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Spontaneously Symmetry-broken Current-carrying States in Nanostructures*

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Abstract

We show that in nanostructure geometry, by approaching the limiting behaviour of a two-dimensional system with large inhomogeneity in the electron density, a spontaneous current-carrying state can develop. We calculate the current patterns of this state and demonstrate that it *cannot* correspond to the chiral state required to support anyon superconductivity.

1. Introduction

Nanostructures provide the possibility for new transport and ground state properties of interacting Fermi systems. In this paper we focus on a repeated two-dimensional (2D) superlattice structure in which 2D conducting layers are separated by wide insulating layers. Now for low electron density (i.e. large r_s) the system spontaneously forms spin polarisations (Ceperley 1978). The issue we address in this paper is whether spontaneously broken current-carrying states (SBCCS) are also a realistic possibility in such an itinerant 2D system? The answer is yes, provided the system is sufficiently inhomogeneous. This is particularly relevant to high-T_c superconducting lattices whose copper–oxygen (CuO) conduction layers are highly 2D, and even more so to the superlattice structures of these materials (see below). For these materials a spontaneously formed chiral ground state (mimicking the properties of the Laughlin ground state in the *presence* of an external magnetic field) has been proposed and from which anyon superconductivity has been suggested to evolve. We show that while a SBCCS is a realistic possibility its properties are those of an orbital ferromagnet and *not* a chiral fluid.

Traditionally, spin polarised ground states are favoured theoretically in a weakly inhomogeneous Fermi liquid over SBCCS because, although χ_L is roughly of the same magnitude as χ_p (Vignale *et al.* 1988), it is of opposite sign to χ_p suggesting that a SBCCS is unstable. Actually, it relates to the difference of broken symmetry in a gauge field (current carrying) and a static field; weakly inhomogeneous SBCCS can be transformed by Galilean invariance, to a lower state. Let us see it more explicitly.

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In the presence of a gauge field $\boldsymbol{A}(\boldsymbol{r})$ and a scalar potential $V(\boldsymbol{r})$, the interacting Hamiltonian $\hat{H}_0$ of a uniform Fermi liquid in d dimensions becomes

$$\hat{H} = \hat{H}_0 + \int 2\mathrm{d}^d r \Big(\hat{n}(\boldsymbol{r})\, V(\boldsymbol{r}) + \mu \hat{\boldsymbol{S}}(\boldsymbol{r}) \cdot \boldsymbol{B}(\boldsymbol{r}) - [e\hat{\boldsymbol{j}}_{\mathrm{p}}(\boldsymbol{r})/c] \cdot \boldsymbol{A}(\boldsymbol{r}) + [e^2 \hat{n}(\boldsymbol{r})/2mc^2]\, |\boldsymbol{A}(\boldsymbol{r})|^2 \Big), \tag{1}$$

where $\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$, $\hat{\boldsymbol{j}}_{\mathrm{p}}(\boldsymbol{r})$ is the paramagnetic current, $\hat{n}(\boldsymbol{r})$ the density and $\hat{S}(\boldsymbol{r})$ the spin operators. The full current operator $\hat{\boldsymbol{j}}(\boldsymbol{r})$ is given by $\hat{\boldsymbol{j}}(\boldsymbol{r}) = \hat{\boldsymbol{j}}_{\mathrm{p}}(\boldsymbol{t}) - [e\hat{n}(\boldsymbol{r})/mc]\boldsymbol{A}(\boldsymbol{r})$.

Generally, the ground state energy $E \equiv \langle \phi_{\mathrm{G}} | \hat{H} | \phi_{\mathrm{G}} \rangle$ is a unique functional of the density $n(\boldsymbol{r}) \equiv \langle \phi_{\mathrm{G}} | \hat{n}(\boldsymbol{r}) | \phi_{\mathrm{G}} \rangle$, the spin density $\boldsymbol{S}(\boldsymbol{r}) \equiv \langle \phi_{\mathrm{G}} | \hat{\boldsymbol{S}}(\boldsymbol{r}) | \phi_{\mathrm{G}} \rangle$ and paramagnetic current $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r}) \equiv \langle \phi_{\mathrm{G}} | \hat{\boldsymbol{j}}_{\mathrm{p}}(\boldsymbol{r}) | \phi_{\mathrm{G}} \rangle$ of the following form:

$$E = \int \mathrm{d}^d r\, E\Big(n(\boldsymbol{r}),\, \boldsymbol{S}(\boldsymbol{r}),\, \nabla \times \big(\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})/n(\boldsymbol{r})\big) \Big), \tag{2}$$

where we assumed that $\boldsymbol{B}(\boldsymbol{r})$ points along one direction (Vignale and Rasolt 1988).

