Aust. J. Phys., 1995, **48**, 1083–92

Modelling the Large Scale Structure of the Universe after COBE*

P. J. Quinn

Mount Stromlo and Siding Spring Observatories,
Australian National University,
Canberra, ACT 0200, Australia.

Abstract

N-body models running on supercomputers have been widely used to explore the development of structure in the expanding Universe. Recent results from the COBE satellite have provided a global normalisation of these models which now allows detailed comparisons to be drawn between observations and model predictions. Some predictions of the cold dark matter primordial perturbation spectrum are now shown to be consistent with surveys of galaxy redshifts.

1. Introduction

Objects in the Universe have well defined scales of mass and linear size. From individual stars with masses of 10^{30} kg and sizes of 10^9 m to galaxies consisting of 10^{11} stars with radii of 10^{19} m to clusters of galaxies consisting of thousands of galaxies with radii of 10^{22} m. Why isn't the Universe uniformly filled with stars? Why are there such large differences in the masses of the Universal building blocks of stars, galaxies and clusters of galaxies? Where do these preferred scales for structure come from? By modelling the physical processes that produce structure in the expanding Universe we hope to discover the important interactions that set universal scales of mass and linear size. On the scales of individual stars, it has been known since the 1950s that nuclear interactions play a key role in determining masses and sizes. The gravity of a star confines the energy release from nuclear burning as does the pressure from electromagnetic effects in the stellar plasma. On the scales of a single galaxy, the energy density from electromagnetic interactions is small compared to gravity and on the scales of clusters of galaxies the binding energy and evolution is totally dominated by gravitational forces. In this paper we shall examine the growth of scales from gravitational processes alone. The motivation for this restricted modelling is that:

- Gravity is currently the dominant force for long range interactions
- The Universe seems to be dominated by dark matter that could possibly be made of massive particles that only interact gravitationally with normal matter

* Refereed paper based on a contribution to the inaugural Australian General Relativity Workshop held at the Australian National University, Canberra, in September 1994.

0004-9506/95/061083$05.00

- Gravity is relatively simple to model and understand theoretically with perturbation techniques.

We will examine the growth of structure via gravitational processes in Section 2 and what demands this places on modelling techniques in Section 3. Section 4 will review the observational normalisation of the models from data on fluctuations in the cosmic microwave background and Section 5 will discuss recent results from modelling the cold dark matter density perturbation spectrum.

2. Gravitational Structure Growth

An initially uniform density Universe with Hubble constant at time t_i of H_i will have zero total gravitational energy if the mass density is given by

$$\rho_{ci} = \frac{3H_i^2}{8\pi G}, \tag{1}$$

which is called the critical density at epoch i. Any spherical region with density greater than ρ_{ci} has negative total gravitational binding energy. The evolution of such a region is shown schematically in Fig. 1. The excess mass inside the spherical region causes mass outside the region to slow with respect to the local Hubble expansion and eventually fall back into the centre of the perturbation. For an initially spatially constant excess density within a spherical perturbation (called a 'top hat'), the density contrast of the perturbed region with respect to the critical density at the time of maximum expansion (the turnaround in Fig. 1) is given by

$$\delta_m = \frac{(\delta_i - 1)^{\frac{1}{2}}}{\delta_i}(\delta_i + 0{\cdot}5) + \tfrac{3}{2}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{\delta_i - 1}{\delta_i}\right)\right], \tag{2}$$

where $\delta = (\rho - \rho_c)/\rho_c$. For small δ_i, top-hat perturbations turn around and stop expanding with the Universe when they reach a density contrast of $(3\pi/4)^2 \sim 5{\cdot}55$.

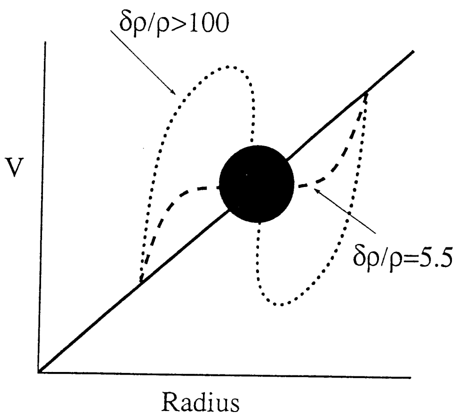

Fig. 1. Schematic evolution of an overdense region in the Hubble (velocity–radius) diagram. The overdensity slows neighbouring material to the point where it stops expanding with the Hubble flow ($\delta\rho/\rho \sim 5$) and eventually collapses to form an isolated virialised structure.

