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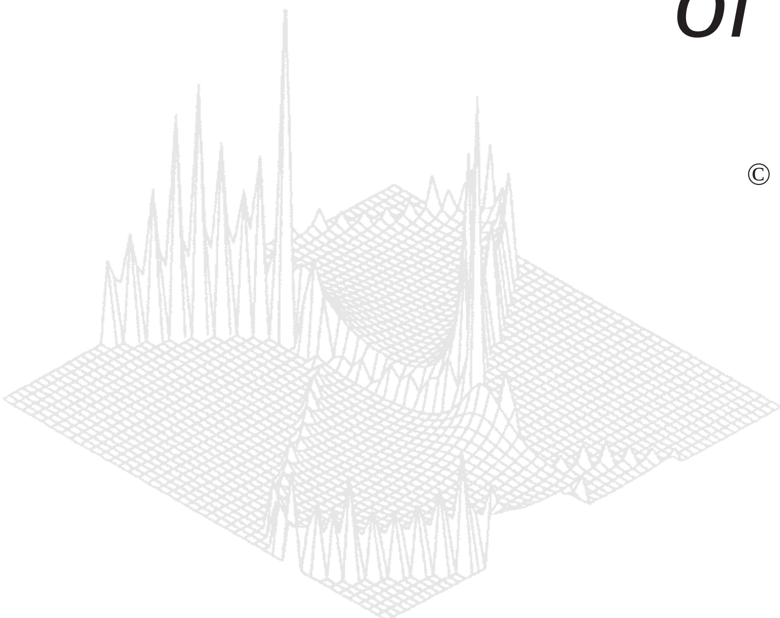

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A New Energy-dependent Δ Width in Coherent Pion Production

P. A. Deutchman

Permanent address: Department of Physics,
University of Idaho, Moscow, ID 83844–0903, USA.

School of Chemistry, Physics, and Earth Sciences,
Flinders University of South Australia, GPO Box 2100,
Adelaide, SA 5001, Australia.

Abstract

A new derivation for the energy-dependent Δ width inside nuclei is presented which includes the all important, energy-dependent nuclear form factor describing the decay of the Δ particle from its harmonic oscillator, bound state back to a captured nucleon and outgoing pion. Additional improvements include relativistic kinematics, generalisation beyond the static limit in the kinematics, and inclusion of the $\Delta N\pi$ energy-dependent vertex form factor. The new decay width provides a reasonable cut-off at high pion momenta and gives the correct momentum dependence in the limit of single-particle decay at low momenta. The results of calculations for the energy-dependent widths and their effects on the energy distributions of exclusive coherent pion production for ${}^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{C} + {}^{12}\mathrm{C}^*$ $(T = 1) + \pi^0$ at incident energies below and above the pion threshold are shown and discussed. The new energy dependence is compared to the free Δ width used previously and it is seen that the new results give a smoother and more realistic shape to the pion energy distributions.

1. Introduction

The study of the excitation and decay of Δ isobars inside nuclei at intermediate energies from various probes continues to provide an exciting challenge both theoretically and experimentally, especially in the search for constructive, coherent pion production. To find such an extreme quantum signature would be considered an exciting developement in the field of nucleus–nucleus collisions at intermediate energies. Questions of the spin–isospin response in nuclei and the relation to coherent pions are of current interest (Gaarde 1996), as well as examinations of Δ excitation and decay by nucleons (Ramachandran and Vidya 1997), light ions (Fernández *et al*. 1995), and heavy ions (Badalà *et al*. 1996). A general review of Δ isobar excitations can be found in Gaarde (1991). This paper is a continuation of a series of papers (Deutchman 1992; Maung *et al*. 1992; Deutchman and Li 1993; Deutchman and Erazmus 1995; Deutchman 1996; Deutchman and Sammarruca 1998) that deals with the constructive coherence in pion production from the collision of moderate equal-mass nuclei at energies above and below the pion threshold, where a nucleon in one nucleus is excited to a Δ isobar while the other nucleus is excited to a spin–isospin giant resonance. In particular, theoretical calculations of the pion energy distributions depend

