Aust. J. Phys., 1995, **48**, 55–69

Meson Supermultiplet Decay Constants

N. R. Jones and R. Delbourgo

Physics Department, University of Tasmania,
GPO Box 252C, Hobart, Tas. 7001, Australia.

Abstract

We use a covariant supermultiplet theory to determine the primary coupling constant associated with several types of two-body meson decay. Despite the diverse range of decays considered, the primary coupling constant is surprisingly uniform. We envisage the extension of the techniques to heavy quark cases, including as preliminary examples the calculation of the D^{*+} and D^{*0} total decay widths with results $57\cdot7\pm1\cdot5$ keV and $42\cdot5\pm2\cdot6$ keV respectively, as well as some predictions about D^* and B^* radiative decays.

1. Introduction

In these heady days of heavy quark effective theories (HQET) we feel it is appropriate to review a supermultiplet scheme developed in the mid-1960s which has great similarities to the HQET in respect of the heavy quark and the accompanying 'brown muck'. In doing so we hope to test the covariant supermultiplet theory for light as well as heavy degrees of freedom, assessing the extent of symmetry breaking and how it manifests itself. We may then apply our techniques to heavy quark examples in future work with considerable confidence.

To determine the matrix elements of currents between hadrons requires knowledge of the hardronic wavefunction in terms of the quark and gluon constituents. A full relativistic treatment of such constituents is impossible because of the infinite degrees of freedom associated with the quarks and gluons. However, the success of the nonrelativistic quark model prompted several workers (Delbourgo *et al.* 1965; Bég and Pais 1965; Sakita and Wali 1965) to construct relativistic spinor fields describing pointlike mesons, incorporating the correct spin, parity, flavour and colour degrees of freedom. Despite later evidence that the mesons were not pointlike objects, these group-theoretical approaches can be shown to be equivalent to the nonrelativistic weak binding limit (Hussain *et al.* 1991).

The basis of our work is a relativistic meson supermultiplet field (Salam *et al.* 1965). The wavefunction describing the meson is dynamically equivalent to a system of two quarks, both of which are on-shell and moving at the same velocity. This differs little from the heavy quark picture which assumes that a meson with a quark much heavier than its light antiparticle partner will have the heavy component almost on-shell and moving at the same velocity as the meson because the light 'brown muck' (Flynn and Isgur 1992) must move with the same velocity. This assumption, along with the hope that weak interactions

0004-9506/95/010055$05.00

will not affect the motion of the heavy quark at small recoil, led to the now popular heavy quark symmetry and decoupling [see Mannel (1993) or Grinstein (1992) for a review].

The reasons for using the supermultiplet scheme are several. Firstly, the scheme automatically incorporates the Zweig rules and duality diagrams, so one can easily determine the Zweig-allowed strong decays. At the same time, it incorporates isospin and field mixing factors so that one can readily normalise the coupling constant of various decays; indeed this makes supermultiplet theory very predictive as one need only know a single coupling constant to predict widths of many seemingly unrelated processes. Secondly, radiative decay modes may be examined by combining the supermultiplet scheme with the vector meson dominance model. This permits the theory to make some quite accurate predictions about photon-mediated decays such as $\omega \to \pi^+\pi^-$. Thirdly, excited mesonic states may be constructed in terms of the supermultiplet field (Delbourgo and Liu 1994) so we are able to further broaden its applications. Finally, the scheme is easily extended to include the c (Delbourgo 1975) and b quark flavoured mesons so that we may venture into the heavy quark arena with little modification.

The recent ACCMOR (1992) and CLEO (1992) Collaborations have renewed interest in the D^* decays due to two major findings. The D^{*+} total width was measured with an upper bound of 131 keV (significantly lower than the 1992 upper bound of $1\cdot 1$ MeV), and the $D^{*+} \to D^+\gamma$ branching fraction appears significantly smaller than earlier measurements. Both these findings are consistent with constituent quark model (Kamal and Xu 1992) and HQET predictions (Colangelo *et al.* 1993). We provide similar calculations within the supermultiplet framework and reproduce these findings with considerably more ease.

2. Supermultiplet Field

Derivation of the wavefunction (Delbourgo *et al.* 1965) proceeds by application of a relativistic boost to the rest-frame spinor $\phi_a^b(\hat{p})$, odd under parity. This leads to a relativistic spinor $\phi_A^B(p)$ satisfying Bargmann–Wigner equations, namely

$$\phi_A^B(p) = \phi_{a\alpha}^{b\beta}(p) = (\not{p} + m)[\gamma^\mu \phi_{\mu a}^b - \gamma_5 \phi_{5a}^b]_\alpha^\beta / 2m\,, \tag{1}$$

where a, b are flavour indices, α, β are spin indices, ϕ_5 corresponds to the pseudoscalar nonet and ϕ_μ to the vector nonet (with $p^\mu \phi_\mu = 0$). One can show (Hussain *et al.* 1991) that such a relativistic spinor is equivalent to describing the meson as a quark–antiquark pair, both members of which are moving at the same velocity as the meson and are therefore both on-shell.

We use the simplest effective interaction Lagrangian

$$\mathcal{L}_{\rm int} = G\Phi_A^B(p_1)[\Phi_B^C(p_2)\,\Phi_C^A(p_3) + (p_2 \leftrightarrow p_3)]\,, \tag{2}$$

as proposed by Delbourgo *et al.* (1965) to describe the three-point coupling between the mesons involved in two-body decays. Here G is a normalisation factor and we note that it has the dimensions of mass. Such an interaction Lagrangian corresponds to a duality diagram (Harari 1969; Rosner 1969) as shown in Fig. 1 with three mesons meeting at a vertex (p_i incoming). Here flavour labels have been included to show how the flavour is automatically conserved, flavour being carried by the line. Contravariant spinor indices correspond to odd parity as they represent the antiquark, while covariant indices have even parity since they represent the quark flavour.

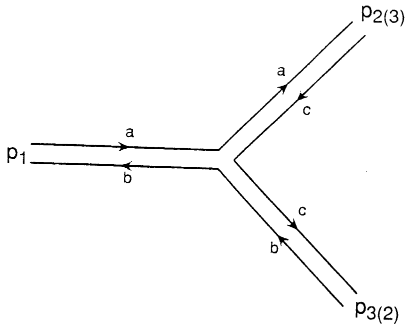

Fig. 1. Quark line or duality diagram.

