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Effect of Electron Inertia on Double Layers in a Relativistic Hot Plasma

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Abstract

The ion-acoustic double layer in a relativistic hot plasma is studied by considering the effect of electron inertia. The critical region is analysed with the help of a combination of the KdV and mKdV equations obtained via a reductive perturbation technique. The profiles of the double layer are explicitly obtained for various values of the plasma parameters and of the electron–ion mass ratio.

1. Introduction

In the last few years various authors have theoretically (Troven 1981, 1986; Schamel 1983; Schamel and Bajurbarua 1983; Tajiri and Nishikawa 1985; Goswami and Bajurbarua 1986; Radu and Chanteur 1986; Bharuthram and Shukla 1985) and experimentally (Allen 1985; Sato *et al.* 1986; Nishida and Hosegawa 1986) investigated double layers (DL) in plasmas. The DLs may be one of the important sources of the accelerating particles in the plasmas of space and astrophysical objects (Alfvén 1986). It was also proposed that energy may be released in solar flares due to the formation of DLs in a current-carrying loop in the solar corona. There is already much evidence of the existence of DLs in the auroral zones (Temerin *et al.* 1982; Kellog *et al.* 1984; Khoskinen *et al.* 1988). Recently, Sutradhar and Bajurbarua (1988), Verheest (1989), Baboolal *et al.* (1988*a*, 1988*b*, 1990), and Hellberg *et al.* (1992) have theoretically investigated DLs under different situations in a plasma and shown that the characteristics of DLs are very interesting in the presence of two-temperature electrons, negative ions, drifting electrons and ions, etc. However, all of these studies are essentially nonrelativistic, but the recent work of Das and Paul (1985), Nejoh (1987*a*, 1987*b*, 1988) Roy Chowdhury *et al.* (1989, 1990*a*, 1990*b*), Salauddin (1990), Chakraborty *et al.* (1992) has shown that the relativistic mass correction effect has a significant contribution to the formation of solitons, shocks, double-layers etc. Consideration of electron streaming has also been observed to be very significant for the study of nonlinear wave propagation in a plasma (Gold 1965: Clemmow and Dougherty 1969; Paul and Bandyopadhyay 1974; Khalil 1988; Paul *et al.* 1994; Mukhopadhyay *et al.* 1993), but in many of the situations noted above, the electrons form the background and are not treated dynamically. In this paper our motivation is to study the formation of a double layer in the presence of

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both electrons and ions streaming in a hot relativistic plasma by taking account of electron inertia.

2. Formulation

We consider a collisionless unmagnetised plasma consisting of warm isothermal electrons and ions. We assume that the electron velocity is relativistic but the ions are nonrelativistic. Moreover, the electrons and ions have constant streaming velocities in the equilibrium state. We consider the electron dynamics in full detail to study the effect of electron inertia. Assuming that the usual hydrodynamic description is possible, we observe that the plasma can be described by the following equations:
Equation of continuity

$$n_t + (nu)_x = 0 \tag{1}$$

Momentum equation

$$u_t + uu_x + \frac{\sigma}{n} p_x + \phi_x = 0 \tag{2}$$

Pressure equation

$$p_t + up_x + 3pu_x = 0 \tag{3}$$

for the ions,

$$n_{et} + (n_e v_e)_x = 0 \tag{4}$$

as the continuity equation for the electrons, and the momentum equation for the electrons,

$$\frac{m_e}{m_i}(v_{e\alpha})_t + v_e(v_{e\alpha})_x + \frac{1}{n_e} n_{ex} - \phi_x = 0\,, \tag{5}$$

which takes account of the electron inertia. Finally we have the Poisson equation,

$$\phi_{xx} = n_e - n, \tag{6}$$

where

$$v_{ex} = (1 - v_e^2/c^2)^{-\frac{1}{2}}\,.$$

In the above expressions u and v_e are the velocities of the ions and electrons, m and m_e their respective masses, n and n_e the number densities, p the thermal pressure of the ions and ϕ the electrostatic potential.