The density field defined by $\delta(\mathbf{x})$ can be expressed as a Fourier expansion with spatial wavenumbers k and complex amplitudes δ_k by

$$\delta(\mathbf{x}) = \frac{V}{(2\pi)^3} \int \delta_k \exp(i\mathbf{k}.\mathbf{x}) \mathrm{d}^3 k \,, \tag{3}$$

where V is the volume over which the transform is obtained (Turner and Kolb 1990). The power spectrum is usually written $P(k) = |\delta_k|^2 = A\mathcal{F}(k)$ where $\mathcal{F}$ is some function of k with normalised amplitude A. Inflationary cosmological models predict $P(k) \sim Ak$ for the spectrum of initial density perturbations at all scales. However, this spectrum is modified when the Universe becomes transparent to radiation and gravity takes over as the dominant force for moving matter around rather than radiation pressure. Before that time all perturbations with scales less than the current horizon scale were not allowed to grow due to radiation pressure. This suppression of growth on small scales causes the large k part of the spectrum to have a negative slope (see Fig. 2). If the Universe is dominated by slow moving (non-relativistic) cold dark matter (CDM) then the asymptotic slopes of the density perturbation spectrum are 1 on large scales, -3 on small scales, with a change over corresponding to the horizon scale at the time of matter–radiation decoupling.

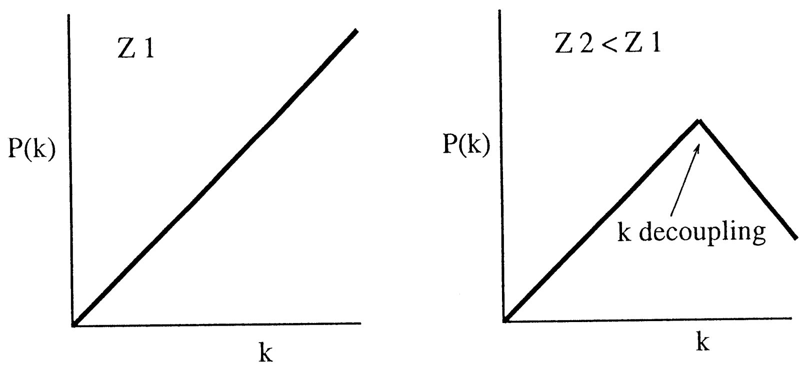

Fig. 2. Inflationary models predict an initial density perturbation spectrum which has a nearly linear dependence on spatial wave number. Before decoupling, perturbations smaller than the horizon are not allowed to grow, which leads to a feature in the spectrum after decoupling at a scale corresponding to the horizon at that time.

In the linear theory for the gravitational growth of density perturbations (see for example Peebles 1993), the perturbation amplitude grows with the expansion factor of the Universe as $\sim 1/(1+z)$ or $time^{\frac{2}{3}}$. For a power law perturbation spectrum $P = Ak^n$, this implies a collapse time for a perturbation of total mass M to be proportional to $M^{(n+3)/4}$. In CDM models, small objects are the first to collapse. These small objects then merge gravitationally to form larger mass objects resulting in what is termed the 'bottom-up' picture of structure formation. If some process erased all the initial power on small scales then we could arrive at a situation where the largest objects form first and smaller scale objects are formed by fragmentation. Such models do not currently seem to be in general agreement with observations of large scale structure.

