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Positron-acoustic Waves in an Electron–Positron Plasma with an Electron Beam

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Abstract

The nonlinear wave structures of large-amplitude positron-acoustic waves are studied in an electron–positron plasma with an electron beam. We present the region where positron-acoustic waves exist by analysing the structure of the pseudopotential. The region depends sensitively on the positron density, the positron temperature and the electron beam temperature. It is shown that the maximum amplitude of the wave decreases as the positron temperature increases, and the region of positron-acoustic waves spreads as the positron temperature increases. The present theory is applicable to analysing large-amplitude positron-acoustic waves associated with positrons which may occur in interplanetary space.

1. Introduction

Contrary to the usual plasma with electrons and positive ions, it is known that nonlinear waves in plasmas having positrons behave differently (Rizzato 1988). In fact, electron–positron plasmas appear in models of the early Universe (Rees 1983), active galactic nuclei (Miller and Witta 1987), pulsar magnetospheres (Michel 1982) and the solar atmosphere (Hansen and Emslie 1988). When positrons are introduced into the plasma, the response of the plasma to disturbances is found to be drastically modified. There have been several reports on solitons with small amplitudes in plasmas with a significant percentage of positrons (Popel *et al.* 1995). An electron–positron plasma is usually characterised as a fully ionised gas consisting of electrons and positrons, the masses of which are equal, as seen in the solar atmosphere (Hansen and Emslie 1988). Nonlinear waves propagating in such plasmas have received a great deal of attention in understanding the plasma wave structures. Studies of nonlinear waves have focused on wave structures, such as solitons, double layers, vortices and so on. We have also suggested that the high-speed streaming particles excite various kinds of nonlinear waves in space plasmas (Nejoh 1992, 1994*a*, 1994*b*; Nejoh and Sanuki 1994, 1995).

On the other hand, the effect of an electron beam on large-amplitude nonlinear waves has not yet been studied in electron–positron plasmas. Hence, in this paper, we investigate the region where large-amplitude positron-acoustic waves exist in the presence of an electron beam with finite temperature and hot electrons and positrons. We use a model where the dynamics of the nonlinear wave motion are governed by the hydrodynamic equations, while the temperature of the positron fluids is finite.

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The purpose of this paper is to derive the pseudopotential for positron-acoustic waves in an electron–positron plasma with an electron beam and to show the dependence of these waves on the positron density and temperature. Stationary nonlinear potential structures can be formulated in terms of an integral equation of the same form as that governing the motion of particles in a potential well. The conditions for the existence of nonlinear positron-acoustic waves will be determined by considering the ratio of the positron to electron density, the ratio of the positron to electron temperature and the normalised potential in an electron–positron plasma.

The structure of this paper is as follows. In Section 2 we present the basic hydrodynamic equations for an electron–positron plasma with an electron beam and derive an energy equation with the pseudopotential. In Section 3 we discuss the condition for large-amplitude positron-acoustic waves to exist on the basis of an energy equation. The dependence of the pseudopotential on the normalised potential, the positron to electron density ratio, the positron to electron temperature ratio and the electron beam temperature are presented. Section 4 gives a concluding discussion.

2. Basic Equations and Formulation

We consider a plasma consisting of electrons, positrons and beam electrons. In order to study one-dimensional propagation of large-amplitude positron-acoustic waves, we describe a set of the fluid equations. Nonlinear wave propagation of low phase velocity is governed by the hydrodynamic equations of the species. We assume that the phase velocity is much smaller than the electron and positron thermal velocities. The continuity equation and the equation of motion for electrons are given by

$$\frac{\partial}{\partial t} n_{\mathrm{e}} + \frac{\partial}{\partial x}(n_{\mathrm{e}} v_{\mathrm{e}}) = 0\,, \tag{1a}$$

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{e}} \frac{\partial}{\partial x}\right) v_{\mathrm{e}} - \frac{\partial \phi}{\partial x} = 0\,. \tag{1b}$$

For positrons we have the equations

$$\frac{\partial n_{\mathrm{p}}}{\partial t} + \frac{\partial}{\partial x} n_{\mathrm{p}} v_{\mathrm{p}} = 0\,, \tag{2a}$$

