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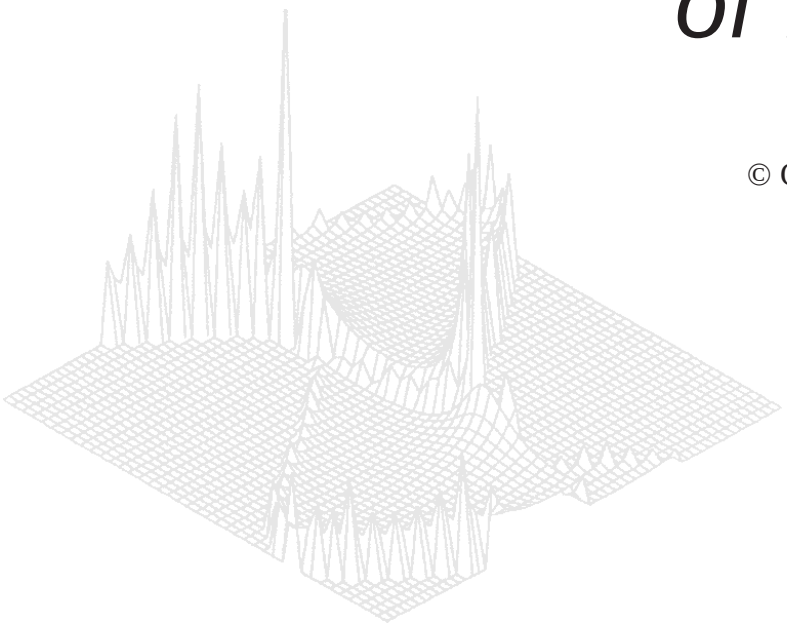

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Electromagnetic Absorption in Two-dimensional Systems under a Magnetic Field and a Unidirectional Periodic Potential*

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Abstract

The absorption of electromagnetic waves by a high mobility two-dimensional electron gas subjected to a magnetic field and a weak periodic potential is investigated. We show that the periodic modulation on the Laudau states has a profound effect on the absorption of electromagnetic waves. We develop a formalism which treats the electron–electron interaction beyond the random-phase-approximation (RPA) and includes the electron-impurity scattering in the lowest order. A RPA dielectric function was employed to study the electromagnetic absorption in modulated systems. Simultaneous excitation of an electron–hole pair with finite momentum contributes significantly to the absorption around and below the cyclotron frequency. Such a process is absent for a uniform electron gas under a magnetic field.

1. Introduction

The absorption of long-wavelength electromagnetic (EM) radiation by plasmas is proportional to $\sum_q F_q \Im m[1/\epsilon_{RPA}(q,\omega)]$ to lowest order in the plasma parameter r_s, where $\Im m$ stands for the imaginary part (Ron and Tzoar 1963; Dubois *et al.* 1962; Perel' and Eliashberg 1960). Here $\epsilon_{RPA}(q,\omega)$ is the random phase approximation (RPA) dielectric response function of the electrons, where ω is the angular frequency of the radiation field and q is the momentum transfer between the electrons and the impurities or low frequency acoustic phonons. The factor F_q is proportional to the square of the electron-impurity (or phonon) interaction, and the plasma parameter is $r_s = (k_F a_B^*)^{-1}$, where k_F is the Fermi wave vector, $a_B^* = \kappa\hbar^2/m^*e^2$ is the Bohr radius, κ is the dielectric constant and m^* is the effective mass.

It was recognised by Hopfield (1965) that, for example for simple metals, the effective electron-impurity interaction is weak and can be treated as a perturbation. However, electron collisions could be important and it may be necessary to consider them beyond the lowest order in r_s. Hopfield then generalised the RPA result and showed that for weak electron-impurity interaction the absorption is proportional to $\sum_q F_q \Im m[1/\epsilon(q,\omega)]$, where F_q is the same form factor as before. However, $\epsilon(q,\omega)$ is the exact dielectric response function of the electron gas.

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A problem has risen in treating the absorption of a two-dimensional electron gas (2DEG) in a strong homogeneous magnetic field (Ando 1976, 1978; Grimes 1978; Ting *et al.* 1977; Gotze and Wolfle 1972; Fukuyama *et al.* 1979). Here the RPA result for the dielectric function gives an absorption proportional to $\Im m[1/\epsilon_{RPA}(q,\omega)]$ which indicates only the existence of absorption lines at the cyclotron frequency and its harmonics. The singular behaviour of $\Im m[\epsilon_{RPA}(q,\omega)]$ makes absorption calculations for $\omega < \omega_c$ unrealistic and thus within the RPA the particle–hole pair excitation cannot be studied. The RPA calculation of absorption due to plasmon excitation is also unrealistic as it is singular at frequencies where the plasma energy is dispersionless ($d\omega_p/dq = 0$). To set up a tractable method to remove these unrealistic features is still an outstanding problem.