Upon substitution of the supermultiplet field (1) into the interaction Lagrangian (2), and using the Dirac trace algebra along with the momentum conditions

$$p_1 + p_2 + p_3 = 0, \quad p_i^2 = m_i^2, \quad \phi(i) = \phi(p_i)\,,$$

we reduce the interaction Lagrangian to

$$\mathcal{L}_{\rm int} = g_{VPP}(p_2 - p_3)^\mu \langle \phi_\mu(1)[\phi_5(2), \phi_5(3)]\rangle \tag{3a}$$

$$+ g_{VVP}\epsilon^{\mu\nu\kappa\lambda} p_{1\kappa} p_{2\lambda} \langle \phi_\mu(1)\{\phi_\nu(2), \phi_5(3)\}\rangle \tag{3b}$$

$$+ g_{VVV}[(p_2 - p_3)^\mu g^{\nu\sigma} m_1 + (p_3 - p_1)^\nu g^{\sigma\mu} m_2 + (p_1 - p_2)^\sigma g^{\mu\nu} m_3$$

$$+ \ 2(p_2 - p_3)^\mu (p_3 - p_1)^\nu (p_1 - p_2)^\sigma / (m_1 + m_2 + m_3)]$$

$$\times \langle \phi_\mu(1)[\phi_\nu(2), \phi_\sigma(3)]\rangle\,, \tag{3c}$$

where $\langle\ \rangle$ stands for a trace over the internal symmetry indices, corresponding to a joining of quark lines in a duality diagram. For instance, such a trace for flavour indices would expand as

$$\phi^b_{\mu a}(1)[\phi^c_{5b}(2)\,\phi^a_{5c}(3) - \phi^c_{5b}(3)\,\phi^a_{5c}(2)]$$

for the vector–pseudoscalar–pseudoscalar (VPP) vertex. The interaction Lagrangian contains three distinct coupling constants, as at this stage we are alert to the possibility that symmetry breaking may affect each piece differently, that is, we have introduced three separate constants depending on the type of decay. However, full supermultiplet symmetry would mean that the constants are related in the following way:

$$2g_{VPP} = m_1 g_{VVP} = m_1 g_{VVV}\,. \tag{4}$$

One should also note that the three-pseudoscalar meson vertex is not present in the interaction Lagrangian, as expected by parity conservation. [This follows automatically in the supermultiplet scheme because the trace over three ϕ_5 fields disappears from equation (2).]

For the moment we only consider strong meson decay so that tree-level calculations in perturbation theory given by (3a, 3b, 3c) suffice. Also we apply the general formula for a two-body decay,

$$\Gamma_{1\to 2,3} = \frac{\lambda^{\frac{1}{2}}(m_1^2, m_2^2, m_3^2)}{16\pi m_1^3(2s_1+1)} \sum_{\text{spins}} |\mathcal{L}_{\text{int}}|^2 , \tag{5}$$

where s_1 is the spin of the parent meson and $\lambda(m_1^2,\ m_2^2,\ m_3^2) = m_1^4+m_2^4+m_3^4-2m_1^2m_2^2 -2m_1^2m_3^2 - 2m_2^2m_3^2$.

Upon substitution of the interaction Lagrangian in the form of equation (3) into the decay rate formula (5), we derive the following widths of the various decays:

$$\Gamma_{V\to PP} = \lambda^{\frac{3}{2}}(m_1^2, m_2^2, m_3^2) g_{VPP}^2/48\pi m_1^5 , \tag{6}$$

$$\Gamma_{V\to VP} = \lambda^{\frac{3}{2}}(m_1^2, m_2^2, m_3^2) g_{VVP}^2/96\pi m_1^3 , \tag{7}$$

$$\Gamma_{V\to VV} = \lambda^{\frac{3}{2}}(m_1^2, m_2^2, m_3^2) g_{VVV}^2 \mathcal{Y}(m_1, m_2, m_3)/192\pi m_1^5 m_2^2 m_3^2 , \tag{8}$$

where

$$\begin{aligned}\mathcal{Y}(m_1,m_2,m_3) &= 9\Bigg(\sum_{1\le i<j\le 3} (m_i+m_j)^2(m_i-m_j)^4 - \sum_{i=1}^{3} m_i^6 \Bigg) \\ &\quad + \prod_{i=1}^{3} m_i \Bigg(98 \sum_{i=1}^{3} m_i^3 - 16 \sum_{1\le i<j\le 3} (m_i+m_j)^3 \Bigg) + 142 \prod_{i=1}^{3} m_i^2 .\end{aligned}$$

We now have an adequate formalism for describing various strong interaction decays amongst ground state mesons. To extend the applications of the supermultiplet theory we invoke the ideas of the vector meson dominance model in order to account for various electromagnetic interactions of our mesons. To do so we make the usual assumption that the coupling between a vector meson flavour singlet and a photon is of the form

$$g_{V\gamma}(k^2{=}0) = em_V^2/g_{VPP} , \tag{9}$$

as shown in Fig. 2. When extrapolating away from $k^2 = 0$ we expect the coupling to decrease and as such have denoted the coupling by em_V^2/g'_{VPP} to allow for such change. That is, we anticipate that $g'_{VPP}(k^2)$ will vary with k^2 due to intermediate virtual particle contributions and its value at $k = 0$ equals g_{VVP} in equation (3).

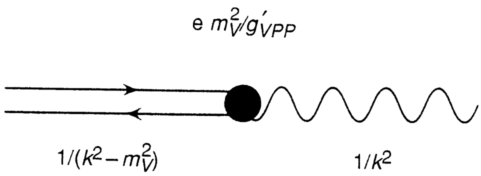

Fig. 2. Vector meson dominance.

The vector meson dominance model, used in conjunction with our decay rate formulae (6→8) give the following rates for the various processes:

$$\Gamma_{V \to l\bar{l}} = (m_V^2 - 4m_l^2)^{\frac{1}{2}}(1 - m_l^2/m_V^2)^{\frac{1}{2}}(e^2/g'_{VPP})^2/12\pi\,, \tag{10}$$

$$\Gamma_{V \to P\gamma} = (m_V^2 - m_P^2)^3(eg_{VVP}/g'_{VPP})^2/96\pi m_V^3\,, \tag{11}$$

$$\Gamma_{P \to V\gamma} = (m_P^2 - m_V^2)^3(eg_{VVP}/g'_{VPP})^2/32\pi m_P^3\,, \tag{12}$$

$$\Gamma_{P \to \gamma\gamma} = m_P^3(e^2 g_{VVP}/g'_{VPP}\, g'_{VPP})^2/64\pi\,, \tag{13}$$

thereby greatly extending the original scope of the supermultiplet scheme. In going from our purely strong interaction decay rates to the radiative ones, we have used the gauge invariance of our interaction Lagrangian and simply substituted a mass of zero for those vectors connecting with the photon. However, the three-vector interaction (3c) is only gauge invariant for the case $m_2 = m_3$, so strictly we should only apply it to radiative examples for which the virtual vector meson satisfies this condition. Unfortunately, since the photon only couples to flavour singlet states, the condition $m_2 = m_3$ also implies that the daughter vector mesons are identical. Due to the F-type coupling between daughter states in the interaction Lagrangian (3c) such decay widths will automatically go to zero. It is for this reason we have not included a $V \to V\gamma$ term above, despite experimental evidence for such (e.g. $\Gamma_{\phi \to \rho\gamma}/\Gamma_{\phi \to \mathrm{all}} < 2\%$, although our zero width prediction does not conflict with this). We now go on to apply the formalism to the ground state mesons.