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{p}} \frac{\partial}{\partial x}\right) v_{\mathrm{p}} + \frac{3\beta}{(1+\alpha)^2} n_{\mathrm{p}} \frac{\partial n_{\mathrm{p}}}{\partial x} + \frac{\partial \phi}{\partial x} = 0\,. \tag{2b}$$

The continuity equation and the equation of motion for an electron beam are written as

$$\frac{\partial}{\partial t} n_{\mathrm{b}} + \frac{\partial}{\partial x}(n_{\mathrm{b}} v_{\mathrm{b}}) = 0\,, \tag{3a}$$

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{b}} \frac{\partial}{\partial x}\right) v_{\mathrm{b}} + \nu \frac{1}{n_{\mathrm{b}}} \frac{\partial n_{\mathrm{b}}}{\partial x} - \frac{\partial \phi}{\partial x} = 0\,. \tag{3b}$$

Poisson's equation is given by

$$\frac{\partial^2\phi}{\partial x^2} = n_\mathrm{e} - n_\mathrm{p} + n_\mathrm{b}\,. \tag{4}$$

Here the parameters $\alpha = n_{\mathrm{b}0}/n_0$, $\beta = T_\mathrm{p}/T_\mathrm{e}$ and $\nu = T_\mathrm{b}/T_\mathrm{e}$ are the ratios between the unperturbed electron beam and background electron densities, between the positron and electron temperatures and between the electron beam and background electron temperatures. The variable n_b is the electron beam density. The densities are normalised by the unperturbed background electron density n_0. The subscripts e, p and b denote electrons, positrons and beam electrons respectively. The space coordinate x, time t, velocities and electrostatic potential ϕ are normalised by the electron Debye length $\lambda_\mathrm{D} = (\epsilon_0\,\kappa T_\mathrm{e}/n_0\,e^2)^{\frac{1}{2}}$, the ion plasma period $\omega_\mathrm{i}^{-1} = (\epsilon_0\,m_\mathrm{p}/n_0\,e^2)^{\frac{1}{2}}$, the positron sound velocity $C_\mathrm{s} = (\kappa T_\mathrm{e}/m_\mathrm{p})^{\frac{1}{2}}$, and $\kappa T_\mathrm{e}/e$, respectively, where m_p, ϵ_0 and e are the positron mass, the permittivity of vacuum and the electric charge respectively. In equilibrium, we have $n_{\mathrm{p}0} = n_0 + n_{\mathrm{b}0}$.

In the linear limit, equations (1)–(4) give rise to the dispersion relation for positron-acoustic waves in this system. We derive the dispersion relation as

$$\omega^2 = \frac{3\beta(2-\alpha)}{2}\,\frac{k^2}{1+\frac{3}{2}\beta k^2}\,, \tag{5}$$

where ω and k are the frequency and wave number.

In order to solve equations (1)–(4) we consider the physical quantities derived in the stationary state. We introduce a variable $\xi = x - Mt$ and assume a stationary state in the moving frame, where M denotes the speed of the nonlinear structure. Integrating (2a) and (2b), we obtain the positron number density as

$$n_\mathrm{p} = \frac{1+\alpha}{\sqrt{1 - \dfrac{2\phi}{M^2 - 3\beta}}}\,. \tag{6}$$

Integrating (3a) and (3b) we get the electron beam density

$$n_\mathrm{b} = \frac{\alpha}{\sqrt{1 - \dfrac{2\phi}{\nu - (v_0 - M)^2}}}\,. \tag{7}$$

where we have used the boundary conditions $n_\mathrm{e}\to 1$, $n_\mathrm{p}\to 1+\alpha$, $n_\mathrm{b}\to\alpha$, $v_\mathrm{e}, v_\mathrm{p}\to 0$, $v_\mathrm{b}\to v_0$ and $\phi\to 0$ at $\xi\to\pm\infty$. From (6) and (7), equation (4) reduces to the nonlinear Poisson equation

$$\frac{\partial^2\phi}{\partial x^2} = \frac{1}{\sqrt{1+\dfrac{2\phi}{M^2}}} - \frac{1+\alpha}{\sqrt{1-\dfrac{2\phi}{M^2-3\beta}}} + \frac{\alpha}{\sqrt{1-\dfrac{2\phi}{\nu-(v_0-M)^2}}}$$

$$\equiv -\frac{\partial V(\phi)}{\partial\phi}, \tag{8}$$

where $V(\phi)$ is the pseudopotential.