For a weak $\boldsymbol{A}(\boldsymbol{r})$ and when we set $V(\boldsymbol{r}) = 0$, then

$$E \equiv \int \mathrm{d}^d r\, E\Big(n_0,\, \boldsymbol{S}(\boldsymbol{r}),\, \nabla \times \big(\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})/n_0\big) \Big) = \tfrac{1}{10} \int \mathrm{d}^d r\, \mathrm{d}^d r'$$

$$\times \Big(S_\alpha(\boldsymbol{r})\, \tilde{\chi}_{\mathrm{p}\alpha\beta}(\boldsymbol{r} - \boldsymbol{r}') S_\beta(\boldsymbol{r}') + \mathcal{V}_\alpha(\boldsymbol{r})\, \tilde{\chi}_{\mathrm{L}\alpha\beta}(\boldsymbol{r} - \boldsymbol{r}')\, \mathcal{V}_\beta(\boldsymbol{r}') + \delta(\boldsymbol{r} - \boldsymbol{r}') \frac{m}{n_0} |\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})|^2 \Big)$$

$$+ \int \mathrm{d}^d r \Big(\mu \boldsymbol{S}(\boldsymbol{r}) \cdot \boldsymbol{B}(\boldsymbol{r}) - \frac{e}{c} \boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r}) \cdot \boldsymbol{A}(\boldsymbol{r}) + \frac{e^2 n_0}{2mc^2} |\boldsymbol{A}(\boldsymbol{r})|^2 \Big), \tag{3}$$

with $\mathcal{V}(\boldsymbol{r}) = \nabla \times \big(\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})/n_0\big)$ and where $\tilde{\chi}$ is related to the inverse of χ.

If the phase transition to spin polarisation or SBCCS is second order, then equation (3) will describe it rigorously. It will occur when $S_\alpha(\boldsymbol{q})\, \tilde{\chi}_{\mathrm{p}\alpha\beta}(\boldsymbol{q})\, S_\beta(\boldsymbol{q}) < 0$ or when $\mathcal{V}_\alpha(\boldsymbol{q})\, \tilde{\chi}_{\mathrm{L}\alpha\beta}(\boldsymbol{q})\, \mathcal{V}_\beta(\boldsymbol{q}) + (m/n_0)\, |\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{q})|^2 < 0$. The difference between spin polarisation and SBCCS is now obvious. From the definition of $\mathcal{V}(\boldsymbol{r})$ the term $(m/n_0)\, |\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{q})|^2$ will clearly not permit such a SBCCS unless the current density is highly nonuniform. We should also add that such considerations apply as well to neutral Fermi liquids like ^{3}He; e.g. we can attach a fictitious charge to ^{3}He and then get rid of it at the end to arrive at (2) and (3). Actually, equation (3) is more appropriate to neutral Fermi liquids since we will be neglecting the energy $B^2/8\pi$ of the electromagnetic field. This will be of little consequence to the 2D and 3D electrons in the normal state (except in the formation of large domain structures). In the superconducting state this term must be included. However, for superfluid ^{3}He, equation (3) is valid both above and below the superfluid transition.

Aside from strong inhomogeneities, 2D is also very important for SBCCS. For example, *unique to 2D* the energy of noninteracting fermions in a uniform magnetic field is *equal* to the energy in its absence, when the Landau levels are filled. This is different to the 3D Landau diamagnetism discussed above and encouraging for SBCCS in 2D. The inhomogeneity further strongly enhances the 2D SBCCS, as can be seen from the work of Hasegawa *et al.* (1989) who put the noninteracting electrons on a lattice. The Hamiltonian of (1) is now

$$\hat{H} = -\sum_{i,j\sigma} t_{ij}\, C^{\dagger}_{i\sigma}\, C_{j\sigma}\,; \qquad t_{ij} = t\, \exp\left(-\mathrm{i}\, 2\pi \int_i^j \boldsymbol{A}\,.\,\mathrm{d}\boldsymbol{\ell}\right), \tag{4}$$

with $\boldsymbol{A}(\boldsymbol{r}) = B(0, x, 0)$. Its ground state energy for several different crystal structures is *lower* than when $B = 0$, for many appropriate ratios of electron densities and magnetic fluxes.