A two-dimensional electronic system subjected to a perpendicular magnetic field and a periodic modulation potential (Weiss *et al.* 1989*a*, 1989*b*, 1993; Winkler *et al.* 1989; Gerhardts *et al.* 1989; Vasilopoulos and Peeters 1989; Zhang 1990; Gerhardts and Zhang 1990; Zhang and Gerhardts 1990; Pfannkuche and Gerhardts 1992; Manolescu and Gerhardts 1995) presents one of the most interesting and challenging problems in physics, mathematics and computer simulation techniques (Hofstadter 1976; Sinai 1970). Since the discovery of Weiss oscillations (Weiss *et al.* 1989*a*) in a weakly modulated system, research in this field has been rapidly expanded. Recent research in the field includes the experimental realisation of the internal structures of Landau bands (Schlosser *et al.* 1996*a*, 1996*b*), quantum chaotic dynamics (Rotter *et al.* 1996). However, to date, the dynamical properties of this subtle system are still far from clear. In a recent work we performed a calculation within the RPA of the dynamical density response function (Stewart and Zhang 1995; Brataas *et al.* 1997; Cui *et al.* 1989). The effect of the periodic potential on the static properties can be summarised as: (i) the sharp Landau levels are broadened in a manner where the width and height of the density of states (DOS) are oscillatory in the magnetic field and level index; (ii) the DOS exhibits inverse-square-root singularities at the band edges; (iii) there is an additional conductivity due to band conduction; and (iv) the magnetoresistivity exhibits commensurability oscillations (or Weiss oscillations). It was revealed (Stewart and Zhang 1995; Brataas *et al.* 1997) that the effect of the periodic potential on the dynamical properties of the system is much more complicated than that on the static properties: (i) it introduces an additional channel in the density response which is an electron–hole pair excitation of finite momentum q_y; (ii) the pair excitation exhibits multiple singularities. The number of singularities depends on the excitation frequencies only; and (iii) the light scattering cross section has an additional peak at the cyclotron frequency and its high harmonics.

In this paper, we demonstrate that an additional weak periodic potential applied to a uniform 2DEG under a magnetic field can have a rather profound effect on the coupling between the electrons and the EM radiation field. As is known, the primary effect of a weak periodic potential is to lift the degeneracy of the original Landau levels. The energy becomes dependent on the centre coordinate of the cyclotron orbit, $x_0 = q_y l^2$, where $l^2 = c\hbar/eB$ is the magnetic length. Though the density of states (DOS) still contains the inverse-square-root singularity, it is now fully integrable and thus makes a finite contribution to the EM absorption. We have calculated the EM absorption for such a periodically modulated system

and the main results are: (i) The x_0 dependent energy dispersion has introduced the electron–hole pair excitation of finite momentum q_y. This pair excitation is the dominant absorption mechanism for $\omega < \omega_c$, where $\omega_c = eB/cm^*$ is the cyclotron frequency. (ii) The absorption due to plasmon excitation is no longer singular because the spectral weight is now distributed continuously over a finite interval of q_y.

2. General Formalism of Electromagnetic Absorption

Let us consider a 2DEG in the x–y plane with a perpendicular magnetic field, $\mathbf{B} = B\mathbf{z}$, where $\mathbf{z}$ is a unit vector in the z-direction. The 2DEG is further subjected to a periodic modulation in the x-direction with a period a. The Hamiltonian is

$$H = H_1 + H_{e-I} + H_\gamma + H_{modu}, \tag{1}$$

where

$$H_1 = \sum_i \frac{(-\mathrm{i}\nabla_i - e\mathbf{A}_i)^2}{m_i} + \tfrac{1}{2}\sum_{i,j} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|}, \tag{2}$$

$$H_{e-I} = \sum_{\mathbf{q}} U_q \sum_{i,j} e^{\mathrm{i}\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{R}_j)}, \tag{3}$$

and

$$H_{modu} = \sum_i V_0 \cos(Kx_i), \tag{4}$$

where $\mathbf{r}_i = (x_i, y_i)$ is the electron position, $\mathbf{p}_i = (-\mathrm{i}\hbar\nabla_i - e\mathbf{A}_i/c)$ is the electron canonical momentum, $\mathbf{A}_i$ is the vector potential of the static magnetic field, $\mathbf{R}_j$ is the impurity coordinate and V_0 is the strength of the modulation with $K = 2\pi/a$. In a heterostructure the doping impurities are at some distance from the 2DEG. We use the electron-impurity interaction $U_q = 2\pi e^2 \exp(-q\alpha)/\kappa q$, where α is the typical distance of the impurities from the 2DEG. The mass of the impurity is treated as infinite. The interaction of an EM wave with the electrons is $H_\gamma = -e\mathbf{E}_\gamma \cdot \sum_i \mathbf{p}_i/m^* \imath\omega$, where $\mathbf{E}_\gamma$ is the transverse electric field and ω is the frequency of the EM wave. We will limit our discussion to low temperatures and set $T = 0$. All results may easily be generalised to finite temperatures.