From (8) we obtain the energy integral

$$\tfrac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)^2 + V(\phi) = 0. \tag{9}$$

The pseudopotential reads

$$-V(\phi) = -M^2\left(1-\sqrt{1+\frac{2\phi}{M^2}}\right) - (M^2-3\beta)(1+\alpha)\left(1-\sqrt{1-\frac{2\phi}{M^2-3\beta}}\right)$$

$$+\alpha\big(\nu-(v_0-M)^2\big)\left(1-\sqrt{1-\frac{2\phi}{\nu-(v_0-M)^2}}\right). \tag{10}$$

It should be noted from (10) that $0<\phi<[\nu-(v_0-M)^2]/2$.

For the positron-acoustic wave to exist the following two conditions must be satisfied:

(i) The pseudopotential must have a local maximum at the point $\phi=0$, and the equation $V(\phi)=0$ should have at least one real solution. This condition leads to the inequality

$$\frac{1}{M^2} + \frac{1+\alpha}{M^2-3\beta} - \frac{\alpha}{\nu-(v_0-M)^2} < 0. \tag{11}$$

We note that the condition (11) is a consequence of the inequality

$$\frac{\mathrm{d}^2V(\phi)}{\mathrm{d}\phi^2} < 0 \quad \text{at} \quad \phi = 0. \tag{12}$$

(ii) Nonlinear acoustic waves exist only when $V(\phi_{\mathrm{M}}) \geq 0$, where the maximum potential ϕ_{M} is determined by $\phi_{\mathrm{M}} = [\nu-(v_0-M)^2]/2$, because $[\nu-(v_0-M)^2]/2 > (M^2-3\beta)/2$ for proper values. The equation $V(\phi)=0$ can have only one real nonzero solution, the solution being positive. This condition can be described as

$$M^2\left(1-\sqrt{1+\frac{\nu-(v_0-M)^2}{M^2}}\right) + (M^2-3\beta)(1+\alpha)\left(1-\sqrt{\frac{\nu-(v_0-M)^2}{M^2-3\beta}}\right)$$

$$-\alpha[\nu-(v_0-M)^2] \geq 0. \tag{13}$$

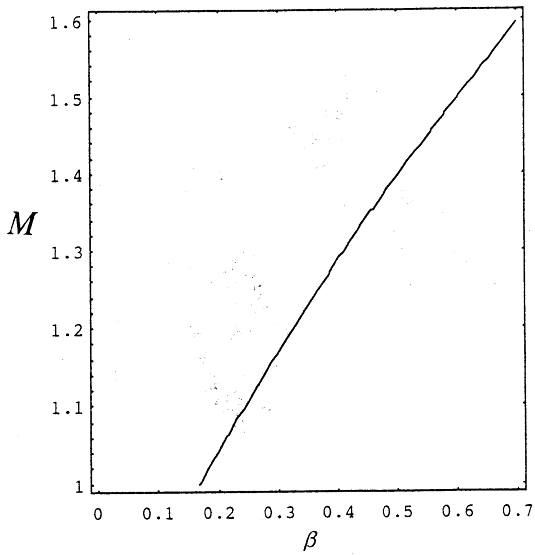

Fig. 1. Dependence of the Mach number M on the positron temperature β for $v_0 = 1 \cdot 4$, $\alpha = 0 \cdot 2$ and $\nu = 0 \cdot 2$.