3. Building Numerical Models

A gravitating system consisting of many individual bodies (stars or galaxies) can be described by a distribution function $f(\mathbf{x}, \mathbf{v}, t)$ which gives the density of the system at all points in the six-dimensional phase space defined by positions and velocities. The evolution of the phase space density is given by the Boltzmann equation (Binney and Tremaine 1987)

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{6} \dot{w}_a \frac{\partial f}{\partial w_a} = \frac{Df}{Dt}, \tag{4}$$

where w_a is any phase space coordinate. If the system is collisionless and particle density is conserved along phase space trajectories, then the evolution of the system is described by the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{6} \dot{w}_a \frac{\partial f}{\partial w_a} = 0. \tag{5}$$

Gravitational interactions are long range and so we are led to ask how collisional are stellar systems of various sizes. For a system of N stars we can compute how long it would take for numerous, weak, long range interactions or several, strong, short range ones, to significantly alter the distribution function. This time scale is called the relaxation time and its ratio to the characteristic dynamical (time to free fall across a system) is given by

$$\frac{T_{\text{relax}}}{T_{\text{crossing}}} \sim 0{\cdot}1 \frac{N}{\ln(N)}. \tag{6}$$

Table 1 shows this ratio for several stellar systems. For galaxies, the relaxation time exceeds the age of the Universe and so they obey a collisionless Boltzmann equation. If we are to model the dynamics of galaxy formation and evolution, as well as the evolution of their cosmological environment, we must ensure the models are collisionless. If we are using a discrete particle (N–body) technique then we must use sufficient particles to make the dynamics collisionless over the course of the evolution period studied which implies at least 10^6 particles in most cases.

Table 1. Relaxation times

N	T_{crossing} (yr)	$T_{\text{relax}}/T_{\text{crossing}}$	$T_{\text{relax}}/T_{\text{Hubble}}$
10^2	10^3	2	10^{-7}
10^6	10^5	10^4	10^{-1}
10^{11}	10^8	10^8	10^6

The amplitude of mass perturbations today at some given scale can be estimated by counting galaxies and equating galaxy number density fluctuations to underlying mass density perturbations. On the present scale of 8 megaparsecs (comparable to the sizes of rich clusters of galaxies), number density fluctuations are of order unity. Since the perturbations grow in the linear domain like the expansion factor of the Universe of $(1+z)$, then at the epoch of matter and radiation decoupling ($z \sim 1000$) these perturbations will have an amplitude of order 10^{-3}. If we wish to imprint a given perturbation amplitude on our initial conditions at some scale then we have to insure that the shot-noise on that scale from the particle distribution is much less than the imposed fluctuation amplitude. This would imply at least 10^5 particles on the 8 megaparces scale. Since we wish to deal with computational volumes comparable to observation samples ($z \leq 1$) then we need many millions of particles to enable us to accurately impose the initial conditions. Furthermore, if we wish to study the internal structure of galaxies simultaneously with larger scale environments we need spatial dynamic ranges from 1 kiloparsec to hundreds of megaparsecs.

N-body models with large numbers of particles and high dynamic range have only recently been made possible by two factors. Firstly, new techniques for finding the accelerations from N bodies based on hierarchical data structures have been developed (Barnes and Hut 1986; Salmon 1990; Salmon *et al.* 1994). These 'tree' codes transform the $\sim N^2$ operations necessary to find all accelerations in a system of N particles into a problem with $\sim N\log(N)$ operations. This computational saving is essential if problems with $N \geq 10^6$ are to be attempted on any of the current generation of supercomputers. Secondly, these tree techniques map efficiently onto parallel computing architectures. Running these codes on machines with $\sim$500 workstation-class nodes can achieve sustained performance in the several Gflop range and allows access to several tens of gigabytes of memory (Salmon 1990).

4. Observational Constraints on Structure Growth

The perturbation spectrum is defined by a normalising amplitude and functional dependence on spatial wavenumber $\mathcal{F}(k)$. The shape of $\mathcal{F}$ is given by theory and the normalisation set by observations of the fluctuation amplitude at the current epoch on a range of spatial scales. There are two problems with normalising the amplitude at the current epoch. Firstly, the tracers (galaxies) have gone through nonlinear development and so linear perturbation growth predictions are in error. Secondly, we are not sure to what extent galaxies trace the underlying total mass perturbations which we know to be dominated by dark matter. What we would like to do is determine the fluctuation amplitudes in the linear domain and where the tracers sample the total gravitational potential.

The cosmic microwave background (CMB) radiation was discovered in 1965. It represents a snapshot of the Universe at the time when the mean free path of photons became comparable to the horizon. For most cosmological models this occurs at $z \sim 1100$ when the Universe had a temperature of approximately 3030 K. At this redshift, three important events occurred:

(1) plasma decouples from the radiation;
(2) ions recombine; and
(3) residual ionisation freezes in.