3. Supermultiplet Method

In the standard way we take the pseudoscalar nonet as:

$$\phi^b_{5a} \xrightarrow{0^-} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}, \tag{14}$$

where

$$\eta_8 = \cos\theta_P\, \eta - \sin\theta_P\, \eta'\,, \tag{15}$$

$$\eta_0 = \sin\theta_P\, \eta + \cos\theta_P\, \eta' \tag{16}$$

as defined in Gasiorowicz (1967) and θ_P is the pseudoscalar mixing angle. The vector nonet is similarly given by

$$\phi^b_{\mu a} \xrightarrow{1^-} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} \end{pmatrix}, \tag{17}$$

where

$$\omega_8 = \cos\theta_V \,\phi - \sin\theta_V \,\omega\,, \tag{18}$$

$$\omega_0 = \sin\theta_V \,\phi + \cos\theta_V \,\omega\,, \tag{19}$$

and θ_V is the vector mixing angle as determined using the Gell-Mann–Okubo (GMO) mass relation. We have chosen to go into some detail as our formalism is not the common one adopted by recent publications (Bramon and Scadron 1990; Particle Data Group 1992). Conversely,

$$\phi = \cos\theta_V \,\omega_8 + \sin\theta_V \,\omega_0 \tag{20}$$

and

$$\omega = -\sin\theta_V \,\omega_8 + \cos\theta_V \,\omega_0\,, \tag{21}$$

where ω_8 and ω_0 masses are determined by the GMO relation (Gasiorowicz 1967). The vector mixing angle is obtained from

$$\tan 2\theta_V = \frac{2[(m_\phi^2 - m_8^2)(m_8^2 - m_\omega^2)]^{\frac{1}{2}}}{2m_8^2 - m_\phi^2 - m_\omega^2}\,, \tag{22}$$

where $3m_8^2 = 4m_{K^*}^2 - m_\rho^2$.

From the vector nonet (17) the octet and singlet fields are expressed in terms of the supermultiplet vector as

$$\sqrt{6}\omega_8 = \phi^1_{\mu 1} + \phi^2_{\mu 2} - 2\phi^3_{\mu 3}, \quad \sqrt{3}\omega_0 = \phi^1_{\mu 1} + \phi^2_{\mu 2} + \phi^3_{\mu 3}\,.$$

Substituting these into equation (20) yields

$$\sqrt{6}\phi = (\cos\theta_V + \sqrt{2}\sin\theta_V)(\phi^1_{\mu 1} + \phi^2_{\mu 2}) + (-2\cos\theta_V + \sqrt{2}\sin\theta_V)\phi^3_{\mu 3}\,.$$

In the case of 'ideal mixing' $\phi = \phi^3_{\mu 3}$ so that

$$\cos\theta_V + \sqrt{2}\sin\theta_V = 0 \quad \text{or} \quad \tan\theta_V = -1/\sqrt{2}$$

leaving us two options for θ_V: either $-\pi/2 < \theta_V < 0$ or $\pi/2 < \theta_V < \pi$. The first case implies $\cos\theta_V = \sqrt{2/3}$, $\sin\theta_V = -1/\sqrt{3}$ so that $\phi = -\phi^3_{\mu 3}$, while the second gives the desired result of $\phi = \phi^3_{\mu 3}$. Thus a suitable solution to equation (22) is in the range $\pi/2 < \theta_V < \pi$. More generally, the solution to (22) is

$$2\theta_V = \tan^{-1}\left(\frac{2[(m_\phi^2 - m_8^2)(m_8^2 - m_\omega^2)]^{\frac{1}{2}}}{2m_8^2 - m_\phi^2 - m_\omega^2}\right) + n\pi\,,$$

where n is any integer.

The above arguments for the determination of the vector mixing angle can be applied to the pseudoscalar nonet with the substitutions $\omega_8 \to \eta_8$, $\omega_0 \to \eta_0$, $\phi \to \eta$, $\omega \to \eta'$. Using the condition $\pi/2 < \theta_V < \pi$ and a similarly derived expression for the pseudoscalar angle, $-\pi/2 < \theta_P < \pi/2$, we obtain the equally likely results

$$\begin{aligned} \theta_V &= 129{\cdot}4^\circ,\ 140{\cdot}6^\circ\,, \\ \theta_P &= -10{\cdot}5^\circ,\ 10{\cdot}5^\circ\,. \end{aligned}$$

The mixing angles we have obtained may seem accurate, but the GMO relation is extremely sensitive to an extra small SU(3) symmetry breaking associated with the *27* representation; a small *27* addition can produce a *major* modification of the angle.

With the correct structure now in place, it is a relatively simple process to test the supermultiplet theory. We wish to calculate the standard coupling constants g_{VPP} and g_{VVP}, examine how similar they are for each process and finally compare the supermultiplet prediction (4) of the relation between them. In practice, we take the decay width and particle masses as input (Particle Data Group 1992) and determine the coupling constant associated with the decay via (6, 7) and (10–13). The simplicity of the supermultiplet method is that isospin and mixing factors are automatically accounted for. One simply chooses an appropriate decay, determines the flavour indices a, b and c using matrices (14, 17) and then uses these in the correct part of the interaction Lagrangian (3) to determine the normalisation factors which arise. For example, in the decay $\rho^+ \to \pi^+\pi^0, a = 1, b = 2, c = 1, 2$ and upon substitution of the fields into equation (3) one finds $g_{\rho^+\pi^+\pi^0} = \sqrt{2}g_{VPP}$, so that the coupling constant we determine for this decay should be divided by the factor $\sqrt{2}$ to obtain the standard coupling constant g_{VPP}. This procedure is repeated for all appropriate physical decays. Mixing is easily accommodated by using the relations (15, 16, 18, 19) to replace the ideal fields by the real mesons in the interaction Lagrangian.

In radiative decays of the type $V \to l\bar{l}$ we allow for the coupling of the photon to the quark. Using the following electromagnetic charge projectors,

$$Q_a^b = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}, \tag{23}$$

we may likewise extract the relevant standard coupling.