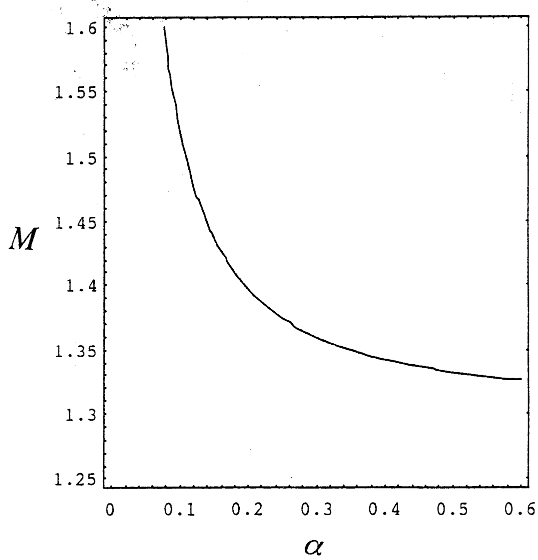

Fig. 2. Dependence of the maximum Mach number M on the electron beam density α for $\beta = 0 \cdot 5$, $v_0 = 1 \cdot 4$, and $\nu = 0 \cdot 2$.

Fig. 1 illustrates the dependence of the maximum Mach number on the positron temperature β, for which the large-amplitude positron-acoustic waves can exist, where $\alpha = 0 \cdot 2$. It is shown that supersonic positron-acoustic waves can propagate under the condition $\beta < M^2/3$. We show the dependence of the Mach number on the beam density in Fig. 2. It turns out that the electron beam density reduces the Mach number. The maximum Mach number and, correspondingly, the maximum amplitude of the positron-acoustic wave depend significantly on

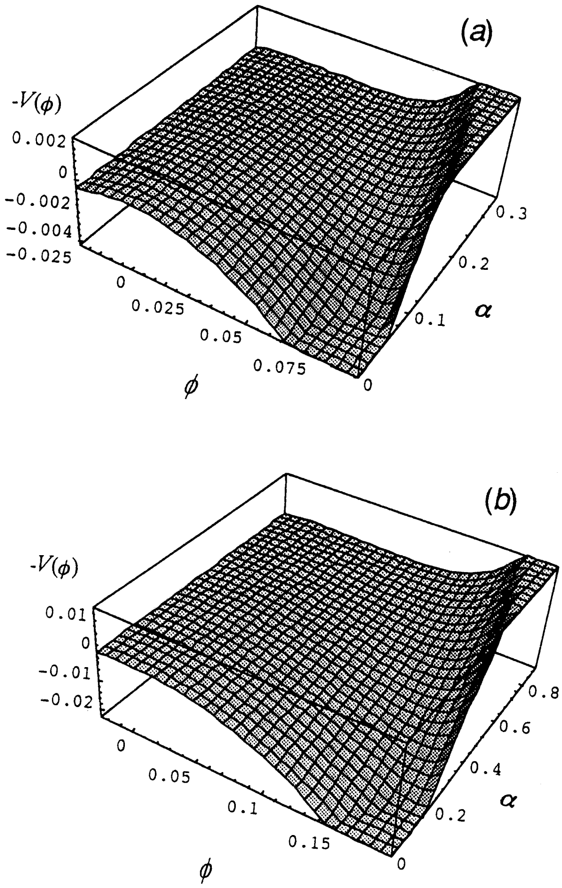

Fig. 3. Bird's eye view of the pseudopotential for the large-amplitude ion-acoustic waves for (a) the case of $M = 1{\cdot}1$, $v_0 = 1{\cdot}4$ and $\beta = 5$ and (b) the case of $M = 1{\cdot}4$, $v_0 = 1{\cdot}4$ and $\beta = 20$.

the parameters α and β. The region of existence is characterised by these conditions.

A complete analytical investigation of the positron-acoustic solitons in the system is possible in the small-amplitude wave limit ($\phi \ll 1$). The specific results can be obtained by expanding $V(\phi)$ in powers of ϕ and keeping up to the third-order terms ϕ^3. Accordingly, equation (10) takes the form

$$V(\phi) \approx \tfrac{1}{2}\left(\frac{1}{M^2} + \frac{1+\alpha}{M^2 - 3\beta} - \frac{\alpha}{\nu - (v_0 - M)^2}\right)\phi^2$$
$$- \tfrac{1}{2}\left(\frac{1}{M^4} + \frac{1+\alpha}{(M^2 - 3\beta)^2} - \frac{\alpha}{[\nu - (v_0 - M)^2]^2}\right)\phi^3 . \qquad (14)$$