The observed temperature of the Universe at this time will be affected by

- motion of the observer (dipole anisotropy)
- potential (i.e. density) fluctuations on the last photon scattering surface (Sachs–Wolfe effect)
- fluctuations in the radiation field

- motion of the last scattering surface
- reionisation

The dipole fluctuation in the CMB radiation is well studied and is visible as blackbody temperature fluctuations of one part in 10^3. The dipole is interpreted as motion of our Galaxy in the rest frame of the CMB with a velocity of order 600 km s^{-1}. The most important other source of temperature fluctuations (in the absence of sources of radiation that predate recombination) is the Sachs–Wolfe effect.

Until 1992 no experiment had seen any temperature fluctuations on small scales down to a limit of $\delta T/T \sim 10^{-5}$. The amplitude of temperature fluctuations due to the primordial density perturbations that resulted in current epoch galaxies can be estimated as

$$\frac{\delta T}{T} \sim (0\cdot 01 - 0\cdot 1)\frac{\delta\rho}{\rho}, \tag{7}$$

where the uncertainty comes from the equation of state. We know already that current regions of size 8 megaparsecs had $\delta \sim 10^{-3}$ at $z \sim 1000$. On larger scales ($\sim$ 100 megaparsecs, corresponding to the resolution of CMB instruments), $\delta \sim 10^{-4}$ and therefore we would expect to see the seeds of structure appear as temperature fluctuations of order one part in 10^5 or 10^6. The fact that these had not been seen was making everyone rather nervous until June 1992 when the COBE satellite team announced the discovery of temperature fluctuations with an amplitude of order $\sim 5 \times 10^{-6}$. This was an important turning point in the history of modern cosmology. We can now normalise our models from the COBE amplitudes in the linear domain and also be sure we are measuring the total mass density fluctuation via the Sachs–Wolfe process. The other important result from COBE is the angular behaviour of these fluctuation which favours a power spectrum with $n \sim 1$, in agreement with inflationary theories.

5. Modelling the Universe after COBE

Armed with parallel supercomputers running hierarchical tree codes for N-body problems and the overall normalisation and spectral constraints placed by COBE, we are now in an excellent position to test cosmological models. We have investigated several large cosmological models of cold dark matter outlined in Table 2 (Zurek *et al.* 1994).

Table 2. Model parameters for the four CDM runs

The values for Q (amplitude of the quadrupole temperature perturbations seen by COBE) follow the definition of Efstathiou *et al.* (1992). Values for $\sigma_8^{\rm linear}$ are for the Efstathiou CDM spectrum integrated over an infinite k space. Actual model initial amplitudes are within 5% of these values at 8 h^{-1} Mpc

Label	N	R_0 (Mpc)	Q (μK)	z_i	$\sigma_8^{\rm linear}$	$\sigma_8^{\rm mass}$	$\sigma_8^{\rm halos}$
cdm250a	17158608	125	9·5	55·5	0·67	0·78	0·68
cdm250b	8783482	125	23·2	110·2	1·61	1·65	1·04
cdm250c	8783482	125	14·6	68·8	1·01	1·06	0·76
cdm100	17154598	50	10·9	62·3	0·75	0·64	0·52

The large models resulted in ~13,000 individual, collapsed dark matter halos with sizes of order 100–200 kiloparsecs consisting of 100–1000 particles each. Fig. 3 shows the distribution of halos in model cdm250c. The small square region is shown with all its particles and with halos marked in Fig. 4.