Radiative modes such as $V \to P\gamma$ require some delicacy in normalising the coupling constant. To elicit a clear understanding of the method, we include an example of the procedure for the decay $\rho^0 \to \eta\gamma$. Firstly, we recognise that ρ^0 is a combination of $\phi^1_{\mu 1}(1)$ and $\phi^2_{\mu 2}(1)$, so we require terms in the interaction Lagrangian (3b) with $a = 1$, $b = 1$ and $a = 2$, $b = 2$. For each of these cases we determine the third flavour index c such that $\phi_\nu(2)$ is a flavour singlet that couples to a photon (thus $c = 1, 2$). Substituting these values into formula (3b) we pick out the uncharged parts:

$$\begin{aligned}
\mathcal{L}_{\rm int} &\propto g_{VVP}[\phi^1_{\mu 1}(1)\{\phi^1_{\nu 1}(2),\ \phi^1_{51}(3)\} + \phi^2_{\mu 2}(1)\{\phi^2_{\nu 2}(2),\ \phi^2_{52}(3)\}] \\
&= g_{VVP}\left[\frac{\rho^0(1)}{\sqrt{2}}2\left(\frac{\rho^0(2)}{\sqrt{2}} + \frac{\omega_8(2)}{\sqrt{6}}\right)\left(\frac{\eta_8(3)}{\sqrt{6}} + \frac{\eta_0(3)}{\sqrt{3}}\right)\right. \\
&\qquad \left. \frac{-\rho^0(1)}{\sqrt{2}}2\left(\frac{-\rho^0(2)}{\sqrt{2}} + \frac{\omega_8(2)}{\sqrt{6}}\right)\left(\frac{\eta_8(3)}{\sqrt{6}} + \frac{\eta_0(3)}{\sqrt{3}}\right)\right] \\
&= \sqrt{\tfrac{2}{3}}g_{VVP}\,\rho^0(1)\,\rho^0(2)[\eta_8(3) + \sqrt{2}\eta_0(3)] \\
&= \sqrt{\tfrac{2}{3}}(\cos\theta_P + \sqrt{2}\sin\theta_P)\,g_{VVP}\rho^0(1)\,\rho^0(2)\,\eta(3)\,,
\end{aligned}$$

where in particular we have used relations (15, 16) to arrive at the final result. The form shows that the coupling between two ρ^0 mesons and a pseudoscalar η is related to the standard VVP coupling by

$$g_{\rho^0\rho^0\eta} = \sqrt{\tfrac{2}{3}}(\cos\theta_P + \sqrt{2}\sin\theta_P)g_{VVP}\,. \tag{24}$$

The virtual vector meson is immediately identifiable as $\rho^0(2)$, and we must necessarily allow for the coupling between this and the photon. Since $\rho^0 = (u\bar{u}+d\bar{d})/\sqrt{2}$, then $g_{\rho^0\gamma} = g_{V\gamma}/\sqrt{2}$, which in turn implies $g'_{\rho^0 PP} = \sqrt{2}g'_{VPP}$ from (9). Consequently, the coupling between a ρ^0, η and photon is related to our standard couplings by

$$\begin{aligned}
g_{\rho^0\eta\gamma} &= e g_{\rho^0\rho^0\eta}/g'_{\rho^0 PP} \\
&= \frac{e\sqrt{\tfrac{2}{3}}(\cos\theta_P + \sqrt{2}\sin\theta_P)g_{VVP}}{\sqrt{2}g'_{VPP}} \\
&= \frac{1}{\sqrt{3}}(\cos\theta_P + \sqrt{2}\sin\theta_P)\frac{e g_{VVP}}{g'_{VPP}}\,.
\end{aligned}$$

In other decays, it is possible that the radiative mode may proceed via more than one virtual vector meson. The above method is still used to determine each virtual vector meson contribution and the appropriate linear combination is taken.

4. Results

The results are presented in Tables 1 and 2. For clarity these tables include the SU(3) factors which we have used to normalise the coupling constant.

Table 1. Results for g_{VPP} determination

Decay	Factor[A]	g_{VPP}
$\rho^{\pm} \to \pi^{\pm}\pi^{0}$	$\sqrt{2}$	$4 \cdot 24 \pm 0 \cdot 05$
$\rho^{0} \to \pi^{+}\pi^{-}$	$\sqrt{2}$	$4 \cdot 30 \pm 0 \cdot 03$
$\rho^{\pm} \to \pi^{\pm}\eta$	$\sqrt{1/6}(\cos\theta_P + \sqrt{2}\sin\theta_P)$	$< 4 \cdot 17 \pm 0 \cdot 16$
$K^{*\pm} \to (K\pi)^{\pm}$	$1, \sqrt{1/2}$	$4 \cdot 59$
$K^{*0} \to (K\pi)^{0}$	$\sqrt{1/2}, 1$	$4 \cdot 55$
$\phi \to K^{+}K^{-}$	$\sqrt{3/2}\cos\theta_V$	$4 \cdot 82 \pm 0 \cdot 05$
$\phi \to K_L^0 K_S^0$	$\sqrt{3/2}\cos\theta_V$	$4 \cdot 99 \pm 0 \cdot 06$
$D^{*+} \to D^{0}\pi^{+}$	1	$< 10 \cdot 2 \pm 1 \cdot 0$
$D^{*+} \to D^{+}\pi^{0}$	$1/\sqrt{2}$	$< 10 \cdot 3 \pm 1 \cdot 1$

Decay	Coupling factor[A]	g'_{VPP}
$\rho^{0} \to e^{+}e^{-}$	$\sqrt{1/2}$	$3 \cdot 57 \pm 0 \cdot 09$
$\rho^{0} \to \mu^{+}\mu^{-}$	$\sqrt{1/2}$	$3 \cdot 41 \pm 0 \cdot 11$
$\omega \to e^{+}e^{-}$	$\sqrt{1/6}\sin\theta_V$	$4 \cdot 40 \pm 0 \cdot 06$
$\omega \to \mu^{+}\mu^{-}$	$\sqrt{1/6}\sin\theta_V$	$> 2 \cdot 70$
$\phi \to e^{+}e^{-}$	$\sqrt{1/6}\cos\theta_V$	$4 \cdot 07 \pm 0 \cdot 05$
$\phi \to \mu^{+}\mu^{-}$	$\sqrt{1/6}\cos\theta_V$	$4 \cdot 46 \pm 0 \cdot 31$
$J/\psi \to e^{+}e^{-}$	$2/3$	$7 \cdot 55 \pm 0 \cdot 29$
$J/\psi \to \mu^{+}\mu^{-}$	$2/3$	$7 \cdot 72 \pm 0 \cdot 31$
$\Upsilon \to e^{+}e^{-}$	$1/3$	$13 \cdot 36 \pm 0 \cdot 53$
$\Upsilon \to \mu^{+}\mu^{-}$	$1/3$	$13 \cdot 47 \pm 0 \cdot 32$
$\Upsilon \to \tau^{+}\tau^{-}$	$1/3$	$11 \cdot 63 \pm 0 \cdot 72$

[A] $\theta_V = 140 \cdot 6^0$, $\theta_P = 10 \cdot 5^0$.