2. Calculation of the SBCCS

We calculated the SBCCS using the results of Vignale and Rasolt (1988). One important aspect of this theory is its functional dependence on $\boldsymbol{\mathcal{V}}(\boldsymbol{r})$, which in a rigorous way transforms the essentially 'surface structure' of the energy of a Fermi system to a bulk property (see e.g. Section 3 of Halperin *et al.* 1989). Very briefly, E in equation (2) is written as

$$E\Big(n(\boldsymbol{r}),\, \nabla \times \big(j_{\mathrm{p}}(\boldsymbol{r})/n(\boldsymbol{r})\big)\Big) = E_{\mathrm{i}}\big(n(\boldsymbol{r}),\, \boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})\big) + \left(n(\boldsymbol{r})\, V(\boldsymbol{r}) - \frac{e}{c}\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})\,.\,\boldsymbol{A}(\boldsymbol{r}) + \frac{e^2 n(\boldsymbol{r})}{2mc}\,|\,\boldsymbol{A}(\boldsymbol{r})\,|^2\right), \tag{5a}$$

where

$$E_{\mathrm{i}}\big(n(\boldsymbol{r}),\, \boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})\big) = T_{\mathrm{s}}\big(n(\boldsymbol{r}),\, \boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})\big) + E_{\mathrm{H}}\big(n(\boldsymbol{r})\big) + E_{\mathrm{xc}}\Big(n(\boldsymbol{r}),\, \nabla \times \big(j_{\mathrm{p}}(\boldsymbol{r})/n(\boldsymbol{r})\big)\Big), \tag{5b}$$

and where T_{s} is the kinetic energy of a noninteracting fermion liquid with density $n(\boldsymbol{r})$ and paramagnetic current $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})$. Here $E_{\mathrm{H}}\big(n(\boldsymbol{r})\big)$ is the Hartree energy and E_{xc} the exchange and correlation. If we choose the Kohn–Sham (1965) approach to T_{s}, then

$$T_{\mathrm{s}}\big(n(\boldsymbol{r}),\, \boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})\big) = -\frac{\hbar^2}{2m}\, 2 \sum_{\boldsymbol{k}} \phi^*_{\boldsymbol{k}}(\boldsymbol{r})\, \nabla^2 \phi_{\boldsymbol{k}}(\boldsymbol{r})\,, \tag{5c}$$

with $n(\boldsymbol{r}) = 2\Sigma_{\boldsymbol{k}} \phi_{\boldsymbol{k}}(\boldsymbol{r}) \phi_{\boldsymbol{k}}(\boldsymbol{r})$ and $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r}) = (\hbar/2m) 2\Sigma_{\boldsymbol{k}} [\phi^*_{\boldsymbol{k}}(\boldsymbol{r}) \nabla \phi_{\boldsymbol{k}}(\boldsymbol{r}) - \nabla \phi^*_{\boldsymbol{k}}(\boldsymbol{r}) \phi_{\boldsymbol{k}}(\boldsymbol{r})]$. Minimising (5) with respect to the ϕ, at fixed $V(\boldsymbol{r})$ and $\boldsymbol{A}(\boldsymbol{r})$ we get (Vignale

and Rasolt 1988)

$$\lim_{A(r)\to 0}\left\{\frac{1}{2m}\left[-i\hbar\nabla+\frac{e}{c}\Big(\boldsymbol{A}(\boldsymbol{r})+\boldsymbol{A}_{xc}(\boldsymbol{r})\Big)\right]\right.$$
$$\left.+\frac{e}{2mc^2}\left[\boldsymbol{A}^2(\boldsymbol{r})-\Big(\boldsymbol{A}(\boldsymbol{r})+\boldsymbol{A}_{xc}(\boldsymbol{r})\Big)^2\right]+V_{\text{eff}}(\boldsymbol{r})+V_{xc}(\boldsymbol{r})\right\}\phi_k(\boldsymbol{r})=\epsilon_k\,\phi_k(\boldsymbol{r}), \quad (6a)$$

$$\frac{e}{c}\boldsymbol{A}_{xc}(\boldsymbol{r})=\frac{1}{n(\boldsymbol{r})}\nabla\times\left(\frac{\delta E_{xc}\big(n(\boldsymbol{r}),\boldsymbol{\mathcal{V}}(\boldsymbol{r})\big)}{\delta\boldsymbol{\mathcal{V}}(\boldsymbol{r})}\right),$$
$$V_{xc}(\boldsymbol{r})=\delta\bar{E}_{xc}\big(n(\boldsymbol{r}),\boldsymbol{j}_p(\boldsymbol{r})\big)/\delta n(\boldsymbol{r}) \quad (6b)$$

