(\bar{r}_1)\, \rho_{\mathrm{i}}(\bar{r}_2)\, \chi^{(\mathrm{C})}_{\mathrm{pt}}(\bar{b},\, \bar{s}_1,\, \bar{s}_2)\,. \tag{10}$$

Using the substitution $\bar{r}' = \bar{r} + (\bar{r}_2 - \bar{r}_1)$ and noting that the vector $\bar{r}_2 - \bar{r}_1$ is fixed during the integration over z, we can write $\chi^{(\mathrm{C})}_{\mathrm{pt}}$ in the form

$$\chi^{(\mathrm{C})}_{\mathrm{pt}}(\bar{b}') = -n \int_{-\infty}^{\infty} \mathrm{d}z'\, \frac{F(\bar{r}')}{\bar{r}'}\,,$$

where $\bar{r}' = \bar{b}' + \hat{k}z'$ and $\bar{b}' = \bar{b} + \bar{s}_2 - \bar{s}_1$.

For the screen function

$$F(r) = \begin{cases} 1, & r \leq a \\ 0, & r > a\,, \end{cases}$$

the point charge Coulomb phase shift $\chi^{(\mathrm{C})}_{\mathrm{pt}}(\bar{b}')$ is given by (Glauber 1959)

$$\chi^{(\mathrm{C})}_{\mathrm{pt}}(\bar{b}') = 2n \ln(b'/2a)\,,$$

i.e.

$$\chi^{(\mathrm{C})}_{\mathrm{pt}}(\bar{b},\, \bar{s}_1,\, \bar{s}_2) = 2n \ln \frac{|\bar{b} + \bar{s}_2 - \bar{s}_1|}{2a}\,. \tag{11}$$

Therefore we get

$$\chi^{(\mathrm{C})}(\bar{b}) = 2n \iint \mathrm{d}\bar{r}_1\, \mathrm{d}\bar{r}_2\, \rho_{\mathrm{t}}(\bar{r}_1)\, \rho_{\mathrm{i}}(\bar{r}_2) \ln(|\bar{b} + \bar{s}_2 - \bar{s}_1|/2a)\,. \tag{12}$$

If we consider only the charge distribution of the target proton and take the incident proton to be a point charge, i.e. we take $\rho_{\mathrm{i}}(\bar{r}_2)$ to be the Dirac function $\delta(\bar{r}_2)$, equation (12) becomes

$$\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) = 2n \int \mathrm{d}\bar{\boldsymbol{r}}_1\, \rho_{\mathrm{t}}(\bar{\boldsymbol{r}}_1) \ln(|\bar{\boldsymbol{b}} - \bar{\boldsymbol{s}}_1|/2a)\,. \tag{13}$$

3. Some Explicit Forms

After long and complicated derivations, which are omitted here, some explicit forms for the pp Coulomb phase shift are obtained and we present the results in the following.

(3a) Uniform Distribution Case

Considering the uniform charge distribution, equation (4), for the incident and target protons, we have the following three forms—depending on the range of b—for the pp Coulomb phase shift. For $0 \le b \le a_{\mathrm{i}}$,

$$\begin{aligned}
\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) = \frac{6n}{\pi a_{\mathrm{i}}^3}\bigg\{ & 2\int_0^{a_{\mathrm{i}}-b} \mathrm{d}u\; u\sqrt{a_{\mathrm{i}}^2-(u-b)^2}\bigg(\ln\frac{a_{\mathrm{t}}+\sqrt{a_{\mathrm{t}}^2-u^2}}{2a} \\
& - \frac{1}{3a_{\mathrm{t}}^3}(a_{\mathrm{t}}^2-u^2)^{3/2} - \frac{1}{a_{\mathrm{t}}}(a_{\mathrm{t}}^2-u^2)^{1/2}\bigg) E\bigg(\frac{\pi}{2},\, \sqrt{\frac{4ub}{a_{\mathrm{i}}^2-(u-b)^2}}\bigg) \\
& + \frac{1}{\sqrt{b}}\int_{a_{\mathrm{i}}-b}^{a_{\mathrm{t}}} \mathrm{d}u\; \sqrt{u}\bigg(\ln\frac{a_{\mathrm{t}}+\sqrt{a_{\mathrm{t}}^2-u^2}}{2a} - \frac{1}{3a_{\mathrm{t}}^3}(a_{\mathrm{t}}^2-u^2)^{3/2} \\
& - \frac{1}{a_{\mathrm{t}}}(a_{\mathrm{t}}^2-u^2)^{1/2}\bigg)\bigg[[a_{\mathrm{i}}^2-(u+b)^2]F\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg) \\
& + 4ubE\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg)\bigg] + \frac{1}{\sqrt{b}}\int_{a_{\mathrm{t}}}^{b+a_{\mathrm{i}}} \mathrm{d}u\; \sqrt{u}\ln(u/2a) \\
& \times \bigg[[a_{\mathrm{i}}^2-(u+b)^2]F\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg) + 4ubE\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg)\bigg]\bigg\},
\end{aligned} \tag{14a}$$