Table 2. Results for g_{VVP} determination

Decay	Factor	g_{VVP} $\times 10^{-2}$ MeV^{-1}
$\phi \to \rho\pi$	$\sqrt{2/3}(\cos\theta_V + \sqrt{2}\sin\theta_V)$	$1 \cdot 062 \pm 0 \cdot 030$
$\rho^{\pm} \to \pi^{\pm}\gamma$	$1/3$	$0 \cdot 923 \pm 0 \cdot 055$
$\rho^{0} \to \pi^{0}\gamma$	$1/3$	$1 \cdot 216 \pm 0 \cdot 156$
$\rho^{0} \to \eta\gamma$	$(\cos\theta_P + \sqrt{2}\sin\theta_P)/\sqrt{3}$	$0 \cdot 984 \pm 0 \cdot 094$
$\omega \to \pi^{0}\gamma$	$(\cos\theta_V - \sin\theta_V)/\sqrt{3}$	$0 \cdot 878 \pm 0 \cdot 033$
$\omega \to \eta\gamma$	$[\sqrt{2}\cos(\theta_V + \theta_P) + \sin\theta_V \cos\theta_P]/3$	$0 \cdot 919 \pm 0 \cdot 165$
$\phi \to \pi^{0}\gamma$	$(\cos\theta_V + \sqrt{2}\sin\theta_V)/\sqrt{3}$	$0 \cdot 731 \pm 0 \cdot 040$
$\phi \to \eta\gamma$	$[\sqrt{2}\sin(\theta_V + \theta_P) - \cos\theta_V \cos\theta_P]/3$	$0 \cdot 603 \pm 0 \cdot 020$
$\phi \to \eta'\gamma$	$[\sqrt{2}\cos(\theta_V + \theta_P) + \cos\theta_V \sin\theta_P]/3$	$< 1 \cdot 69$
$K^{*\pm} \to K^{\pm}\gamma$	$1/3$	$0 \cdot 905 \pm 0 \cdot 045$
$K^{*0} \to K^{0}\gamma$	$-2/3$	$0 \cdot 733 \pm 0 \cdot 036$
$J/\psi \to \eta_c\gamma$	$4/3$	$0 \cdot 308 \pm 0 \cdot 073$
$\eta' \to \rho^{0}\gamma$	$(\sqrt{2}\cos\theta_P - \sin\theta_P)/\sqrt{3}$	$0 \cdot 693 \pm 0 \cdot 041$
$\eta' \to \omega\gamma$	$-[\sin\theta_P \sin\theta_V + \sqrt{2}\sin(\theta_P + \theta_V)]/3$	$0 \cdot 698 \pm 0 \cdot 051$
$\pi^{0} \to \gamma\gamma$	$\sqrt{2/3}$	$0 \cdot 915 \pm 0 \cdot 053$
$\eta \to \gamma\gamma$	$\sqrt{2/3}(\sqrt{2}\sin\theta_P + \cos\theta_P)/3$	$0 \cdot 870 \pm 0 \cdot 056$
$\eta' \to \gamma\gamma$	$\sqrt{2/3}(\sqrt{2}\cos\theta_P - \sin\theta_P)/3$	$0 \cdot 730 \pm 0 \cdot 055$
$\eta_c \to \gamma\gamma$	$8/9$	$0 \cdot 484 \pm 0 \cdot 337$

[A] $\theta_V = 140 \cdot 6^0$, $\theta_P = 10 \cdot 5^0$.

Table 1 summarises the results of our investigation into the coupling between a vector meson and two pseudoscalar mesons. The first half of Table 1 displays purely strong interaction decays, while the second lists the coupling constant g'_{VPP} obtained from vector meson dominance extrapolation. Two important features are apparent. Firstly, we have found that the coupling is far more regular than previously believed by those persons who deprecate light quark symmetry. Secondly, the form of the symmetry breaking is now very clear. As the mass of the parent vector meson increases (as we go down each half of the table), so does the coupling, apparently following the simple rule $g_{VPP} \approx 0\cdot154\, m_V^{\frac{1}{2}}$ (for m_V in units MeV). Similarly, the mass-shell constants g'_{VPP} follow such a relation, except that the constant of proportionality is approximately $(0\cdot136{\pm}0\cdot003)$ MeV$^{-\frac{1}{2}}$ by a weighted mean method (and an error scale factor of 4, following the Particle Data Group's handling of errors). This result complies with the known scaling law behaviour for f_V as $m_V \to \infty$ (Grinstein 1992). Thus if the matrix elements of quark currents between a given vector meson and the vacuum state are defined by

$$< 0|\bar{q}_2\, \gamma_\mu\, q_1|V> \; = f_V\, m_V\, \epsilon_\mu\,,$$

then it is well known (Reinshagen and Rückl 1993) that $f_V \propto |\psi(0)|/m_V^{\frac{1}{2}}$ as $m_V \to \infty$. Translating to our terminology $f_V = em_V/g'_{VPP}$, we verify this prediction and, importantly, we find that the result is also supported in the light meson sector. Admittedly, the $\omega \to e^+e^-$ has a very high g'_{VPP}, but since the width of $\omega \to \mu^+\mu^-$ is only known to an upper bound (providing a lower-bound estimate of g'_{VPP}), the anomaly remains unsubstantiated.

Table 2 predominantly lists the results from studying radiative decays to obtain estimates of g_{VVP} using vector meson dominance. The first entry in the table

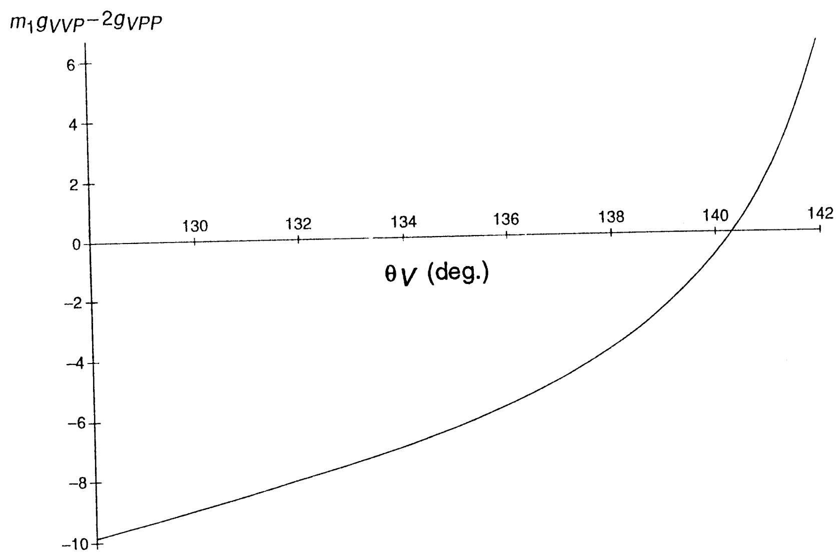

Fig. 3. Support for the supermultiplet symmetry condition $2g_{VPP} = m_1 g_{VVP}$ in the decays $\phi \to K^0 K^0$ and $\phi \to \rho\pi$.