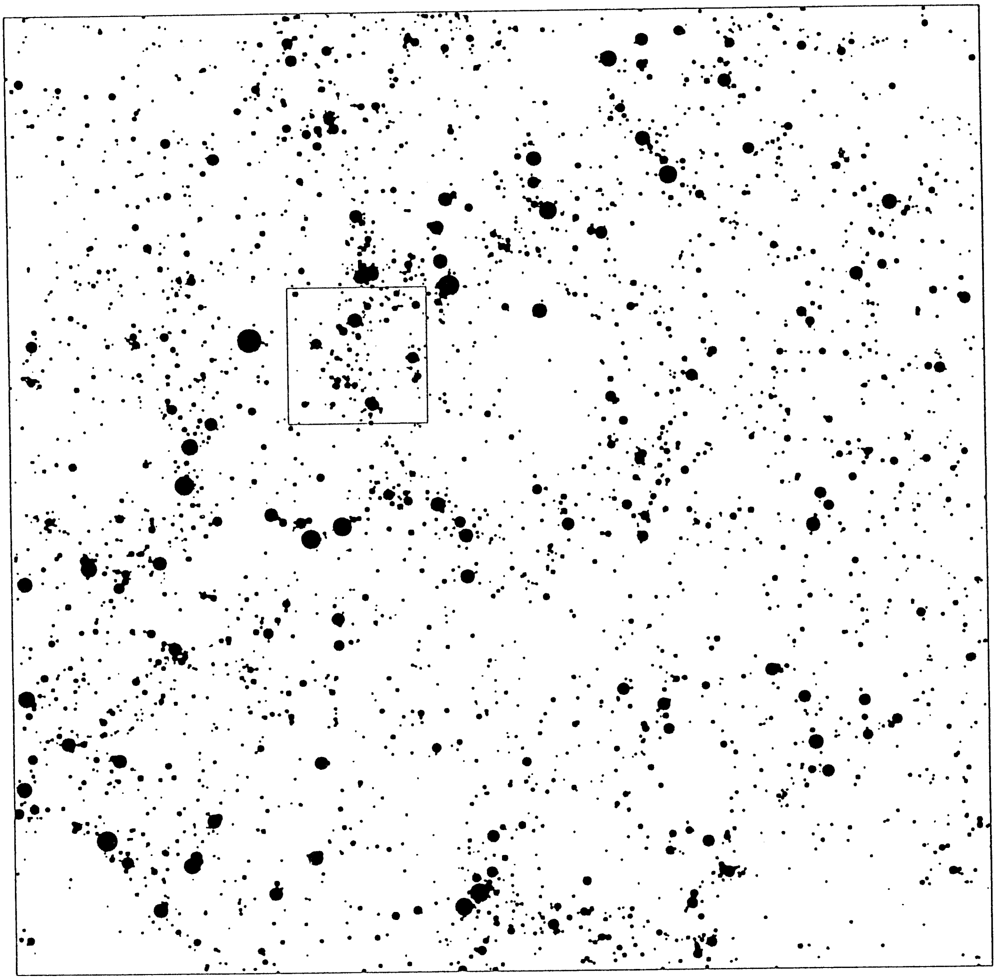

Fig. 3. A plot of the projected locations of halos in model cdm250c. The size of the square is 180 Mpc across. Sizes of dots are approximately proportional to the masses of the halos. The small square indicates the region shown in Fig. 4.

One of the perceived problems with CDM has been the assertion that the observed random pairwise motions of galaxies in the Universe (the random motion of galaxies superimposed on the Hubble flow) was approximately one half that produced in COBE normalised CDM models. If true, this would be a major difficulty for CDM. In the models previously used to compare CDM to observations (e.g. Davis *et al.* 1985), galaxies were represented as point masses due to limitations on particle numbers by computation time and N–body algorithm. Using sufficient particles to provide many hundreds per galaxy, we have shown that compact systems (halos and presumably the luminous galaxies they contain)