In order to obtain the nonlinear evolution equation we have adopted the reductive perturbation method and introduced the stretched variables

$$\xi = \varepsilon^{\frac{1}{2}}(x - \lambda t)\,, \tag{7a}$$

$$\tau = \varepsilon^{\frac{3}{2}} t\,, \tag{7b}$$

where λ is to be the phase velocity of the wave. The physical quantities are expanded as follows:

$$n = 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \dots ,$$

$$n_{\mathrm{e}} = 1 + \varepsilon n_{\mathrm{e}}^{(1)} + \varepsilon^2 n_{\mathrm{e}}^{(2)} + \dots ,$$

$$v = v^{(0)} + \varepsilon v^{(1)} + \varepsilon^2 v^{(2)} + \dots ,$$

$$v_{\mathrm{e}} = v_{\mathrm{e}}^{(0)} + \varepsilon v_{\mathrm{e}}^{(1)} + \varepsilon^2 v_{\mathrm{e}}^{(2)} + \dots ,$$

$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \dots ,$$

$$p = p_0 + \varepsilon p^{(1)} + \varepsilon^2 p^{(2)} + \dots . \tag{8}$$

We further assume that the basic equations are supplemented by the following boundary conditions as $|x| \to \infty$:

$$n \to 1, \quad v \to v_0, \quad n_{\mathrm{e}} \to 1, \quad v_{\mathrm{e}} \to v_{\mathrm{e}0}, \quad \phi \to 0 . \tag{9}$$

Using equations (7) to (9) in (1) to (5), we obtain information about the first-, second- and third-order perturbations of various physical quantities. From the first-order equations, the dispersion relation is obtained as

$$(v_0 - \lambda)^2 + \frac{m_{\mathrm{e}}}{m}(v_{\mathrm{e}0} - \lambda)^2 + \frac{m_{\mathrm{e}}}{m}\frac{3v_{\mathrm{e}0}^2}{2c^2}(v_{\mathrm{e}0} - \lambda)^2 = 1 + 3\sigma , \tag{10}$$

which we solve for the phase velocity and get

$$\lambda = \frac{v_0 + \gamma v_{\mathrm{e}0}}{1+\gamma} \pm \frac{(u_0 + \gamma v_{\mathrm{e}0})^2 - G(1+\gamma)^{\frac{1}{2}}}{1+\gamma} , \tag{11}$$

$$\gamma = \frac{m_{\mathrm{e}}}{m}\left(1 + \frac{3v_{\mathrm{e}0}^2}{2c^2}\right) ,$$

$$G = v_0^2 + \gamma v_{\mathrm{e}0}^2 - (1 \, + \, 3\sigma) .$$

From the higher order equation, the KdV equation is derived as

$$\phi_{1t} = \frac{Q}{R(v_0 - \lambda)^2 - 3\sigma p_0}\phi_1 \phi_{1\xi} + \frac{P}{R}\phi_{1\xi\xi\xi} = 0 , \tag{12}$$

where

$$Q = \, - (v_0 - \lambda)^3 - 3(v_0 - \lambda) + 27\sigma p_0^2 + \frac{m_{\mathrm{e}}}{m}(v_0 - \lambda)(v_{\mathrm{e}0} - \lambda)^2 ,$$

$$R = 2(v_0 - \lambda)^2 ,$$

$$P = 1 - \frac{m_{\mathrm{e}}}{m}(v_{\mathrm{e}0} - \lambda)^2 \left(1 - \frac{3v_{\mathrm{e}0}}{2c^2}\right)^2 (v_0 - \lambda) . \tag{13}$$

We note that the nonlinear term in (12) vanishes when $Q = 0$ and the KdV soliton no longer exists. In order to consider ion waves under such a condition, we scale the space–time variables in different manner. We set

$$\xi = \varepsilon(x - \lambda t)\,, \tag{14a}$$

$$\tau = \varepsilon^3 t\,. \tag{14b}$$

Then, adopting the usual procedure (Nejoh 1987*a*, 1988) we arrive at the mKdV equation,

$$\phi_{1t} + \frac{B}{2N}\phi_1^2\,\phi_{1\xi} + \frac{1}{2N}\phi_{1\xi\xi\xi} = 0\,, \tag{15}$$