[with $V_{\text{eff}}(\boldsymbol{r})=V(\boldsymbol{r})+V_H(\boldsymbol{r})$]. The dependence on $\boldsymbol{S}(\boldsymbol{r})$ [in equation (2)] is removed because we are searching below for SBCCS; i.e. when $\boldsymbol{A}(\boldsymbol{r})$ and $\boldsymbol{B}(\boldsymbol{r})\to 0$. There is one additional technical point concerning $\lim \boldsymbol{A}(\boldsymbol{r})\to 0$ in (6a) or (5a). We need to consider the internal energy E_i in (5a), or the solution of (6), bearing an infinitesimal $\boldsymbol{A}(\boldsymbol{r})$ to fix the gauge! If we use the Thomas–Fermi approach, then T_s in (5b) becomes a direct functional of $n(\boldsymbol{r})$ and $\boldsymbol{j}_p(\boldsymbol{r})$ which simplifies the search for a possible SBCCS considerably (see below).

Now from (6) the transition to a SBCCS is clearly driven by a self-induced gauge field $\boldsymbol{A}_{xc}(\boldsymbol{r})$ whose origin is the exchange and correlation functional $E_{xc}\big(n(\boldsymbol{r}),\nabla\times[\boldsymbol{j}_p(\boldsymbol{r})/n(\boldsymbol{r})]\big)$. A realistic search for SBCCS then requires a realistic form for E_{xc}; our conclusions will crucially depend on a reasonable form (both in magnitude and structure) for E_{xc}. We derive E_{xc} from the energy of a uniform 2D two-component electron gas in arbitrary but uniform $B(\boldsymbol{r})=B$. In the range of ν where the Laughlin liquid state is valid we used our results for the energy of a 2D two-component quantum plasma (Rasolt *et al.* 1985) at $\nu=\frac{2}{3}$, $\frac{2}{5}$ and 2. In the limit when $\nu\to\infty$ we use the Tanatar and Ceperley (1989) $E_{xc}^{\infty}(n_0)$. Our Padé form is then

$$\bar{E}_{xc}(n_0,\nu)=\frac{E_{xc}^{0}(n_0,\nu)+a\nu^4E_{xc}^{\infty}(n_0)}{1+a\nu^4}, \quad (7)$$

where a is chosen to closely reproduce the values of $\bar{E}_{xc}(n_0,\nu)$ at $\nu=\frac{2}{9},\frac{2}{5},2$. Our form of E_{xc} is finally given by noting that for an arbitrary but uniform $\boldsymbol{B}$ and a uniform ground state, $\boldsymbol{j}(\boldsymbol{r})=\boldsymbol{j}_p(\boldsymbol{r})-(en_0/mc)\boldsymbol{A}(\boldsymbol{r})=0$. So $\nabla\times\big(j_p(\boldsymbol{r})/n_0\big)=(e/mc)\nabla\times\boldsymbol{A}(\boldsymbol{r})=(e/mc)\boldsymbol{B}$. From the definition $\nu\equiv n_0\,2\pi\ell^2=n_0\,2\pi\,\hbar c/eB$ and the usual extension (Hohenberg and Kohn 1964) from $n_0\to n(\boldsymbol{r})$ and from $\boldsymbol{j}_p(\boldsymbol{r})$ (appropriate to a uniform B) to arbitrary $\boldsymbol{j}_p(\boldsymbol{r})$, we get the corresponding local current density approximation (LCDA) form for $E_{xc}\big(n(\boldsymbol{r}),\nabla\times(\boldsymbol{j}_p(\boldsymbol{r})/n(\boldsymbol{r}))\big)$. We make two additional observations: (1) There is no Zeeman splitting in (7) for the fictitious $\boldsymbol{B}(\boldsymbol{r})\approx\nabla\times\big(\boldsymbol{j}_p(\boldsymbol{r})/n(\boldsymbol{r})\big)$. (2) For an inhomogeneous system the true $E_{xc}\big(n(\boldsymbol{r}),\nabla\times(\boldsymbol{j}_p(\boldsymbol{r})/n(\boldsymbol{r}))\big)$ is clearly smoothed out and the discontinuities in $\bar{E}_{xc}$ are not very important.