is for the decay $\phi \to \rho\pi$, and leads to a direct determination of g_{VVP}, not via a radiative transition. In fact, we use it to test the supermultiplet prediction $2g_{VPP} = m_1 g_{VVP}$, the results of which are shown in Fig. 3. This figure shows the sum $m_1 g_{VVP} - 2g_{VPP}$ plotted against vector mixing angle θ_V, and clearly demonstrates that the supermultiplet rule is satisfied at $\theta_V \approx 140 \cdot 3°$, very close to the accepted value $\theta_V = 140 \cdot 6°$, and it is for this reason that we have used this value in all our calculations.

In the case of radiative decays, where we know that the coupling is related to the ratio g_{VVP}/g'_{VPP}, we have used the relationship $g'_{VPP} \approx 0 \cdot 136 m_V^{\frac{1}{2}}$, which is well supported by the data in Table 1. Importantly, this relation applies to the virtual vector meson so that for decays mediated via the ideal field ω_8, we have to use its mass of approximately 931 MeV. The data show that once again the coupling is quite regular, but now the symmetry breaking appears to obey a power-law relation $g_{VVP} \propto m_1^{-n}$, where $\frac{1}{2} < n < \frac{3}{2}$.

5. Predictions

With a clearer understanding of the effects of symmetry breaking on the coupling constant, we may now confidently determine the decay rates for non-Zweig, allowed decays. In particular we study the decays $\omega \to \pi^+\pi^-$ and $\phi \to \pi^+\pi^-$, both of which are mediated by a virtual photon coupling between the parent vector meson and a ρ meson (electromagnetic mixing). Thus in the case $\omega \to \pi^+\pi^-$, we have the overall coupling of

$$g_{\omega\pi\pi} = e^2 m_\rho^2 g_{\rho\pi\pi} / g'_{\omega PP}(m_\omega^2 - m_\rho^2) g'_{\rho PP} \,,$$

and for $\phi \to \pi^+\pi^-$ we have

$$g_{\phi\pi\pi} = e^2 m_\rho^2 g_{\rho\pi\pi} / g'_{\phi PP}(m_\phi^2 - m_\rho^2) g'_{\rho PP} \,.$$

Using

$$\begin{aligned} g_{\rho\pi\pi} &= \sqrt{2}(0 \cdot 1537 \pm 0 \cdot 002) m_\rho^{\frac{1}{2}} \,, \\ g'_{\rho PP} &= \sqrt{2}(0 \cdot 136 \pm 0 \cdot 003) m_\rho^{\frac{1}{2}} \,, \\ g'_{\omega PP} &= \sqrt{6}(0 \cdot 136 \pm 0 \cdot 003) m_\omega^{\frac{1}{2}} / \sin\theta_V \,, \\ g'_{\phi PP} &= \sqrt{6}(0 \cdot 136 \pm 0 \cdot 003) m_\phi^{\frac{1}{2}} / \cos\theta_V \,, \end{aligned}$$

we predict

$$\begin{aligned} \Gamma_{\omega \to \pi^+\pi^-} &= (1 \cdot 66 \pm 0 \cdot 16) \times 10^{-2} \text{ MeV} \,, \\ \Gamma_{\phi \to \pi^+\pi^-} &= (5 \cdot 88 \pm 0 \cdot 55) \times 10^{-4} \text{ MeV} \,, \end{aligned}$$

which compare favourably with the presently accepted values

$$\begin{aligned} \Gamma_{\omega \to \pi^+\pi^-} &= (1 \cdot 86 \pm 0 \cdot 25) \times 10^{-2} \text{ MeV} \,, \\ \Gamma_{\phi \to \pi^+\pi^-} &= (3 \cdot 5 \pm 2 \cdot 8) \times 10^{-4} \text{ MeV} \,. \end{aligned}$$

The symmetry-breaking effects we have observed also lead to a measurable consequence in the radiative decays of heavy mesons. We begin by re-examining the decays $K^{*\pm} \to K^{\pm}\gamma$ and $K^0\gamma$. Experimentally, the K^* branching fraction is

$$\Gamma_{K^{*0} \to K^0\gamma}/\Gamma_{K^{*+} \to K^+\gamma} = 2\cdot 31 \pm 0\cdot 29\,,$$

and allowing for phase space factors, this translates into a coupling constant ratio of

$$|g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}| = 1\cdot 514 \pm 0\cdot 095\,,$$

and as such is far from the exact SU(3) ratio of 2. Under the supermultiplet scheme, one can show that the decays proceed via two intermediate vector mesons, ρ^0 and ω_8. Following the procedure we described for determining the normalisation factors, one finds

$$g_{K^{*+}K^+\gamma} = e\left(\frac{g_{K^{*+}\rho^0 K^+}}{g'_{\rho^0 PP}} + \frac{g_{K^{*+}\omega_8 K^+}}{g'_{\omega_8 PP}}\right), \tag{25}$$

$$g_{K^{*0}K^0\gamma} = e\left(\frac{g_{K^{*0}\rho^0 K^0}}{g'_{\rho^0 PP}} + \frac{g_{K^{*+}\omega_8 K^0}}{g'_{\omega_8 PP}}\right). \tag{26}$$