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sensitively on the Δ-decay width. It was realised that the free Δ-decay width previously used was inadequate and unphysical for the pions resulting in the decay of the Δ inside nuclei in intermediate-energy, nucleus–nucleus collisions. A more realistic energy dependence of the nuclear Δ decay needed to be found. In this work, the focus will be on the decay of the excited Δ particle inside the nucleus and, in particular, the energy dependence of the Δ width is derived where the all important nuclear form factor is included in this model for the first time. Previous calculations have used the low energy free Δ-decay width obtained from Guet *et al.* (1989) which contains the p-wave penetration factor p_π^3 (Brown and Weise 1975), where p_π is the pion momentum; however, this width is of course independent of any nuclear effects. The need for a decay width that describes the decay of a Δ-isobar inside a nucleus has been discussed by Jain and Kundu (1996) who used an energy parametrisation that describes the decay of a Δ which is produced in proton–nucleus collisions at intermediate energies. This parametrisation is identical to that used by Gaarde (1992) in his discussion of Δ production and decay in light-ion reactions on ^{12}C. These parametrisations work phenomenologically, but the analytic forms do not explicitly exhibit nuclear properties in a transparent manner. In slightly different work, on the $\pi^+ d \to pp$ reaction going through a Δ excitation, Canton *et al.* (1996) developed their own phenomenological energy-dependent parametrisation of the isobar width having concluded that the energy dependence of the relativistically improved free isobar decay model (RIIM) (Oset *et al.* 1982) is oversimplified. Finally, a general discussion of nuclear effects on the Δ width can be found in the text by Ericson and Weise (1998), where the free Δ-decay width becomes modified by a change in the available phase space because of the presence of the nucleus; however, an explicit form for the energy dependence of the Δ width is not presented. It is the purpose of this work to embark on a program to explicitly derive the energy-dependence for the Δ width in the presence of a medium-sized nucleus where the Δ-hole states are produced in a quantum constructively coherent manner. The energy-dependent nuclear form factor will now be included in the nuclear Δ width for the first time in this overall calculation.

As a final comment, it is noted that the development of the description of pion coherent production due to nucleus–nucleus collisions by a microscopic, quantum theory is quite challenging. As a way to systematically proceed, a hierarchy of importance was considered. First, it is known that cross sections are quite sensitive to nuclear structure. Since the original motivation was to establish a link between pion production and the coherence coming from nuclear structure, it was considered prudent to first solve the nuclear structure problem for both nuclei, without adding in the complications of distortion. Then, from the progress of that work, it became clear that all energy dependencies needed to be examined and this examination led to the work described in this paper. The next feature of importance would be to include pion distorted waves. It is expected that the cross sections would be somewhat sensitive to pions that are created in the surface regions of nuclei undergoing peripheral collisions, but perhaps not as important as more central collisions where pions that are created deep inside nuclei have a greater likelihood of being absorbed. Peripheral collisions are considered to be important in our general approach, because nuclei are less likely to fragment as they would in more central collisions. The nuclear structure information would be lost under fragmentation. The question of pion absorption and distortion is now being examined. Finally, after the pion absorption and distortion questions are settled, it is envisioned to include the distorted waves describing the nucleus–nucleus relative motion. It is certainly expected that internuclear distortion would play a role, but one would like to have the pion effects in place before examining distortion of the relative motion. This hierarchy provides a program whereby one can learn of these effects as one proceeds. This is still a work in progress, but it is envisioned to include all distortion effects in the future.

2. Derivation of the New Δ Width

The formal expression to first order for the differential decay width (Rodberg and Thaler 1967) is given by

$$\frac{1}{2\pi}\, d\Gamma_{A(\Delta)}(\epsilon_\pi) = |\langle F|V_{\Delta N\pi}|N\rangle|^2 \;\; d\rho \;\; , \tag{1}$$

where a nucleus containing an unstable isobar decays as $A(\Delta) \to A+\pi$ with decay width $\Gamma_{A(\Delta)}(\epsilon_\pi)$ of total pion energy ϵ_π. The Born approximation is also used to describe the scattering states. The intermediate state is $|N\rangle = |A(\Delta)\Phi_{\mathbf{P}_N} 0_\pi\rangle$, where $|A(\Delta)\rangle$ is the internal nuclear structure, $|\Phi_{\mathbf{P}_N}\rangle$ is the plane-wave scattering state of the nucleus of intermediate momentum $\mathbf{P}_N$, and $|0_\pi\rangle$ is the Fock space of no pions. The final state is $|F\rangle = |A_0\Phi_{\mathbf{P}_F}\, 1_\pi\rangle$, where $|A_0\rangle$ is the final nucleus in its ground state with final momentum $\mathbf{P}_F$ described by the plane wave $|\Phi_{\mathbf{P}_F}\rangle$, and a plane-wave pion of momentum $\mathbf{p}_\pi$ in the Fock space $|1_\pi\rangle$. The differential two-body density of states is given by

$$d\rho = \frac{V}{(2\pi\hbar)^3}\, d^3p_\pi\, d^3P_A\, \delta(\mathbf{P}_{A(\Delta)} - \mathbf{P}_A - \mathbf{p}_\pi)\, \delta(E_{A(\Delta)} - E_A - \epsilon_\pi) \;\; , \tag{2}$$

where the energy and momentum of the Δ nucleus is $E_{A(\Delta)}$ and $\mathbf{P}_{A(\Delta)}$, which decays into the final nucleus of energy and momentum E_A and $\mathbf{P}_A$ and final pion of energy and momentum ϵ_π and $\mathbf{p}_\pi$, and V is the integration volume. After integrating over d^3P_A, which eliminates the momentum δ-function, and then changing variables in the energy δ-function to a momentum δ-function containing the pion momentum p_π, with a final integration over dp_π, the two-body density of states becomes