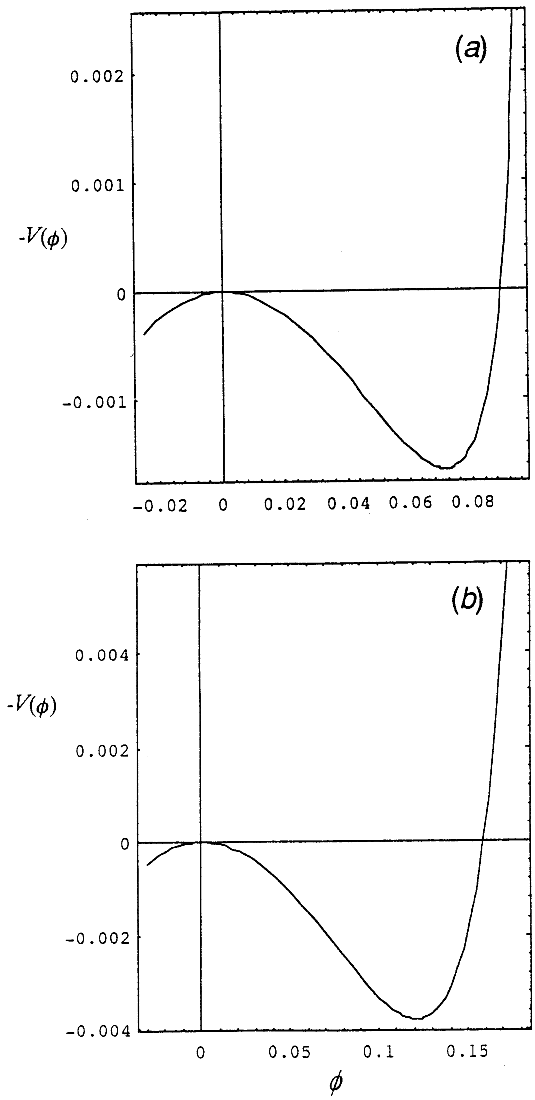

Fig. 4. Pseudopotential $V(\phi)$ against the electrostatic potential ϕ for (*a*) $M = 1{\cdot}1$, $v_0 = 1{\cdot}4$, $\beta = 5$ and $\alpha = 0{\cdot}95$, and (*b*) $M = 1{\cdot}4$, $v_0 = 1{\cdot}4$, $\beta = 20$ and $\alpha = 0{\cdot}97$.

Then, integrating (9) with (14), we obtain a soliton solution

$$\phi = \frac{\dfrac{1}{M^2} + \dfrac{1+\alpha}{M^2 - 3\beta} - \dfrac{\alpha}{\nu - (v_0 - M)^2}}{\dfrac{1}{M^4} + \dfrac{1+\alpha}{(M^2 - 3\beta)^2} + \dfrac{\alpha}{[\nu - (v_0 - M)^2]^2}}$$

$$\times\, \mathrm{sech}^2\left(\tfrac{1}{2}\sqrt{\frac{1}{M^2} + \frac{1+\alpha}{M^2 - 3\beta} - \frac{\alpha}{\nu - (v_0 - M)^2}}(\xi - \xi_0)\right).$$

It should be noted that the positron-acoustic soliton exists in the limiting case with $\phi \ll 1$.

We study the nonlinear potential structures for positron-acoustic waves on the basis of equations (9)–(14) in the following section.

3. Pseudopotential Structure and the Region of Large-amplitude Positron-acoustic Waves

We consider the nonlinear wave structures of large-amplitude positron-acoustic waves for the case where the positron temperature and electron beam temperature and density are important. We show a bird's eye view of the pseudopotential $V(\phi)$ in Fig. 3*a* for the case $\nu = 0\cdot 2$, $M = 1\cdot 5$, $v_0 = 1\cdot 4$ and $\beta = 0\cdot 5$. Fig. 4*a* illustrates the dependence of $V(\phi)$ on the potential ϕ when $M = 1\cdot 5$, $\beta = 0\cdot 5$ and $\alpha = 0\cdot 16$. When the beam temperature increases, we show a bird's eye view of the pseudopotential in Fig. 3*b* for $\nu = 0\cdot 4$, $M = 1\cdot 5$, $v_0 = 1\cdot 4$ and $\beta = 0\cdot 5$. The pseudopotential $V(\phi)$ versus ϕ in this case is also illustrated in Fig. 4*b* for $M = 1\cdot 5$, $\beta = 0\cdot 5$ and $\alpha = 0\cdot 65$.