The interaction of an EM wave with the electrons is

$$H_\gamma = \frac{e}{m}\mathbf{P}\cdot\mathbf{A}_\gamma\,, \tag{5}$$

where $\mathbf{P}$ is the total canonical momentum of the system and $\mathbf{A}_\gamma$ is the vector potential of the radiation field. The energy loss rate of the EM field can be written as

$$R_q = 2\pi \sum_{F,I} \rho_I |\langle F|H_\gamma|I\rangle|^2 \delta(E_F - E_I - \omega)\,, \tag{6}$$

where F and I represent the final and initial state, and ρ_I is the statistical weight for the initial state,

$$\rho_I = \exp[\beta(\Omega + \mu N_I - E_I)]\,, \tag{7}$$

where $\beta = 1/kT$, μ is the chemical potential and

$$e^{-\beta\Omega} = \sum_I e^{-\beta(E_I - \mu N_I)}\,. \tag{8}$$

In the rest of this paper we develop a method to calculate the matrix element in equation (6), $\langle F|H_\gamma|I\rangle$. First we would like to express the momentum in coordinates appropriate to circular polarisation

$$\mathbf{P} = e_+ P_+ + e_- P_- + e_z P_z\,, \tag{9}$$

where $e_\pm = \sqrt{\frac{1}{2}}(e_x \pm e_y)$ and $P_\pm = \sqrt{\frac{1}{2}}(P_x \pm P_y)$. Therefore we have the following commutation rule:

$$[P_+, P_+] = [P_-, P_-] = 0\,, \tag{10}$$

$$[P_+, P_-] = m\omega_c\,. \tag{11}$$

Since we choose the magnetic field to be along the z-direction and the radiation field is in the x–y plane, the energy loss rate can be written as

$$R_q = \frac{2\pi^2}{m^2} \sum_{F,I} \rho_I |\langle F|P_+ A_\gamma^- + P_- A_\gamma^+ |I\rangle|^2 \delta(E_F - E_I - \omega)\,. \tag{12}$$

The matrix element $\langle F|P_\pm|I\rangle$ can be obtained as the following:

$$\begin{aligned}\langle F|P_\pm|I\rangle &= \frac{\langle F|[H, P_\pm]|I\rangle}{E_F - E_I} \\ &= \frac{1}{\hbar\omega}\langle F|[H, P_\pm]|I\rangle\,,\end{aligned} \tag{13}$$

where we use the eigen-equation for the final and initial state:

$$H|F\rangle = E_F|F\rangle\,, \tag{14}$$

$$H|I\rangle = E_I|I\rangle\,, \tag{15}$$

and the energy denominator can be replaced by ω because of the δ-function in equation (6). The commutator of $P_\pm$ with the electron–electron interaction is zero and the other three commutators are given as

$$[P_\pm, P^2] = \pm 2m\omega_c P_\pm \,, \tag{16}$$

$$[H_{e-I}, P_\pm] = q_\pm \sum_{i,j} U_q e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{R}_j)} \,, \tag{17}$$

and

$$[H_{modu}, P_\pm] = \frac{K_\pm}{\sqrt{2}} V_0 \sum_i \sin(K_\pm x_i) \,. \tag{18}$$

Using equations (16)–(18), we have the self-consistent equation for the matrix element $\langle F|H_\gamma|I\rangle$ and the solution is

$$\begin{aligned}\langle F|P_\pm|I\rangle = {} & \frac{q_\pm}{\omega \pm \omega_c}\langle F|V_q \sum_{i,j} e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{R}_j)}|I\rangle \\ & + \frac{iV_0}{\sqrt{2}}\frac{K_\pm}{\omega \pm \omega_c}\langle F|\sum_i \sin(K_\pm x_i)|I\rangle \,. \end{aligned} \tag{19}$$