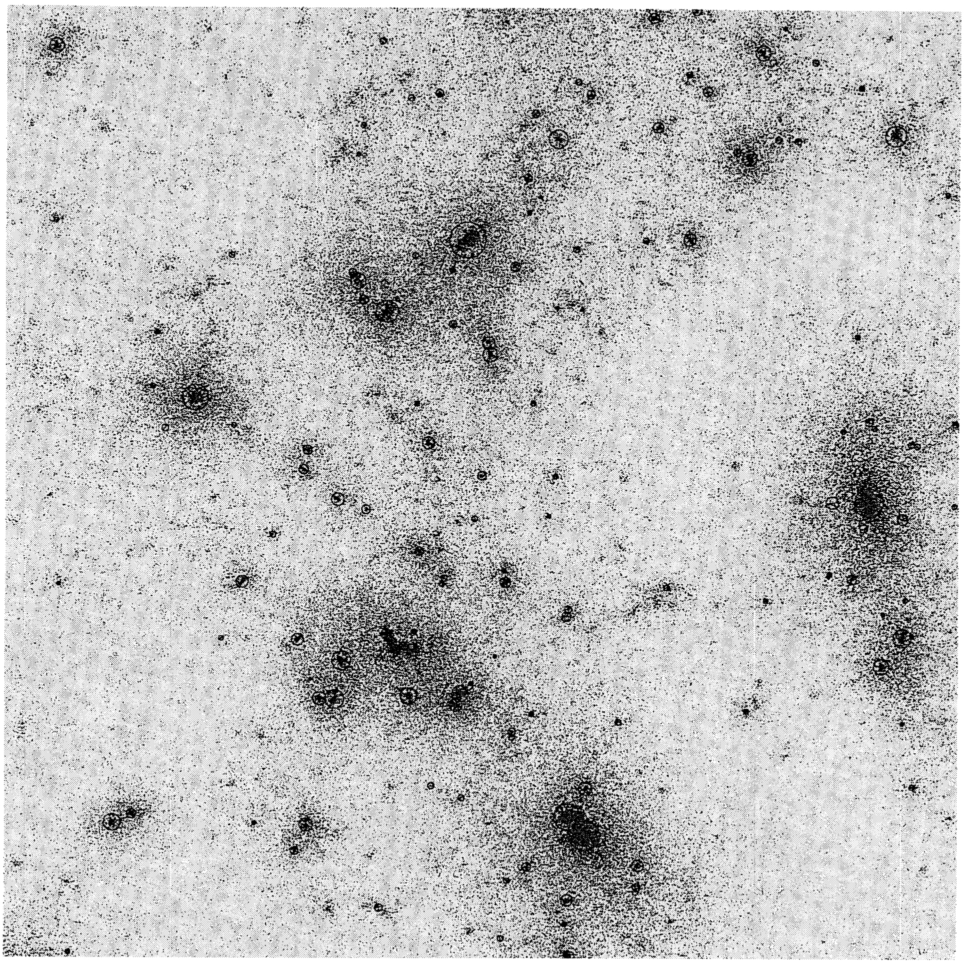

Fig. 4. A high–resolution plot of every particle (about 250,000) in one 50th of the total system with circles representing the location of galaxy halos.

move more slowly than the free particles in the model, see Fig. 5. This lowering of the motion of halos relative to the bulk of the uncondensed mass is due to halos loosing kinetic energy relative to particles via dynamical friction and merging (Zurek *et al.* 1994). At a scale of one megaparsec, halos move with random motions of order 70% of that of free particles. Although in the correct direction, this effect is not large enough to solve the factor of two difference between CDM and observations. We then tried to apply the same techniques used by Davis and Peebles (1983) to measure the peculiar pairwise motion from galaxy redshift catalogues to catalogue-like subsamples of the CDM models. By doing this we discover that the derived mean pairwise peculiar motion is highly sensitive to the clustering content of the catalogue. Large clusters of galaxies bias the derived peculiar motion to larger values. When examining the galaxy redshift catalogues, Davis and Peebles excluded the Virgo cluster which is a nearby rich cluster that dominates the redshift surveys. Once the clusters are corrected for, COBE normalised CDM and the galaxy catalogues give consistent values of the pairwise peculiar motion of galaxies. This result, obtained with high resolution N-body models, has removed one of the major stumbling blocks to the CDM theory.