The equivalent form in the LCDA for T_s in (5b) can be derived; it is given by

$$T_s\big(n(\boldsymbol{r}),\boldsymbol{j}_p(\boldsymbol{r})\big) = T_0\big(n(\boldsymbol{r})\big) + \left(\tfrac{1}{2}\hbar\,|\,\boldsymbol{\mathcal{V}}(\boldsymbol{r})\,|\,\delta n(\boldsymbol{r}) - \frac{\pi}{2}\frac{\hbar^2}{m}\big(\delta n(\boldsymbol{r})\big)^2 + \frac{m\,|\,\boldsymbol{j}_p(\boldsymbol{r})\,|^2}{2n(\boldsymbol{r})}\right);$$

$$\delta n(\boldsymbol{r}) = n(\boldsymbol{r}) - \frac{Nm\,|\,\boldsymbol{\mathcal{V}}(\boldsymbol{r})\,|}{\pi\hbar}, \qquad N \equiv \text{integ part of } \frac{\pi\hbar}{m}\frac{n(\boldsymbol{r})}{|\,\boldsymbol{\mathcal{V}}(\boldsymbol{r})\,|}, \tag{8}$$

where $T_0\big(n(\boldsymbol{r})\big)$ is the kinetic energy of a 2D electron gas in the absence of $\boldsymbol{B}$. The last term in (8) is equivalent to (3), now for arbitrary magnitude of currents in the SBCCS in the presence of a nonuniform $n(\boldsymbol{r})$ derived in the LCDA.

For $V_{\text{eff}}(\boldsymbol{r})$ in (6a) we take the following geometry. We put one spherical well potential at the four corners of a square (with sides of length $7a_0$; $a_0 = \hbar^2/me^2$) and another spherical well at the centre of the four sides. We fix the electron density at one electron per square and increase the depth of the wells to create a stronger and stronger inhomogeneity (see our initial discussion). We take two depth wells with effective band structure masses of $m^*/m \approx 10\cdot3$ and $\approx 17\cdot3$ and replace the m by m^* in (7) and (8).†

To find the nature of the ground state we minimise (5) [using equations (7) and (8)] with respect to $n(\boldsymbol{r})$ and $\boldsymbol{j}_p(\boldsymbol{r})$ in the $\lim \boldsymbol{A}(\boldsymbol{r}) \to 0$. {Actually, a posteriori $\boldsymbol{j}_p(\boldsymbol{r})$ is very small. So we solve the Kohn–Sham equations for $n(\boldsymbol{r})$ above the SBCCS [i.e. (6) with $\boldsymbol{A}_{xc}(\boldsymbol{r}) = 0$] and insert $n(\boldsymbol{r})$ in (5) to minimise with respect to $\boldsymbol{j}_p(\boldsymbol{r})$ below the SBCCS transition.} To facilitate the calculation, and in particular the $\nabla \times \big(\boldsymbol{j}_p(\boldsymbol{r})/n(\boldsymbol{r})\big)$ term, we work in a plane-wave basis. The increasing inhomogeneity (i.e. m^*) then requires a larger and larger basis set. The density $n(\boldsymbol{r})$ above the SBCCS transition is calculated using up to 225 plane waves of reciprocal lattice vectors $\boldsymbol{k}$ in $\phi_k(\boldsymbol{r})$ of (6). Below the SBCCS transition the current is expanded as $\boldsymbol{j}_p(\boldsymbol{r}) = \Sigma_{\boldsymbol{K}}\,\boldsymbol{a}_{\boldsymbol{K}}[C_{\boldsymbol{K}}\cos(\boldsymbol{K}\,.\,\boldsymbol{r}) + S_{\boldsymbol{K}}\sin(\boldsymbol{K}\,.\,\boldsymbol{r})]$, where $\boldsymbol{K}\,.\,\boldsymbol{a}_{\boldsymbol{K}} = 0$. The total energy change of (8) was calculated by numerical integration of the unit cell of a 61 × 61 point mesh. Equation (8) was then minimised with respect to all the $C_{\boldsymbol{K}}$ and $S_{\boldsymbol{K}}$ using a 'conjugate gradients method'.