for $a_{\mathrm{i}} \le b \le a_{\mathrm{i}}+a_{\mathrm{t}}$,

$$\begin{aligned}
\chi^{(\mathrm{C})}(\bar{\boldsymbol{b}}) = \frac{6n}{\pi a_{\mathrm{i}}^3\sqrt{b}}\bigg\{ & \int_{b-a_{\mathrm{i}}}^{a_{\mathrm{t}}} \mathrm{d}u\; \sqrt{u}\bigg[\ln\bigg(\frac{a_{\mathrm{t}}+\sqrt{a_{\mathrm{t}}^2-u^2}}{2a}\bigg) \\
& - \frac{1}{3a_{\mathrm{t}}^3}(a_{\mathrm{t}}^2-u^2)^{3/2} - \frac{1}{a_{\mathrm{t}}}(a_{\mathrm{t}}^2-u^2)^{1/2}\bigg]\bigg[[a_{\mathrm{i}}^2-(u+b)^2]F\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg) \\
& + 4ubE\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg)\bigg] + \int_{a_{\mathrm{t}}}^{b+a_{\mathrm{i}}} \mathrm{d}u\; \sqrt{u}\ln\bigg(\frac{u}{2a}\bigg)\bigg[[a_{\mathrm{i}}^2-(u+b)^2] \\
& \times F\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg) + 4ubE\bigg(\frac{\pi}{2},\, \sqrt{\frac{a_{\mathrm{i}}^2-(u-b)^2}{4ub}}\bigg)\bigg]\bigg\},
\end{aligned} \tag{14b}$$

and finally, for $b \geq a_{\rm i} + a_{\rm t}$,

$$\chi^{(\rm C)}(\bar{\boldsymbol{b}}) = 2n \ln(b/2a)\,, \tag{14c}$$

where $F(\phi, K)$ and $E(\phi, K)$ are the elliptic integrals of first and second kinds.

(3b) Exponential Distribution Case

In the case of a charge density function of the form

$$\rho_\ell(\bar{\boldsymbol{r}}) = \frac{1}{8\pi a_\ell^3}\exp(-r/a_\ell), \qquad \ell = {\rm t, i}\,, \tag{15}$$

when we consider the charge distribution of the target proton only, equation (13), we have

$$\chi^{(\rm C)}(\bar{\boldsymbol{b}}) = 2n\{\ln(b/2a) + \tfrac{1}{2}[K_0(b/a_{\rm t}) + (b/a_{\rm t})K_1(b/a_{\rm t})]\}\,, \tag{16}$$

$K_\nu(Z)$ is the modified Bessel function of the third kind.

The asymptotic forms of this formula are simple and may be useful. For $b/a_{\rm t} \to \infty$, since

$$K_\nu(b/a_{\rm t}) \approx (\pi a_{\rm t}/2b)^{1/2}\exp(-b/a_{\rm t})\,,$$

we get

$$\chi^{(\rm C)}(\bar{\boldsymbol{b}}) \approx 2n[\ln(b/2a) + \tfrac{1}{2}(\pi a_{\rm t}/2b)^{1/2}(1 + b/a_{\rm t})\exp(-b/a_{\rm t})]\,. \tag{17a}$$

Also, for $b/a_{\rm t} \to 0$, since $K_0(b/a_{\rm t}) \approx -\ln(b/2a_{\rm t})$ and $K_1(b/a_{\rm t}) \approx a_{\rm t}/b$, we have

$$\chi^{(\rm C)}(\bar{\boldsymbol{b}}) = 2n\{\ln(b/2a) + \tfrac{1}{2}[1 - \ln(b/2a_{\rm t})]\}\,. \tag{17b}$$