Here $K_\pm = (K_x \pm \mathrm{i}K_y)/\sqrt{2} = K/\sqrt{2}$. We use equation (19) in equation (12) and then the final result for the energy loss rate can be written as

$$\begin{aligned} R = {} & \frac{2\pi}{\hbar}\frac{e^2}{m^2}\frac{1}{\omega^2}\sum_F \delta(E_F - E_0 - \hbar\omega) \\ & \times \left| \sum_{\mathbf{q}} \left(\frac{q_- E_+}{\omega + \omega_c} + \frac{q_+ E_-}{\omega - \omega_c} \right) U_q \langle F| \sum_{i,j} e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{R}_j)}|0\rangle \right. \\ & \left. + \frac{\mathrm{i}KV_0}{\sqrt{2}} \left(\frac{E_+}{\omega + \omega_c} + \frac{E_-}{\omega - \omega_c} \right) \langle F| \sum_i \sin(Kx_i)|0\rangle \right|^2 , \end{aligned} \tag{20}$$

where $|0\rangle$ is the ground state of the many-electron system, $q_\pm = (q_x \pm \mathrm{i}q_y)/\sqrt{2}$, and $E_\pm = (E_x \pm \mathrm{i}E_y)/\sqrt{2}$. This expression is exact to second order in the impurity potential U_q and the modulation potential V_0 including all electron–electron interactions, and is valid for any strength of the magnetic field. The electron scattering matrix contains two terms. The first term, proportional to $n_I q U_q$, is due to electron scattering off the impurity potential and the second term, proportional to KV_0, is due to the electron scattering off the modulation potential. Here n_I is the density of impurities. In typical experimental situations, $V_0 \sim 0{\cdot}5 - 2$ meV, $K \sim 2 - 4 \times 10^5$ cm^{-1} and $n_I = 2 - 5 \times 10^{11}$ cm^2. Therefore, the ratio is $n_I q U_q / KV_0 \gg 1$ and we shall neglect the electron scattering from the weak periodic potential (for typical α values of around 50–100 Å). Furthermore, since we are seeking a result which is in the lowest order of the electron-impurity interaction, i.e. $|U_q|^2$, the many-body states $|F\rangle$ and $|I\rangle$ can be regarded as only having electron coordinates. In this case $\sum_j e^{-i\mathbf{q}\cdot\mathbf{R}_j}$ can be taken out of the matrix element and an average taken over the random impurity ensemble $\sum_{j,j'} e^{-i\mathbf{q}\cdot\mathbf{R}_j} e^{i\mathbf{q}'\cdot\mathbf{R}_{j'}}/A = n_I \delta_{\mathbf{q},\mathbf{q}'}$, where A is the area of the two-dimensional

system. On the other hand, if the impuritites are far away from the 2DEG in high-mobility samples and the modulation potential is sufficently strong, the second term in equation (19) is dominant. In this case we see that the long wavelength conductivity is given by the charge-density excitations along the x-axis at wave vector K due to the scattering with the periodic modulation. The long wavelength conductivity is then directly proportional to $\mathrm{Im}[1/\epsilon(K,\omega)]$ with a prefactor which gives a resonance at the cyclotron frequency. The charge-density excitations for this system have been studied (Stewart and Zhang 1995) and we will therefore concentrate on the first case here.

3. Dynamical Resistivity

From the definition of the dielectric function in terms of the system response to the external potential, the expression for the imaginary part of the reciprocal of the longitudinal dielectric function is

$$\Im m\left[\frac{1}{\epsilon(q,\omega)}\right] = \frac{4\pi^2 e^2}{q^2}(1-e^{-\beta\omega})\sum_{F,I} e^{\beta(\Omega+\mu N_I - E_I)} \times |\langle F|\sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i}|I\rangle|^2 \delta(E_F - E_I - \omega)\,. \tag{21}$$

We can relate the energy loss rate to the conductivity by $\hbar\omega R(\omega) = 2A\Re e\big[\mathbf{j}(\omega)\cdot\mathbf{E}^*(\omega)\big]$. The current can be written as $\mathbf{j} = \sigma\cdot\mathbf{E}$, where σ is the conductivity, and we thus obtain the real part of the diagonal element of the conductivity tensor:

$$\sigma^R_{xx} = \eta\sum_{\mathbf{q}}(q_x^2\omega^2 + q_y^2\omega_c^2)\frac{|U_q|^2}{V_q}\left[-\Im m\frac{1}{\epsilon(\mathbf{q},\omega)}\right], \tag{22}$$

$$\sigma^R_{yy} = \eta\sum_{\mathbf{q}}(q_x^2\omega_c^2 + q_y^2\omega^2)\frac{|U_q|^2}{V_q}\left[-\Im m\frac{1}{\epsilon(\mathbf{q},\omega)}\right], \tag{23}$$

where $V_q = 2\pi e^2/\kappa q$. We have set the temperature to zero. The prefactor η is

$$\eta = n_I\left(\frac{e}{m^*}\right)^2\frac{1}{\omega(\omega^2-\omega_c^2)^2}\,. \tag{24}$$

It is possible to generalise the above results to include the imaginary parts of the conductivity and to obtain results for the relaxation time by comparing the results with the standard Drude conductivity. Due to the unidirectional modulation potential the conductivity is now in general anisotropic. However, we found that the anisotropy in the conductivity is not significantly close to the cyclotron frequency, $\omega\sim\omega_c$, even if the dielectric function is strongly anisotropic. The anisotropy is increasing for frequencies away from the cyclotron frequency.