For $m^*/m \approx 10\cdot3$ we found a well defined minimum with finite $C_{\boldsymbol{K}}$ and $S_{\boldsymbol{K}}$; an SBCCS clearly appears. In Fig. 1 we detail its current pattern. We observe a clear vortex–antivortex structure. For $m^*/m \approx 17\cdot3$ (Fig. 2) the $V_{\text{eff}}(\boldsymbol{r})$ potential is so strong that it forces the vortex–antivortex structure into one single vortex. Other details are self-explanatory in the figure captions. The implication of these results to the incompressible chiral states, whose many-body gauge field mimics an external uniform magnetic field, is immediate. For example, taking the current patterns of Figs 1 and 2 and calculating the corresponding $\boldsymbol{A}_{xc}(\boldsymbol{r})$ in (6), we find that the form of $\boldsymbol{A}_{xc}(\boldsymbol{r})$ bears *no resemblance* to a uniform magnetic field. All this was argued in general and rigorously by Rasolt and Vignale (1990) and more recently by Tomboulis (1992) in the tJ model.

† As a side comment about the choice of one electron per unit square we note that strong instabilities toward a Pauli antiferromagnet alignment occur at half-filling and strong periodic potentials. We therefore checked that the SBCCS is stable away from half-filling (i.e. less or more than one electron per unit square) where Fermi surface nesting is not an issue and where the instability towards a Pauli antiferromagnetic or orbital antiferromagnetic (see below) structure does not predominate.

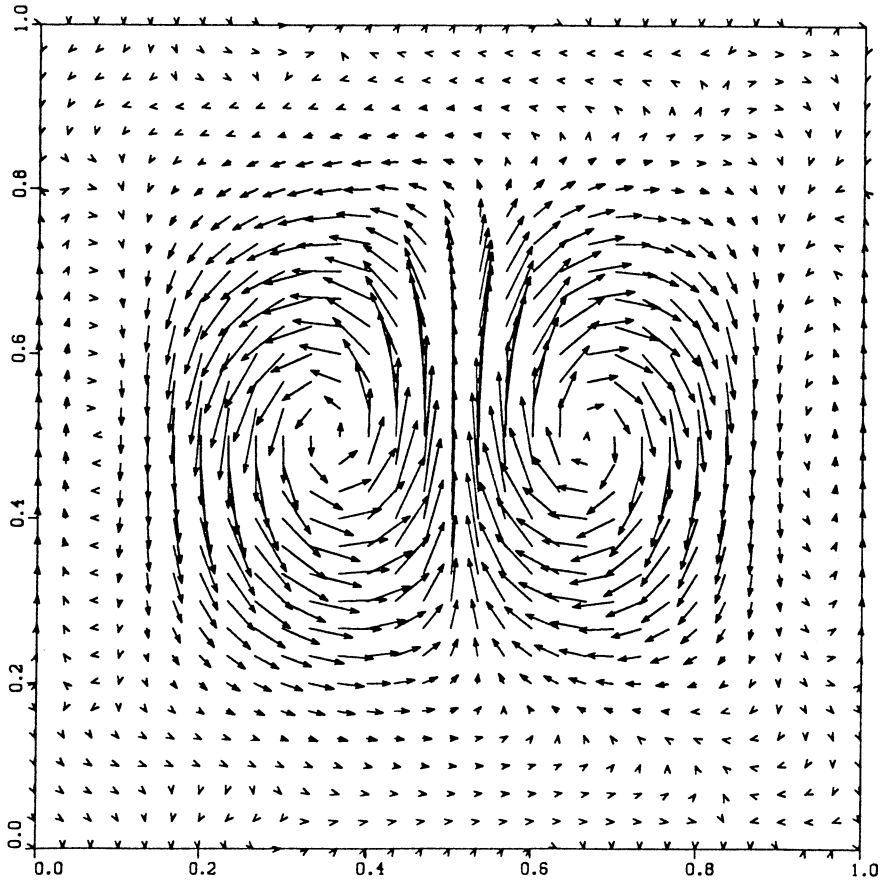

Fig. 1. Current patterns in a unit cell of a symmetry-broken current-carrying state in a two-dimensional nonuniform interacting electron gas with $m^*/m \approx 10{\cdot}3$ (see text). The corresponding $V_{\text{eff}}(\boldsymbol{r})$ in equation (6a) is made up of two spherical wells, one at the corner and one at the centre of the squares (see text). The two spherical wells have depths $V_1 = 0$ and $V_2 = -1{\cdot}0$ (in atomic units) and radii $R_1 = 0{\cdot}278a_0$ and $R_2 = 0{\cdot}222a_0$. The square corner is located in the centre of the panel. The effective dipole moment $\boldsymbol{\mu}$ per square $|\boldsymbol{\mu}| \equiv \hbar e\alpha/2mc$, with $\alpha = m \int \mathrm{d}^2 r\, \boldsymbol{j}(\boldsymbol{r}) \times \boldsymbol{r}$; the α we get is $\approx 5{\cdot}2 \times 10^{-3}$.