In the general case, where the charge distributions of the incident and target protons are considered, we have

$$\begin{aligned}
\chi^{(\rm C)}(\bar{\boldsymbol{b}}) = 2n\Big\{&\ln(b/2a) + \tfrac{1}{2}[K_0(b/a_{\rm t}) + (b/a_{\rm t})K_1(b/a_{\rm t})] + (1/4a_{\rm t}^3\, b)\\
&\times \bigg(\int_0^b {\rm d}b'\, b' K_1(b'/a_{\rm t})\Big[bb' I_0(b'/a_{\rm i})\, K_0(b'/a_{\rm i}) + 2(b^2 + b'^2)\Big(K_1(b/a_{\rm i})\\
&\times I_1(b'/a_{\rm i}) + \sum_{n=1}^{\infty}(2n+1)K_{2n+1}(b/a_{\rm i})\, I_{2n+1}(b'/a_{\rm i})\Big) - 2bb'\sum_{n=1}^{\infty} 2nK_{2n}(b/a_{\rm i})\\
&\times I_{2n}(b'/a_{\rm i})\Big] + \int_b^{\infty} {\rm d}b'\, b' K_1(b'/a_{\rm t})\Big[bb' I_0(b/a_{\rm i})\, K_0(b/a_{\rm i}) + 2(b^2 + b'^2)\\
&\times \Big(I_1(b/a_{\rm i})\, K_1(b'/a_{\rm i}) + \sum_{n=1}^{\infty}(2n+1) I_{2n+1}(b/a_{\rm i})\, K_{2n+1}(b'/a_{\rm i})\Big) - 2bb'\\
&\times \sum_{n=1}^{\infty} 2n I_{2n}(b/a_{\rm i})\, K_{2n}(b'/a_{\rm i})\Big]\bigg)\Big\}\,,
\end{aligned} \tag{18}$$

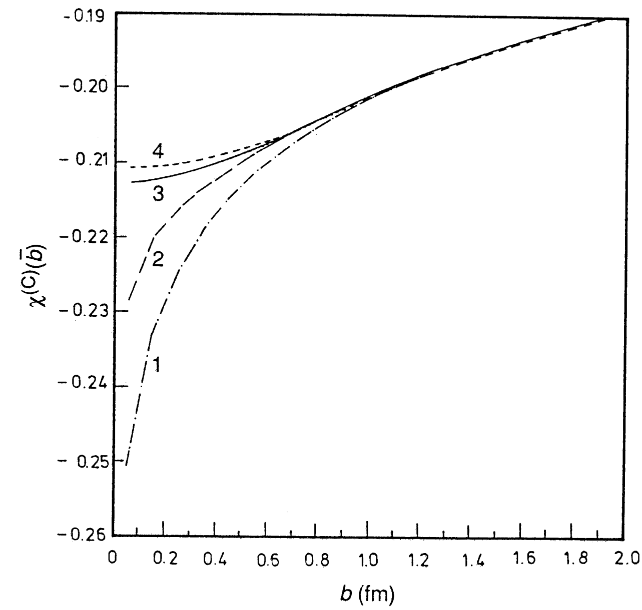

Fig. 1. pp Coulomb phase shift for different forms of the charge distribution function of the target proton. The incident proton is considered to be a point charge. Curves 1, 2, 3 and 4 correspond to the point charge, exponential, Gaussian and uniform forms respectively.

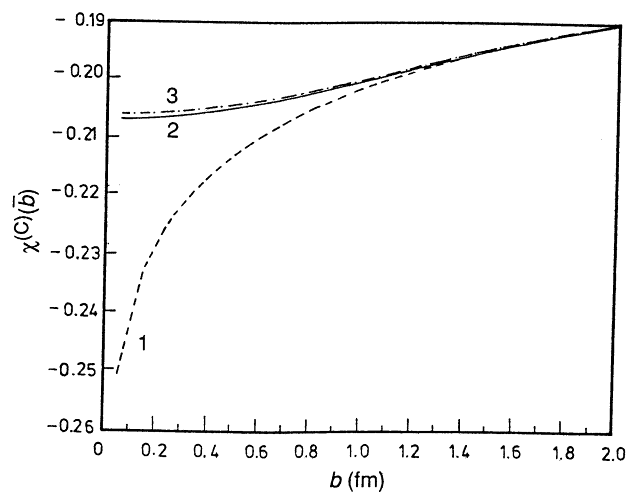

Fig. 2. pp Coulomb phase shift taking into account the charge distribution of both the incident and target protons. Curves 1, 2 and 3 represent the point charge, Gaussian and uniform distributions respectively.

where $I_\nu(z)$ is the modified Bessel function of the first kind.