where

$$\begin{aligned} B &= \frac{1}{\beta}(R\alpha_4 - \alpha_{\rm R}) + \frac{1}{\beta(v_0 - \lambda)}[(\beta\alpha_1 - v_0 - \lambda)\alpha_2 + \sigma\alpha_3] \\ &= \frac{1}{\beta}\bigg(- \frac{18\alpha\sigma}{\beta^4}p_0 - \frac{2}{\beta^3} - \frac{5}{2\beta^3} + \frac{3m_{\rm e}}{m}\frac{\lambda(1 - 2v_{\rm e0})}{2c^2\,\beta^3} \\ &\quad - \frac{18\sigma p_0}{\beta^3} - \frac{3}{2\beta^4}\{\beta_+(\beta + \sigma p_0) + 3\sigma p_0(\beta - 2\sigma p_0)\} + \frac{18\sigma}{\beta^4}p_0\,\beta_+\bigg)\,, \\ \alpha &= \frac{m_{\rm e}}{m}(v_{\rm e0} - \lambda)^2\left(1 + \frac{3v_{\rm e0}}{2c^2}\right), \\ \beta &= (v_0 - \lambda)^2 - 3\sigma p_0\,, \\ \beta_+ &= (v_0 - \lambda)^2 + 3\sigma p_0\,. \end{aligned}$$

On the other hand, near the critical situation a combination of both the KdV and mKdV equations will be valid, which we rewrite as

$$\phi_1 + \alpha\phi_1\phi_1 + \frac{\beta}{2}\phi_1^2\phi_{1\xi} + \tfrac{1}{2}\phi_{1\xi\xi\xi} = 0\,. \tag{16}$$

3. Solution for the Double Layers

To obtain the solution of equation (16) we introduce a new variable $\eta = \xi - u\tau$ (in the stationary frame of the nonlinear wave, u being constant) so we get

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 + \Psi(\phi, \alpha, \beta, u) = 0\,. \tag{17}$$

Here Ψ is regarded as a potential function

$$\Psi = \frac{\beta}{\sigma}\phi^2\left(\phi^2 + \frac{4\alpha}{\beta} - \frac{12u}{\beta}\right). \tag{18}$$

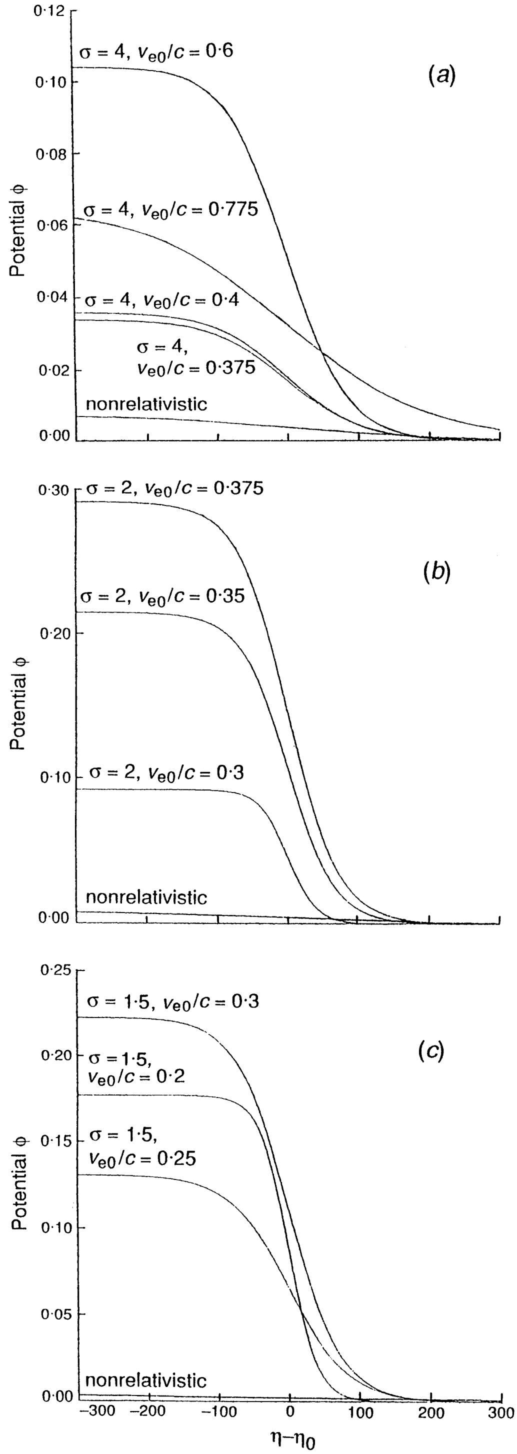

Fig. 1. Form of the shock wave in the relativistic situation for various values of v_{e0}/c and σ.