For absorption in the system we now need a model for the dielectric function in a 2DEG with a perpendicular magnetic field and a periodic modulation potential. We use the results obtained within the RPA. In order to simplify the discussion we now consider the case of integer filling of the Landau levels. The dielectric

function in the case of a partially filled Landau band has been discussed in (Stewart and Zhang 1995; Brataas *et al.* 1997). The single-particle energy, to first order in the modulation potential, is given as

$$E_n(x_0) = \hbar\omega_c\big(n + \tfrac{1}{2}\big) + U_n \cos Kx_0\,, \tag{25}$$

where $U_n = V_0 L_n(\mathcal{H}) \exp(-\mathcal{H}/2)$, $\mathcal{H} = (Kl)^2/2$, and $L_n(\mathcal{H})$ is a Laguerre polynomial. This is a good approximation if the cyclotron resonance energy and the Fermi energy are not too small compared with the modulation potential. The imaginary part of the dielectric function is (Stewart and Zhang 1995; Brataas *et al.* 1997)

$$\begin{aligned}\Im m[\epsilon(\mathbf{q},\omega)] = \frac{2\hbar\omega_c}{qa_B^*} \sum_{m'=0}^{n_F} \sum_{m=x}^{\infty} C_{m+m',m'} \\ \times \big[\theta\big(\Delta U_{mm'}^2 - (\Delta E_m^-)^2\big)/\sqrt{\Delta U_{mm'}^2 - (\Delta E_m^-)^2} \\ - \theta\big(\Delta U_{mm'}^2 - (\Delta E_m^+)^2\big)/\sqrt{\Delta U_{mm'}^2 - (\Delta E_m^+)^2}\big]\,,\end{aligned} \tag{26}$$

where $n_F + 1$ is the number of occupied Landau bands and $x = n_F + 1 - m'$ is the lower limit of the second summation. Further, $x_0' = q\sin(\vartheta)l^2$, where ϑ is the angle between the wave vector $\mathbf{q}$ and the x-axis and the transition matrix is $(n' < n)$

$$C_{n,n'} = \frac{n'!}{n!} X^{n-n'} e^{-X} [L_{n'}^{n-n'}(X)]^2\,, \tag{27}$$

where $X = (ql)^2/2$ and $L_n^m(X)$ is an associated Laguerre polynomial. The real part of the dielectric function is (Stewart and Zhang 1995; Brataas *et al.* 1997)

$$\begin{aligned}\Re e[\epsilon(\mathbf{q},\omega)] = 1 + \frac{2\hbar\omega_c}{qa_B^*} \sum_{m'=0}^{n_F} \sum_{m=x}^{\infty} C_{m+m',m'} \\ \times \big[\theta\big((\Delta E_m^-)^2 - \Delta U_{mm'}^2\big)/\Delta E_m^- \sqrt{1 - (\Delta U_{mm'}/\Delta E_m^-)^2} \\ + \theta\big((\Delta E_m^+)^2 - \Delta U_{mm'}^2\big)/\Delta E_m^+ \sqrt{1 - (\Delta U_{mm'}/\Delta E_m^+)^2}\big]\,,\end{aligned} \tag{28}$$

where $\theta(x)$ is the Heaviside function. In (25) and (27) we have introduced

$$\Delta U_{mm'}^2 = U_{m+m'}^2 - -2U_{m+m'}U_{m'}\cos Kx_0' + U_{m'}^2 \tag{29}$$

and $\Delta E_m^{\pm} = m\hbar\omega_c \pm \hbar\omega$. For the subsingularities in $\Im m[\epsilon(q,\omega)]$ we see that for positive frequencies they occur at $\Delta E_m^- = \pm\Delta U_{mm'}$ if subband m' is occupied and subband $m + m'$ is empty. Now, since the conductivity in equations (21) and (22) in principle contains contributions from all (q,ϑ) due

to scattering from the impurities the subsingularities will occur in the interval $||U_{m+m'}| - |U_{m'}|| \leq |\Delta E_m^-| \leq ||U_{m+m'}| + |U_{m'}||$ and will be smoothed out. The calculated structure of both the real and imaginary parts of the dielectric function is shown in Fig. 1.