In this regard we note that we can imagine configurations of densities and $\boldsymbol{A}_{\text{xc}}(\boldsymbol{r})$ within the continuum Lagrangian more closely related to the lattice models, i.e. the electrons are forced to occupy only a certain region of the unit cell. For example, consider unit cells where the electrons are *totally* excluded from some region in the centre of the square and further imagine that the $\boldsymbol{A}_{\text{xc}}(\boldsymbol{r})$ mimics a uniform field $\boldsymbol{B}$ outside this region and a field $\boldsymbol{B}(\boldsymbol{r})$ inside this region, such that the total 'flux' $\int 2\nabla \times (\boldsymbol{A}_{\text{xc}}(\boldsymbol{r}))\, \mathrm{d}S$ equals zero. This is not a chiral state, in that the Wilson loop is not proportional to the area (Halperin *et al.* 1989; Rasolt and Vignale 1990). Nevertheless, if we take the limit of the excluded region to zero the electron outside this region would experience an environment 'similar' to a uniform external magnetic field. Such states were not ruled out by Rasolt and Vignale (1990). However, the gauge field has specific requirements.

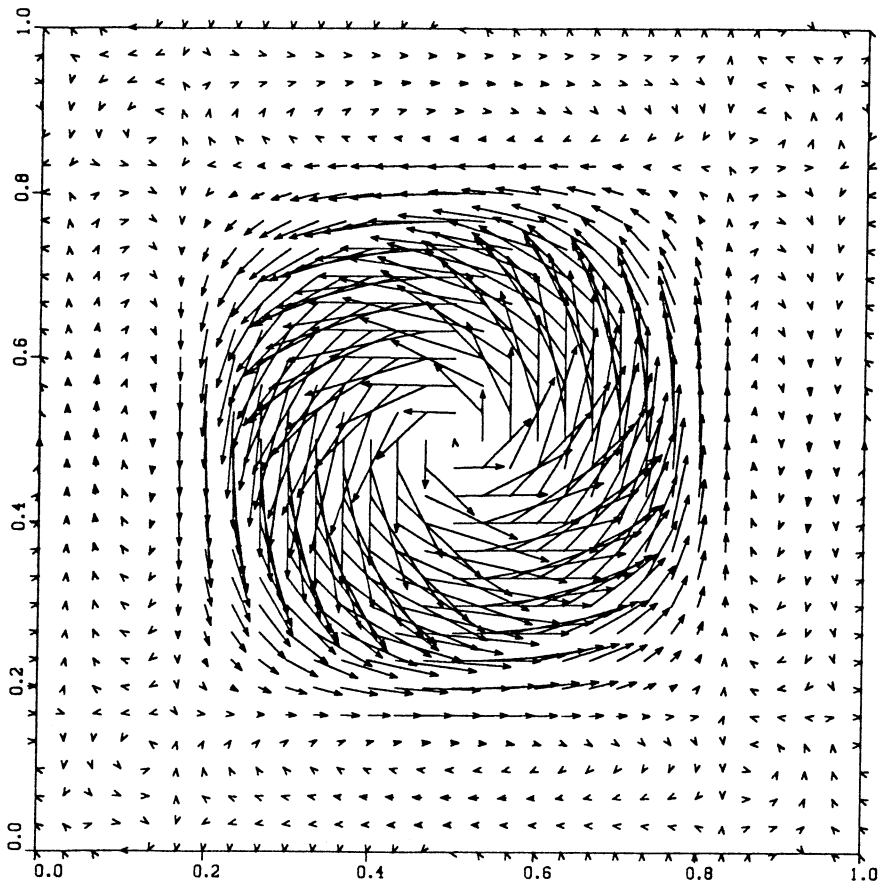

Fig. 2. The same as Fig. 1 but with $m^*/m \approx 17{\cdot}3$, $V_1 = -1{\cdot}0$ and $V_2 = -2{\cdot}0$.