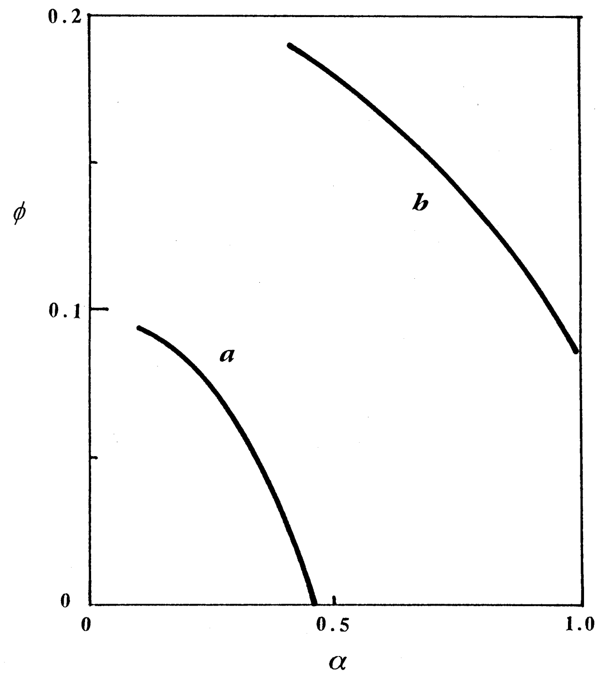

Fig. 5. The ϕ–α plane containing the ion-acoustic wave in the case of (*a*) $M = 1\cdot 1$, $v_0 = 1\cdot 4$ and $\beta = 5$, and (*b*) $M = 1\cdot 4$, $v_0 = 1\cdot 4$ and $\beta = 20$. The region where the large-amplitude ion-acoustic waves exists lies in the lower region bounded by the two curves.

In Fig. 5 we illustrate the region of existence of the positron-acoustic waves as a function of the ratio α of beam electron density to background electron density, in the case of $\nu = 0\cdot 2$, $M = 1\cdot 5$, $v_0 = 1\cdot 4$ and $\beta = 0\cdot 5$ (Fig. 5*a*).

Positron-acoustic waves propagate in the lower region bounded by the curve and do not exist in other regions. We show the region of existence in the ϕ–α plane in Fig. 5*b* for $\nu = 0{\cdot}4$, $M = 1{\cdot}5$ and $\beta = 0{\cdot}5$. Positron-acoustic waves exist in the lower region bounded by the curve.

It turns out that large-amplitude ion-acoustic waves can propagate under the two conditions mentioned above.

4. Concluding Discussion

The nonlinear wave structures of large-amplitude positron-acoustic waves have been studied in an electron–positron plasma with an electron beam. We have presented the region where supersonic positron-acoustic waves exist on the basis of the fluid equations by analysing the structures of the pseudopotential. Typical results are shown in Figs 1–5. The results are summarised as follows:

(1) Supersonic positron-acoustic waves can propagate in an electron–positron plasma according to an analysis of the maximum Mach number. The presence of positrons increases the Mach number and the effect of the electron beam density reduces the Mach number.

(2) Conditions for the existence of positron-acoustic waves depend sensitively on the positron density and temperature, the electrostatic potential and the electron beam density and temperature.

(3) The region for existence of large-amplitude positron-acoustic waves spreads as the electron beam temperature increases.

The present investigation predicts new findings on large-amplitude positron-acoustic waves in an electron–positron plasma with an electron beam. The positron-acoustic wave events associated with an electron beam have been frequently observed in the solar atmosphere. Hence, referring to the present study, we can understand the properties of these positron-acoustic waves where positrons and an electron beam exist in space. Although we have not referred to any specific observations, the present theory is applicable in analysing large-amplitude positron-acoustic shock and solitary waves associated with positrons and electron beams which may occur in interplanetary space. The large-amplitude positron-acoustic waves presented here are a new nonlinear wave mode and they will be detected by observational techniques in the future.

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