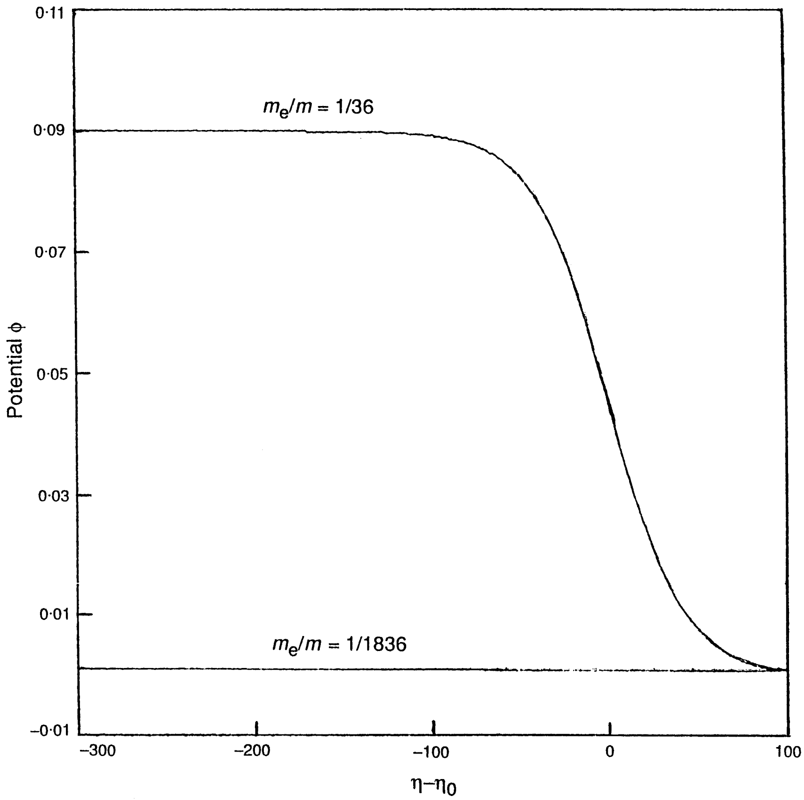

Fig. 2. Form of the shock wave in the nonrelativistic case for two values of the electron–ion mass ratio m_e/m.

We now use the boundary conditions

$$\phi \to 0, \quad \frac{d\phi}{d\eta} \to 0, \quad \frac{d^2\phi}{d\eta^2} \to 0 \quad \text{as } \eta \to \infty . \tag{19}$$

For the formation of the double layers, the potential Ψ should behave as follows:

$$\Psi(\phi, u) \to 0 \quad \text{as } \phi \to 0 \text{ and } \phi \to \phi_m , \tag{20}$$

$$\frac{d\Psi(\phi, u)}{d\phi} \to 0 \quad \text{as } \phi \to \phi_m , \tag{21}$$

$$\frac{d^2\Psi(\phi, u)}{d\phi^2} \to 0 \quad \text{as } \phi \to 0 \text{ and } \phi \to \phi_m , \tag{22}$$

where ϕ_m is the maximum value of ϕ. A simple integration of equation (17) leads to

$$\phi(\alpha,\beta,\phi_m) = \tfrac{1}{2}\phi_m(\alpha,\beta)\left[1-\tanh\left\{\tfrac{1}{2}\left|-\frac{\beta}{\sigma}\phi_m(\alpha,\beta)\right|(\eta-\eta_0)\right\}\right]. \tag{23}$$

Now in practice two types of double layers can be formed. For compressive double layers $\phi_m(\alpha,\ \beta)$ is positive. From the above equations we see that this is only possible when $\beta < 0$, because $\alpha > 0$. A rarefactive double layer is formed when $\phi_m(\alpha,\ \beta)$ is negative, that is, when $\beta > 0$. It therefore seems that in a relativistic plasma with streaming, both of these kinds of double layers are possible. To have an explicit idea we take recourse to numerical computation. Here we evaluate the form of the shock wave for various values of the parameters such as the temperature σ, v_{e0}/c, and m_e/m, the ratio of electron and ion masses.