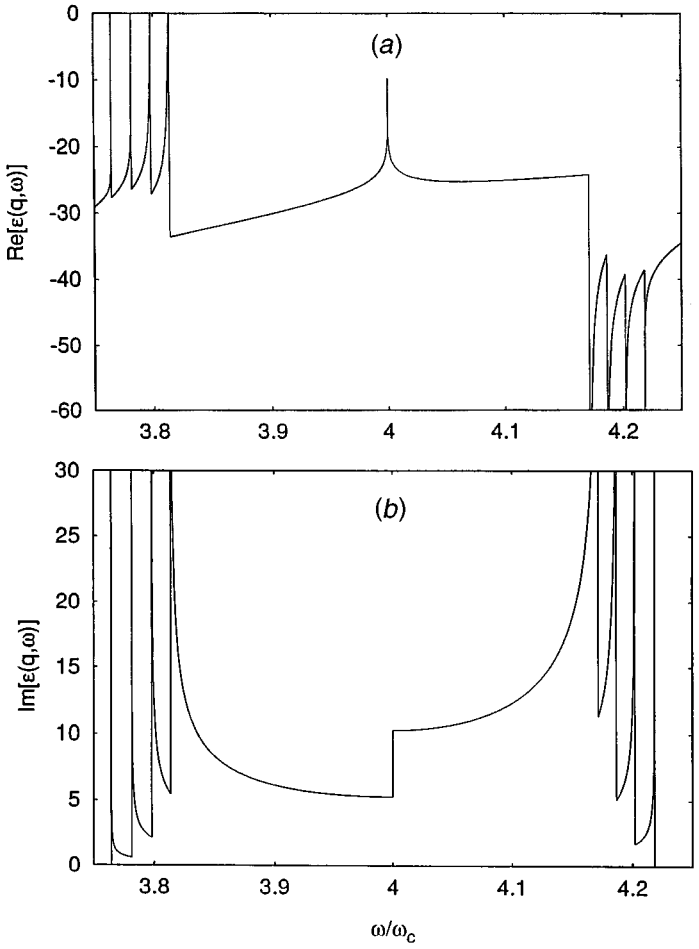

Fig. 1. (a) Real part of the dielectric function and (b) imaginary part of the dielectric function. Here $V_0 = 1{\cdot}0$ meV, $q_x = 0{\cdot}2k_F, q_y = 0$, $a = 300$ nm, $E_F = 10$ meV, and the filling factor is $5{\cdot}5$.

The EM absorption of the system consists of contributions from both single-particle excitations and plasmon excitations. We may write these two terms separately as

$$-\Im m[1/\epsilon(\mathbf{q},\omega)] = P\frac{\epsilon_I}{\epsilon_I^2 + \epsilon_R^2} + \pi\delta(\epsilon(\mathbf{q},\omega))\,. \tag{30}$$

From this equation we may immediately conclude that, in the absence of a periodic modulation potential, the EM response of a system with sharp Landau levels does not contain the contribution due to the particle–hole pair excitation.

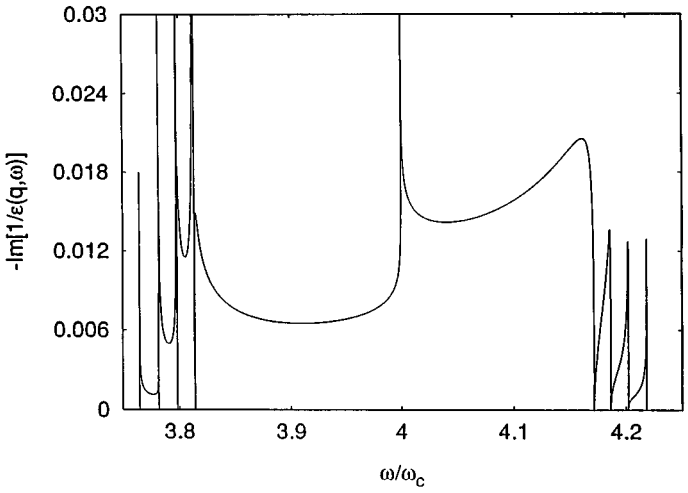

Fig. 2. Plot of $-\Im m[1/\epsilon(q_x,\omega)]$ as function of frequency. All parameters are the same as in Fig. 1.

The imaginary part of $1/\epsilon(\mathbf{q},\omega)$ for some typical parameters is shown in Fig. 2. The contribution to the EM response due to plasmon excitation in such a system usually has sharp peaks due to the fact that the spectral weight is concentrated for a give value of the photon wavenumber. The application of a modulation potential has broadened the originally sharp Landau level. The broadening is both B-dependent and level index-dependent. The centre-coordinate-dependent energy dispersion has two important effects on the EM response of the system: (i) The particle–hole pair excitation channel has now been opened and thus a finite absorption occurs for frequencies around ω_c and each of its harmonics ($n\omega_c$ where $n = 1, 2, 3, \ldots$). Therefore, in a modulated 2DEG, EM absorption is finite even for frequencies less than ω_c. (ii) The spectral weight in plasmon excitation is no longer only dependent on the magnitude of the photon wavenumber, but it is now also dependent on the direction of the photon wavenumber. The spread of the spectral weight along different directions (specified by the angular variable ϑ) removes the singular behaviour (or smears out the sharp peaks) in the aborption coefficient. The contribution from collective excitations to the conductivity component σ_{xx}^R is now given as

$$\sigma_{xx}^{Rc} = \tfrac{1}{2}\eta(\omega)\int_0^{2\pi}\frac{d\vartheta}{2\pi} \times \sum_{q^*}\left[q(q_x^2\omega^2 + q_y^2\omega_c^2)\frac{|U_q|^2}{V_q}\left|\frac{\partial\epsilon(q,\vartheta,\omega)}{\partial q}\right|^{-1}\right]_{q=q^*}. \tag{31}$$