4. Discussion

As an example, calculations of the Coulomb phase shift for proton–proton scattering at 1 GeV according to equations (1), (5) and (16), where the charge distribution of the target proton only is considered, are represented in Fig. 1. Also, calculations according to equations (3) and (17), for example, where the charge distributions of the incident and target protons are considered, are represented in Fig. 2. Fig. 3 represents the different forms of proton charge distribution that are taken into account in the calculations of the Coulomb phase shift. The parameters a_i and a_t for the incident and target protons are taken to be the same. For uniform, Gaussian and exponential distributions the parameter a_t (or a_i) is equal to $1\cdot07$, $0\cdot678$ and $0\cdot237$ fm respectively. These values correspond to the value $0\cdot82$ fm for the r.m.s. radius of the proton, which is known from electron scattering. The radius parameter of the screened Coulomb potential a is taken to be the Bohr radius ($=0\cdot529\times10^5$ fm). At an energy of 1 GeV the constant n is $0\cdot0083$.

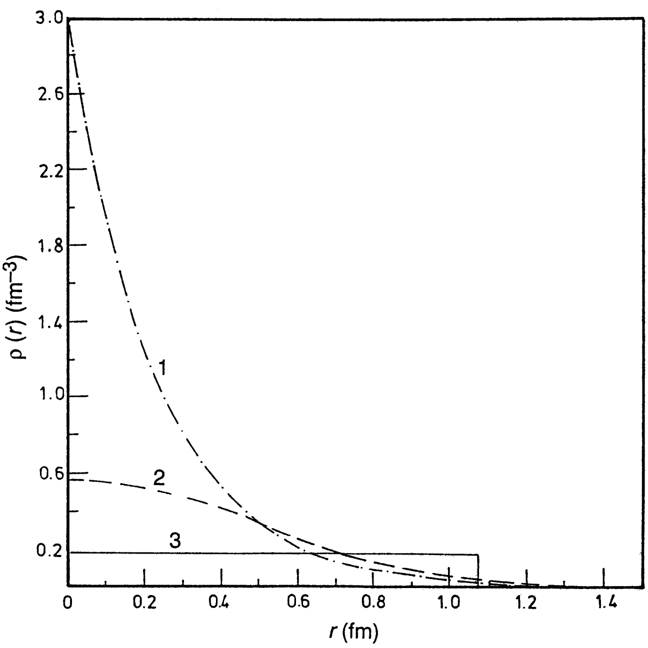

Fig. 3. Charge distribution functions where curves 1, 2 and 3 represent the exponential, Gaussian and uniform forms respectively.

Figs 1 and 2 show that the pp Coulomb phase shift depends on the form of the charge distribution function for impact parameter values less than the radius of the proton where we have the largest absolute values of the phase shift function. For impact parameter values larger than this radius, the Coulomb phase shift is essentially the same for all forms of charge density and is equal to the point charge phase shift.

The charge distribution effect leads to a decrease in the absolute value of the Coulomb phase shift, and a more concentrated charge has the largest absolute value of the Coulomb phase shift. We also note that the difference between the phase shifts of the uniform and Gaussian distributions is very small, especially when we consider the charge distributions of the two colliding particles.

These results can be interpreted as follows. The transformation from the point charge to the charge distribution case leads to a decreasing Coulomb force on the proton in the region where the charge density has not vanished. Therefore, the Coulomb effect decreases, i.e. the absolute value of the Coulomb phase shift decreases in this region which corresponds to b less than or equal to the radius of the proton. At the same time, the finite size of the proton has some effect on the calculation of proton scattering at high energy and to obtain more accurate results for the Coulomb effect we must consider the charge distribution of the proton (Franco and Varma 1975). Thus, we can conclude that for b less than or equal to the radius of the proton, the incident proton must pass through the target proton in some way, otherwise the charge distribution has no effect.

The most realistic distribution of these three is the exponential form, which gives good agreement with the experimental data on charge form factors and cross sections of pp scattering at medium and high energy (Elton 1961; Kuroda and Miyazaya 1973; Schiz *et al.* 1981; Fajardo *et al.* 1981). The corresponding form factor for this distribution—the dipole form factor $1/(1+\mu t)^2$ where μ is a constant and t is the square momentum transfer—is used to describe pp scattering where the composite model of particles is used (Kuroda and Miyazaya 1973; Wakaizumi 1978). In this model the proton is made of structureless objects (quarks) which are distributed over an extension (Harrington and Pagnamenta 1968).

Therefore, to describe the Coulomb part of pp scattering we can also use the same composite model with multiple scattering theory, as in the case of the strong interaction. Transmission of the incident proton through the target proton in this situation may be understood as a multi-scattering process.

Of course, this composite model of the proton is used essentially in pp scattering at very high energy, where the incident proton (with very short wavelength) is more sensitive to the internal structure of the target proton. But, since the Coulomb phase shift is sensitive to the form of the proton charge distribution at any energy, it is sensitive to the internal structure of the proton at this energy and we can use the same model to interpret the results.

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