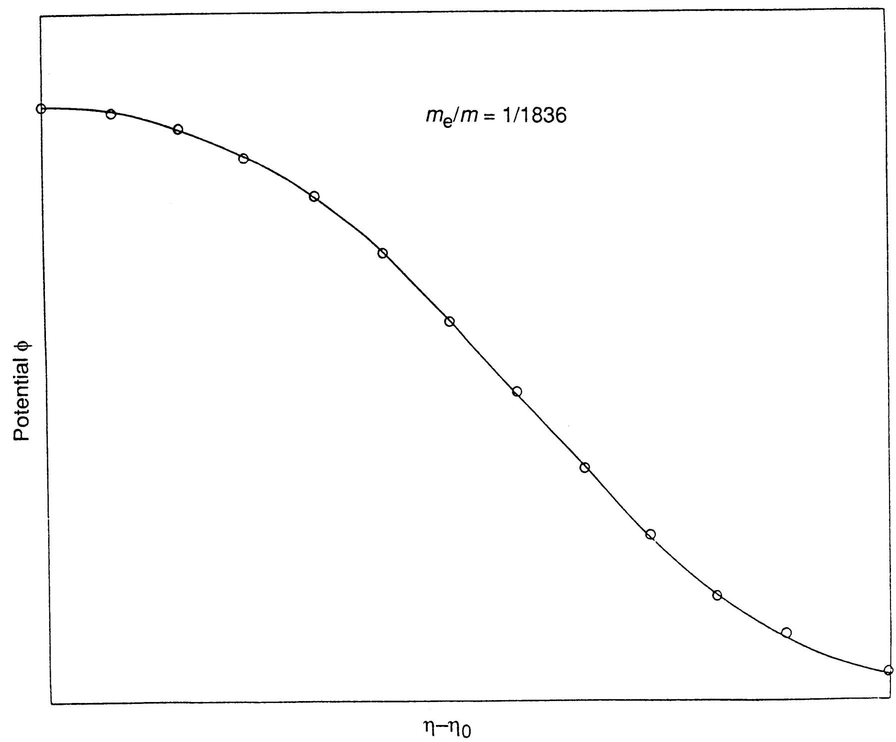

Fig. 3. Form of the shock wave in the nonrelativistic case for $m_e/m = \frac{1}{1836}$ at an (arbitrary) increased scale.

We have considered three possible values of the mass ratio, namely $\frac{1}{10}$, $\frac{1}{36}$ and $\frac{1}{1836}$. Although the first two ratios are unphysical, these help to simulate the situations where the heavier particle is not a proton. In Figs 1*a* and 1*b* we show the form of the shock wave for medium values of v_{e0}/c with $\sigma = 4$ and $\sigma = 2$. On the other hand in Fig. 1*c* the forms are plotted for $\sigma = 1{\cdot}5$. In each case it is seen that the nonrelativistic situation appears as a straight line, because on the same scale its order of magnitude is much less. Hence in Fig. 2

we have plotted the nonrelativistic situation separately for $m_{\rm e}/m = \frac{1}{36}$ and $\frac{1}{1836}$. One clearly visible feature of these diagrams is the increase in magnitude of the shock wave maxima with an increase in the value of $m_{\rm e}/m$. It may also be noted that the case of $m_{\rm e}/m = \frac{1}{1836}$ again appears as a straight line when plotted on the same scale as that of $m_{\rm e}/m = \frac{1}{36}$. The reason for this is again that the magnitude of the shock wave is much reduced in this case (see Fig. 3).

4. Conclusion

In the above analysis we have considered in detail the effects of relativity and inertia on the shape of the shock wave. Since in many of the plasma theories we usually take the electrons to form the background, we never encounter the inertia effect. Only when one considers the proper dynamics does it appear. Our computation clearly demonstrates why it is difficult to detect experimentally a shock or double-layer phenomenon in a plasma.

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