The solutions q^* are now given by $\epsilon(q^*,\vartheta,\omega) = 0$. All singularities will thus be smoothed out in the angular integral. We illustate this behaviour in Fig. 3, where we show the dispersion $\omega(q)$ of the plasmon for two values of ϑ $(= 0, \pi/2)$. The system parameters used in the calculation are to be described below. The areas

below the dotted lines are the regimes of single-particle excitations where the collective excitations (plasmon) cannot exist. For $\vartheta = 0$ this regime is constant as a function of q ($q_y = 0$, $q_x = q$), but for $\vartheta = \pi/2$ ($q_y = q$, $q_x = 0$) this regime is increasing as a function of q. The angular dependence of the plasmon energy is larger for small frequency differences, $\omega(q) - \omega_c$, as should be expected from the expression for the real part of the dielectric function (26) since the angular dependent terms in the denominator are more important then. The highest peak in the dispersion has a rather weak angular dependence, since the dispersive part of energies in the real part of the dielectric function is less pronounced for larger energy differences, $\omega - \omega_c$. This means that the peak in the conductivity within the RPA around this frequency will remain sharp after the integration in (29), while the other peaks will be more smoothed out. However, this main peak can easily fall in the particle–hole region with a slight increase of V_0.

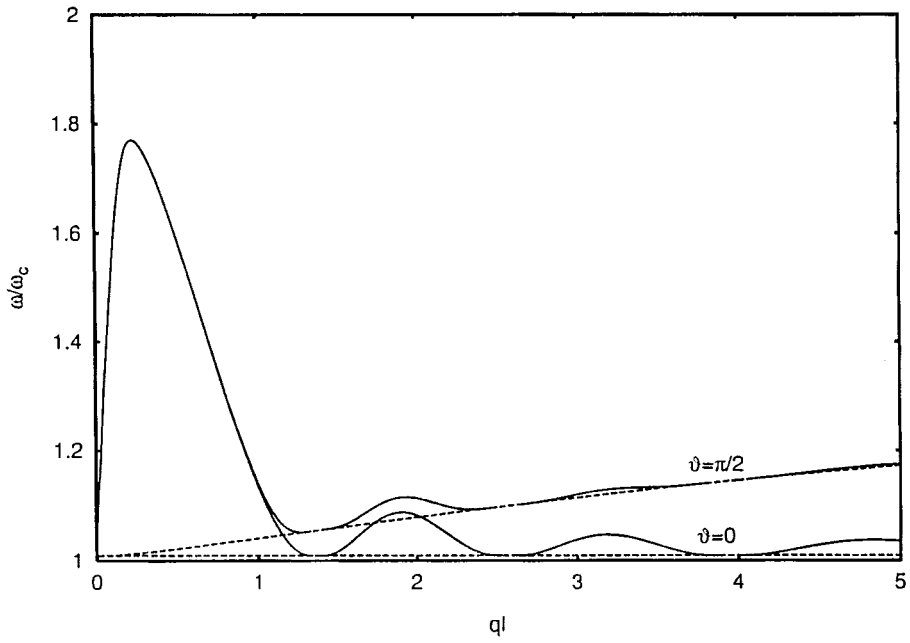

Fig. 3. Dispersion of the plasmon (solid lines) and upper energy for single particle excitations (dashed lines) for $\vartheta = 0$ and $\Pi/2$. The system parameters are described in the text.

4. Results and Discussion

We have used the following GaAs parameters in our numerical calculation: $m^* = 0{\cdot}067m$ and $\kappa = 13$ (where m is the electron mass). This gives an effective Bohr radius of $a_B^* = 102$ Å. The magnetic field is $B = 2$ T giving a Landau energy $\hbar\omega_c = 3{\cdot}5$ meV and the four lowest Landau levels are filled giving a 2D electron density of $3{\cdot}9 \times 10^{11}$ cm^{-2}. The modulation potential is $V_0 = 0{\cdot}5$ meV with a period of $a = 3000$ Å, and the typical distance from the impurities to the 2DEG is $\alpha = 100$ Å.