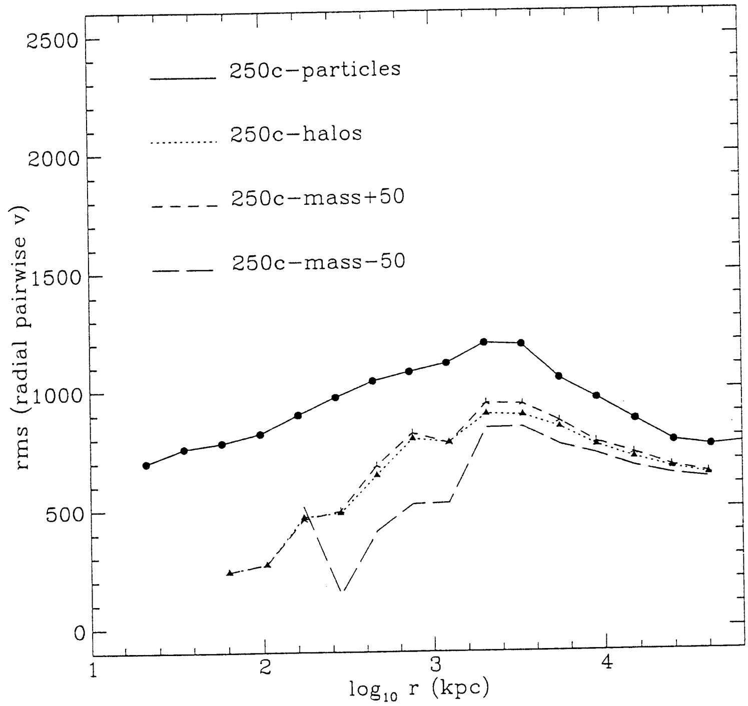

Fig. 5. One-dimensional relative peculiar velocity of halos and particles in the model with closest to COBE normalisation. There is a significant scale-dependent velocity bias.

6. Conclusion and Summary

The growth of structure in the Universe proceeds from density perturbation seeds laid down in the inflationary Universe to the present day via gravitational instability. A given model is defined by its density perturbation spectrum and overall normalisation. Modelling the time development of these spectra requires high performance parallel supercomputers and very large numbers of particles so that the dynamic range can be appropriate to current galaxy catalogues. The discoveries of the COBE satellite have set the overall normalisation of possible perturbation spectra (such as cold dark matter) and have demonstrated that the very largest scales are consistent with a primordial perturbation spectrum given by inflation. We have used hierarchical N-body techniques to evolve COBE normalised CDM models. These models have demonstrated that CDM does not produce an inconsistent amplitude of the pairwise random motions of galaxies. Rather, the methods used to derive the peculiar motions from the data have been flawed by regarding the Universe as 'isothermal' and ignoring the greater random motions of galaxies found in regions of large overdensity (like current epoch galaxy clusters).

While gravitational N-body models can tell us a great deal about the dynamics of structure formation, they only model one of the physical processes necessary to form galaxies. We know galaxies consist of stars and gas which are dissipated structures, i.e. they have lost binding energy via electromagnetic radiation. We cannot therefore account for the characteristic scales of galaxies and stars via gravity alone. We need to include extra physical processes (such as hydrodynamics, MHD and radiative transport) into our modelling. While it is currently possible to include hydrodynamical physics into the gravitational N-body codes, we do not even know how to parametrise the process of star formation. The interplay of magnetic, radiative, gravitational and nuclear physics in the process of star formation is very difficult to understand even in the simplest of geometries and compositions. It is actually possible that we may learn a great deal about the star formation process by modelling gravity and hydrodynamics alone and then forcing the resulting 'galaxies' to form stars in such a way as to produce galaxies like those observed. Although supercomputers and algorithms may improve significantly, the physics of star formation is the most important missing link in developing a complete picture of structure formation.

References

Barnes, J., and Hut, P. (1986). *Nature* **324**, 446.

Binney, J., and Tremaine, S. (1987). 'Galactic Dynamics' (Princeton University Press).

Davis, M., and Peebles, P. J. E. (1983). *Astrophys. J.* **267**, 465.

Davis, M., Efstathiou, G., Frenk, C. S., and White, S. D. M. (1985). *Astrophys. J.* **292**, 371.

Efstathiou, G., Bond, J. R., and White, S. D. M. (1992). *Mon. Not. R. Astron. Soc.* **258**, 1.

Peebles, P. J. E. (1993). 'Physical Cosmology' (Princeton University Press).

Salmon, J. K. (1990). Parallel Hierarchical N-body Methods. Ph.D. thesis, Caltech.

Salmon, J. K., Warren, M. S., and Winckelmans, G. S. (1994). *Int. J. Supercomputer Applications* **8**, 129–42.

Turner, M. S., and Kolb, E. W. (1990). 'The Early Universe', Frontiers in Physics' (Addison–Wesley: New York).

Zurek, W. H., Quinn, P. J., Salmon, J. K., and Warren, M. S. (1994). *Astrophys. J.* **431**, 559.

Manuscript received 31 January, accepted 13 July 1995