If one assumes that g'_{VPP} is constant, then

$$\begin{aligned} g_{K^{*+}K^+\gamma} &= e\left(\frac{1/\sqrt{2}}{\sqrt{2}} + \frac{-1/\sqrt{6}}{\sqrt{6}}\right)(g_{VVP}/g'_{VPP}) \\ &= g_{VP\gamma}/3\,, \\ g_{K^{*0}K^0\gamma} &= e\left(\frac{-1/\sqrt{2}}{\sqrt{2}} + \frac{-1/\sqrt{6}}{\sqrt{6}}\right)(g_{VVP}/g'_{VPP}) \\ &= -2g_{VP\gamma}/3\,, \end{aligned}$$

and we arrive at the exact SU(3) prediction. If instead we use a symmetry-breaking g'_{VPP} we must substitute $g'_{\rho^0 PP} = \sqrt{2}Cm_{\rho^0}^{\frac{1}{2}}$ and $g'_{\omega_8 PP} = \sqrt{6}Cm_{\omega_8}^{\frac{1}{2}}$ in equations (25) and (26). Thus

$$\frac{g_{K^{*0}K^0\gamma}}{g_{K^{*+}K^+\gamma}} = -\frac{m_{\rho^0}^{-\frac{1}{2}} + m_{\omega_8}^{-\frac{1}{2}}/3}{m_{\rho^0}^{-\frac{1}{2}} - m_{\omega_8}^{-\frac{1}{2}}/3} = -1\cdot 87\,,$$

and notice that the result is independent of C, the constant of proportionality between g'_{VPP} and $m_V^{\frac{1}{2}}$. Although not matching the experimental result, it is an improvement on exact SU(3). Actually, the most satisfactory explanation of the symmetry-breaking mechanism comes from Bramon and Scadron (1989). They attribute the deviation from exact SU(3) to the constituent mass difference between the strange and non-strange quarks in the loop of a quark triangle diagram. As $K^* \to K\gamma$ excite both strange and non-strange quarks, such a difference must be accounted for. With these corrections, the experimental ratio is found to match theoretical estimates very well. We intend to apply the method to heavier meson cases in future work (Liu and Jones 1995).

Let us continue to use the 'mass variation principle' of g'_{VPP} in the heavy meson sector. Upon application to the D^* and B^* mesons, we obtain

$$\frac{g_{D^{*0}D^0\gamma}}{g_{D^{*+}D^+\gamma}} = \frac{3m_{\rho^0}^{-\frac{1}{2}} + m_{\omega_8}^{-\frac{1}{2}} + 4m_{J/\psi}^{-\frac{1}{2}}}{-3m_{\rho^0}^{-\frac{1}{2}} + m_{\omega_8}^{-\frac{1}{2}} + 4m_{J/\psi}^{-\frac{1}{2}}} \approx -60\,, \tag{27}$$

$$\frac{g_{B^{*0}B^0\gamma}}{g_{B^{*+}B^+\gamma}} = \frac{-3m_{\rho^0}^{-\frac{1}{2}} + m_{\omega_8}^{-\frac{1}{2}} + 2m_{\Upsilon}^{-\frac{1}{2}}}{3m_{\rho^0}^{-\frac{1}{2}} + m_{\omega_8}^{-\frac{1}{2}} + 2m_{\Upsilon}^{-\frac{1}{2}}} \approx -0\cdot 34$$

which are significantly different from the exact SU(5) predictions of

$$g_{D^{*0}D^0\gamma}/g_{D^{*+}D^+\gamma} = 4\,,$$

$$g_{B^{*0}B^0\gamma}/g_{B^{*+}B^+\gamma} = 0\,,$$

and as such require better experimental data to test the results.

In addition to these relative decay rate predictions, the supermultiplet scheme can easily be adapted to decay width calculations. Scattered amongst Tables 1 and 2 are various constants determined by extending the supermultiplets to include the charm and bottom quark mesons. In particular, using the upper bound of 131 keV for the D^{*+} decay width (ACCMOR Collab. 1992), we have found $g_{VPP} < 10$. Conversely, we can use our knowledge of the effects of symmetry breaking to predict the VPP coupling constant for D^{*+}. We find

$$g_{VPP}(D^{*+}) \approx (0\cdot 1537 \pm 0\cdot 002)(2010)^{\frac{1}{2}} = 6\cdot 89 \pm 0\cdot 09\,, \tag{28}$$

surprisingly similar to a heavy quark prediction of 7±1 by Colangelo *et al.* (1994*a*). We can use the g_{VPP} value for D^{*+} to calculate the total decay width of the $D^{*+} \rightarrow PP$ channels. Using the supermultiplet method, we can predict all the possible decays of D^{*+} into two pseudoscalars; however, phase space restricts the processes to $D^{*+} \rightarrow D^0\pi^+$ and $D^{*+} \rightarrow D^+\pi^0$ so that the width must be

$$\begin{aligned}\Gamma_{D^{*+}\rightarrow PP} &= \Gamma_{D^{*+}\rightarrow D^+\pi^0} + \Gamma_{D^{*+}\rightarrow D^0\pi^+} \\ &= g_{VPP}^2[\lambda^{\frac{3}{2}}(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2) + \lambda^{\frac{3}{2}}(m_{D^{*+}}^2, m_{D^+}^2, m_{\pi^0}^2)]/48\pi m_{D^{*+}}^5 \\ &= 57\cdot 7 \pm 1\cdot 5\ \text{keV}\,.\end{aligned}$$

We compare this with the radiative width $D^{*+} \rightarrow D^+\gamma$ in the following branching fraction:

$$
\begin{aligned}
\frac{\Gamma_{D^{*+}\to D^+\gamma}}{\Gamma_{D^{*+}\to PP}} &= \frac{(g_{VP\gamma}/g_{VPP})^2(m_{D^{*+}}^2 - m_{D^+}^2)^3/96\pi m_{D^{*+}}^3}{[\lambda^{\frac{3}{2}}(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2) + \lambda^{\frac{3}{2}}(m_{D^{*+}}^2, m_{D^+}^2, m_{\pi^0}^2)]/48\pi m_{D^{*+}}^5} \\
&\approx \frac{[e(-3m_{\rho^0}^{-\frac{1}{2}} + m_{\omega_8}^{-\frac{1}{2}} + 4m_{J/\psi}^{-\frac{1}{2}})/(6\times 0{\cdot}1361)]^2(m_{D^{*+}}^2 - m_{D^+}^2)^3}{\lambda^{\frac{3}{2}}(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2) + \lambda^{\frac{3}{2}}(m_{D^{*+}}^2, m_{D^+}^2, m_{\pi^0}^2)} \\
&\approx (9{\cdot}48 \pm 0{\cdot}43)\times 10^{-5},
\end{aligned}
$$

where in particular we have used the supermultiplet prediction $g_{VVP}/g_{VPP} = 2/m_{D^{*+}}$ and we have confidence in our prediction to this order. This finding implies that the dominant decay modes in D^{*+} decay are the PP channels we derived, and as such approximate well to the full width. Thus the radiative branching fraction for $D^{*+} \to D^+\gamma$ is relatively small. This is not inconsistent with recent measurements by CLEO which measured a fraction of $(1{\cdot}1{\pm}1{\cdot}4{\pm}1{\cdot}6)\%$ (CLEO Collab. 1992). However, our prediction does conflict with other theoretical models (Kamal and Xu 1992; Colangelo *et al.* 1994*b*; O'Donnell and Xu 1994; Jain *et al.* 1994). Branching fraction calculations for the two PP channels yield