For example, from equation (6b), $\nabla . \big(n(\boldsymbol{r})\, A_{\mathrm{xc}}(\boldsymbol{r})\big) = 0$ and furthermore, since the gauge field is a *many body* gauge field, $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})$ must equal the physical current $\boldsymbol{j}(\boldsymbol{r})$. In the above configuration this implies that just outside the excluded regions the electron velocities go to infinity; this is not possible.

Next, there are other points of interest which we wish to address. An interacting version of (4) has been a subject of many recent publications, i.e. the Hubbard Hamiltonian

$$\hat{H} = \lim_{A(r)\to 0} \Big(- \sum_{i,j\sigma} t_{ij}\, C^{\dagger}_{i\sigma}\, C_{j\sigma} + \tfrac{1}{2} U \sum_{i\sigma} \hat{n}_{i\sigma}\, \hat{n}_{i-\sigma} \Big) \tag{9}$$

or the tJ model (Tomboulis 1992) which is a transformed version of (9). A wide range of broken symmetry ground states have been suggested (Anderson *et al.* 1989). Obviously, the orbital antiferromagnetic configuration (Schulz 1989) and the 'chiral' configuration (Affleck and Marston 1987) are of particular relevance to this paper. Both have been understood to depend on Fermi surface nesting of (9); are the patterns of $\boldsymbol{j}(\boldsymbol{r})$ in Fig. 1 the continuum version of this, or are

they something new? Fermi surface nesting leads to doubling of the unit cell (i.e. of the square). We increased the size of the super cell to four squares and found no change in our SBCCS of Fig. 1. It is then clear that this SBCCS is a new type of ground state generated by a new type of gauge field [equations (6) and (7)] appropriate to a continuum strongly-inhomogeneous electron gas. This state is actually closely related to SBCCS as suggested for the ^{3}He A phase (Volovik 1984), heavy fermion superconductivity (Sigrist *et al.* 1989), and the anyon phase (Halperin *et al.* 1989). Concerning these states, we make the following observation. In these SBCCS a relation is suggested between $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})$ and $n(\boldsymbol{r})$ given by $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r}) \approx \nabla n(\boldsymbol{r}) \times \hat{\boldsymbol{\ell}}$ [$\hat{\boldsymbol{\ell}}$ is a unit vector perpendicular to the plane of current flow $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r})$]. However, the relation $\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r}) \approx \nabla n(\boldsymbol{r}) \times \hat{\boldsymbol{\ell}}$ is favoured with a local moment picture in mind (Volovik 1984). The SBCCS, however, has a global order which from our exact gauge structure is unlikely to sustain such a relation in any limiting range of the SBCCS. This can be argued using equation (10) of Vignale *et al.* (1992); details will be presented elsewhere.

3. Conclusions

We conclude with some experimental implications. The high-T_{c} superconducting materials $LaCuO_4$ and $Y_1Ba_2Cu_3O_7$ are naturally two-dimensional; even more so are their superlattice structures (Rasolt *et al.* 1991). However, our interest in SBCCS in those materials is probably preempted by a superconducting or antiferromagnetic transition at low temperature. It would be interesting, for example, to create artificial periodic insulating metallic–insulating heterojunctions (or quantum wells), or even insulating-doped semiconductors–insulating heterojunctions, with increasingly strong electron density variation in the conducting plane to look for this SBCCS. The experimental observations follow the same suggestions made for the search for the anyon state (Halperin *et al.* 1989). If the state is an orbital ferromagnetic from plane to plane, the magnetic moment per unit cell, although smaller (see Fig. 1) than the 15 G predicted in the anyon state, is still readily observable. For a distance of $30a_0$ between planes, we calculate the induction to be $B \approx 1{\cdot}0$ G for Fig. 1 and $B \approx 11{\cdot}5$ G for Fig. 2. The other possibilities and subtleties in muons, neutron scattering, LEED and Faraday rotations are all equally applicable, but with considerably lower cross sections (see Halperin *et al.* 1989; Dzyloshinskii, unpublished). For example, for something like a high-T_{c} material we estimate the Faraday rotation to be $\approx 10^{-5}$–10^{-3} radians per light wavelength, still within the sensitivity of the Stanford group (Spielman *et al.* 1993). A preference for orbital antiferromagnetic alignment (Rojo and Canright 1991) and its subtleties in optical measurement would also follow this SBCCS.

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