We consider the resistivity ρ_{yy} around ω_c. The region of single-particle excitations is determined by transitions between Landau levels 3 and 4, where

$U_3 = 0{\cdot}11\omega_c$ and $U_4 = 0{\cdot}10\omega_c$. In principle, the sum $|U_3 + U_4| = 0{\cdot}21\hbar\omega_c$ gives the maximum frequency around ω_c for single-particle excitations and $\hbar\omega_c + |U_3 - U_4| = 1{\cdot}01\hbar\omega_c$ gives the minimum frequency for which the plasmon may exist. However, the spectrum is effectively cut-off if q_y is larger than $1/\alpha$, where α is the typical distance from the impurities to the 2DEG. The maximum argument for Kx_0' is thus $Kx_0' \sim 2\pi l^2/a\alpha \approx 0{\cdot}71$ leading to a narrower band of single-particle excitations around ω_c. We show in Fig. 4 the resistivity ρ_{yy} as a function of frequency. The solid line displays the total resistivity including contributions from both single-particle and collective excitations. For frequencies less than the cyclotron frequency only single-particle excitations contribute (the dashed line which merges with the solid line slightly above ω_c). At frequencies slightly larger than ω_c, $\omega = 1{\cdot}01\omega_c$ (see above), the collective excitations (dotted line) will contribute to the conductiviy and their contribution is dominant at high frequencies. The resistivities ρ_{xx} (not shown) and ρ_{yy} are both very similar even though the anisotropy of the dielectric function is large. As already mentioned, this is due to the fact that we have integrated over the angular variable for both σ_{xx} and σ_{yy}.

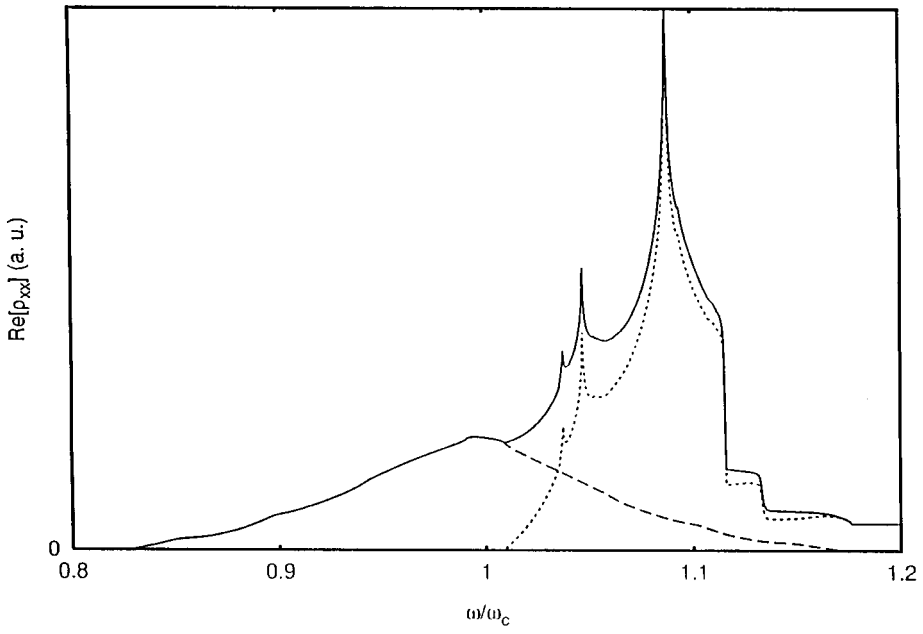

Fig. 4. Resistivity ρ_{yy} as a function of frequency (solid line). The dashed lines give the single-particle excitations (low energy part) and the collective contributions (high energy part). The system parameters are described in the text.

We would like to point out that in a realistic system the Landau levels are not infinitely sharp in the absence of periodic modulation. There are always unavoidable disorders which broaden the Landau levels. Therefore the particle–hole pair excitation can make a nonzero contribution to the EM absorption, though a theory beyond RPA is required to include such collision broadenings. However, the physical origin of this commonly studied collision broadening (without modulation

potential) and of the modulation broadening discussed here are completely different. The former is known as the lifetime effect and the latter is the dispersive effect. The particle–hole excitations due to collision broadening and to modulation broadening and their respective contribution to the EM absorption are qualitatively different: (i) the former is still isotropic for random disorders while the latter is anisotropic; (ii) the EM absorption due to the former usually has a Lorentzian-type spectral distribution which is relatively sharp, while that due to the latter is definitely non-Lorentzian. The angular redistribution of the spectral weight due to plasmon excitation is also a completely new mechanism compared with the collisional damping of plasmons in unmodulated systems. The central result of this work is the novel absorption mechanism due to modulation-induced level broadening. The RPA dielectric function is employed in our work, but the qualitative physical picture should remain unchanged if a higher order interaction is included in the dielectric function.

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