$$\Gamma_{D^{*+}\to D^0\pi^+}/\Gamma_{D^{*+}\to \mathrm{all}} \approx 68{\cdot}8\%\,,$$

$$\Gamma_{D^{*+}\to D^+\pi^0}/\Gamma_{D^{*+}\to \mathrm{all}} \approx 31{\cdot}2\%\,,$$

which compare well with other models and the experimentally determined results from CLEO (1992):

$$
\begin{aligned}
\Gamma_{D^{*+}\to D^0\pi^+}/\Gamma_{D^{*+}\to \mathrm{all}} &= (68{\cdot}0 \pm 1{\cdot}4 \pm 2{\cdot}4)\%\,, \\
\Gamma_{D^{*+}\to D^+\pi^0}/\Gamma_{D^{*+}\to \mathrm{all}} &= (31{\cdot}0 \pm 0{\cdot}4 \pm 1{\cdot}6)\%\,.
\end{aligned}
$$

We are able to employ similar methods in the decays of the D^{*0} vector meson. In this instance, the possible PP decay channels are restricted by phase space to $D^{*0} \to D^0\pi^0$. We calculate this width to be $27{\cdot}2{\pm}0{\cdot}7$ keV, where we used $g_{D^{*0}D^0\pi^0} = (0{\cdot}1537 \pm 0{\cdot}002)(2007{\cdot}1)^{\frac{1}{2}}/\sqrt{2}$. To determine the radiative width $D^{*0} \to D^0\gamma$, we apply relation (27) along with a small correction for the change in phase space to derive

$$\Gamma_{D^{*0}\to D^0\gamma} \approx 3672 \times \Gamma_{D^{*+}\to D^+\gamma} = (20{\cdot}1 \pm 1{\cdot}1)\ \mathrm{keV}\,.$$

Thus the total D^{*0} width is $(47{\cdot}3{\pm}1{\cdot}3)$ keV, although the result is sensitive to the supermultiplet prediction $g_{VVP}(D^*)/g_{VPP}(D^*) = 2/m_{D^*}$. Consequently we predict the following branching fractions:

$$\Gamma_{D^{*0}\to D^0\pi^0}/\Gamma_{D^{*0}\to \mathrm{all}} \approx (57{\cdot}5 \pm 2{\cdot}2)\%\,,$$

$$\Gamma_{D^{*0}\to D^0\gamma}/\Gamma_{D^{*0}\to \mathrm{all}} \approx (42{\cdot}5 \pm 2{\cdot}6)\%\,,$$

which are in fair agreement with the CLEO data:

$$\Gamma_{D^{*0}\to D^0\pi^0}/\Gamma_{D^{*0}\to \mathrm{all}} = (64 \pm 2{\cdot}4 \pm 4{\cdot}5)\%\,,$$

$$\Gamma_{D^{*0}\to D^0\gamma}/\Gamma_{D^{*0}\to \mathrm{all}} \approx (36 \pm 2{\cdot}4 \pm 4{\cdot}5)\%\,.$$

5. Conclusions

This study of two-body meson decays has shown that the supermultiplet method unifies meson decays quite well, even for the light quarks. The most significant finding is that the coupling between mesons is susceptible to symmetry-breaking mechanisms, but in a *regular* way, allowing us to successfully extrapolate to decay rates for other processes. In particular, the methods are readily applicable to heavy quark examples, as highlighted by our examination of D^* processes.

Acknowledgments

The authors would like to thank Dr Dongsheng Liu for many discussions, along with Dr Dirk Kreimer for insights into extrapolating the vector dominance model from $k^2 = 0$ to other values.

References

ACCMOR Collab., S. Barlag *et al.* (1992). *Phys. Lett.* B **278**, 480.
Bég, M. A. B., and Pais, A. (1965). *Phys. Rev.* **137**, 1514.
Bramon, A., and Scadron, M. D. (1989). *Phys. Rev.* D **40**, 3779.
Bramon, A., and Scadron, M. D. (1990). *Phys. Lett.* B **234**, 346.
CLEO Collab., F. Butler *et al.* (1992). *Phys. Rev. Lett.* **69**, 2041.
Colangelo, P., Fazio, F. D., and Nardulli, G. (1993). *Phys. Lett.* B **316**, 555.
Colangelo, P., Nardulli, G., Deandrea, A., Di Bartolomeo, N., Gatto, R., and Feruglio, F. (1994*a*). On the coupling of heavy mesons to pions in QCD. UGVA-DPT 1994/06-856.
Colangelo, P., Fazio, F. D., and Nardulli, G. (1994*b*). *Phys. Lett.* B **334**, 175.
Delbourgo, R. (1975). *Phys. Lett.* B **59**, 357.
Delbourgo, R., and Liu, D. (1994). *Phys. Rev.* D **49**, 5979.
Delbourgo, R., Rashid, M. A., Salam, A., and Strathdee, J. (1965). *In* 'High-Energy Physics and Elementary Particles', p. 455 (International Atomic Energy Agency: Vienna.)
Flynn, J. M., and Isgur, N. (1992). *J. Phys.* G **18**, 1627.
Gasiorowicz, S. (1967). 'Elementary Particle Physics' (John Wiley: New York).
Grinstein, B. (1992). *Annu. Rev. Nucl. Part. Sci.* **42**, 101.
Harari, H. (1969). *Phys. Rev. Lett.* **22**, 562.
Hussain, F., Körner, J. G., and Thompson, G. (1991). *Ann. Phys.* **206**, 334.
Jain, P., Momen, A., and Schechter, J. (1994). Heavy meson radiative decays and light vector meson dominance. SU-4240-581.
Kamal, A. N., and Xu, Q. P. (1992). *Phys. Lett.* B **284**, 421.
Liu, D., and Jones, N. R. (1995). In preparation.
Mannel, T. (1993). *Chin. J. Phys.* **31**, 1.
O'Donnell, P. J., and Xu, Q. P. (1994). Strong and radiative D^* decays. UTPT-94-15.
Particle Data Group, J. J. Hernandez *et al.* (1992). *Phys. Rev.* D **45**, S1.
Reinshagen, S., and Rückl, R. (1993). On heavy meson decay constants. CERN-TH-6879-93.
Rosner, J. L. (1969). *Phys. Rev. Lett.* **22**, 689.
Sakita, B., and Wali, K. C. (1965). *Phys. Rev.* B **139**, 1355.
Salam, A., Delbourgo, R., and Strathdee, J. (1965). *Proc. R. Soc. London* **284**, 146.

Manuscript received 13 September, accepted 22 November 1994