$$d\rho = \frac{V}{(2\pi\hbar c)^3}\;\frac{(p_\pi c)^2\epsilon_\pi E_A\, d\Omega_\pi}{[(p_\pi c)(\epsilon_\pi + E_A) + (P_{A(\Delta)}c)\epsilon_\pi \cos\Theta_{\pi A(\Delta)}]} \;\; , \tag{3}$$

where $\Theta_{\pi A(\Delta)}$ is the angle between the Δ–nucleus momentum $\mathbf{P}_{A(\Delta)}$ and pion momentum $\mathbf{p}_\pi$. Since the Δ–nucleus momentum is not directly measurable, it is necessary to replace $\mathbf{P}_{A(\Delta)}$ by the momentum of the outgoing nucleus $\mathbf{P}_{A'}$. If one looks at the overall problem, which is an equal mass nucleus–nucleus collision producing a Δ nucleus and subsequently decaying as $A+A \to A(\Delta)+A' \to A+\pi+A'$, where A is a nucleus in its ground state, $A(\Delta)$ is the nucleus with a Δ isobar replacing a nucleon, and A' is the excited nucleus that does not have a Δ created inside it. Then by choosing the nucleus–nucleus rest frame to measure all kinematic quantities, one can set $\mathbf{P}_{A(\Delta)} = -\mathbf{P}_{A'}$, and the two-body phase space becomes

$$d\rho = \frac{V}{(2\pi\hbar c)^3}\ f_A(\epsilon_\pi, \Omega_\pi)\, d\Omega_\pi \ , \tag{4}$$

where the phase-space factor is

$$f_A(\epsilon_\pi, \Omega_\pi) = \frac{(p_\pi c)^2 \epsilon_\pi E_A}{(p_\pi c)(\epsilon_\pi + E_A) + (P_{A'} c)\epsilon_\pi \cos\Theta_{\pi A'}} \ . \tag{5}$$

Here $P_{A'}$ is the momentum magnitude of the outgoing nucleus A' and the angle $\Theta_{\pi A'}$ between $\mathbf{p}_\pi$ and $\mathbf{P}_{A'}$ is related to outgoing pion and nuclear angles by the usual expression $\cos\Theta_{\pi A'} = \cos\Theta_{A'}\cos\theta_\pi + \sin\Theta_{A'}\sin\theta_\pi\sin(\Phi_{A'} - \phi_\pi)$, where $\Theta_{A'}$, $\Phi_{A'}$ are the polar and azimuthal angles of the outgoing nucleus A' and θ_π, ϕ_π are the polar and azimuthal angles of the outgoing pion.

The Δ-nucleus state is a coherent linear combination of Δ-hole states which is

$$|A(\Delta)\rangle = \sum_{\Delta h}\ x_{\Delta h}(A)\,|\Delta h; L_A J_A M_A T_A T_Z(A)\rangle \ , \tag{6}$$

where the Δ-hole coefficients are $x_{\Delta h}$, and the ket states are coupled in angular momentum and isospin of the Δ-hole states to produce the angular momentum and isospin of the nucleus $A(\Delta)$, as has been discussed in Deutchman and Li (1993). The one-body, Δ-decay interaction operator $V_{\Delta N\pi}$ has been described in Maung *et al.* (1992) and the matrix element $\langle F|V_{\Delta N\pi}|N\rangle$ has been previously calculated in the general problem of coherent pion production in the collision of equal-mass nuclei where a Δ isobar can be created in either the projectile or target with subsequent decay of the Δ (Deutchman 1992). The amplitude for Δ excitation to either nucleus and excitation to a giant spin–isospin resonance in the other nucleus is given by a sum of projectile and target formation and decay amplitudes as

$$\begin{aligned} C_{FI} = &\frac{A^{T^*}_{P(\Delta)\to P\pi}}{(\epsilon_\pi + m_n c^2 - m_\Delta c^2) + \frac{i}{2}\Gamma_{P(\Delta)}(\epsilon_\pi)} \\ &+ \frac{A^{P^*}_{T(\Delta)\to T\pi}}{(\epsilon_\pi + m_n c^2 - m_\Delta c^2) + \frac{i}{2}\Gamma_{T(\Delta)}(\epsilon_\pi)} \ , \end{aligned} \tag{7}$$

where $m_n c^2$ and $m_\Delta c^2$ are the rest masses of the nucleon and Δ isobar at resonance and separate widths are needed for the projectile and target. The formal expression to first order for the finite width integrated over the density of states and summed over final states except for the intermediate state is

$$\Gamma_{A(\Delta)}(\epsilon_\pi) = 2\pi \sum_{F\neq N} \int d\rho\ |\langle F|V_{\Delta N\pi}|N\rangle|^2 \ . \tag{8}$$

After integrating over the two-body decay phase space and summing over the final states, the Δ width becomes

$$\Gamma_{A(\Delta)}(\epsilon_\pi) = \frac{2\pi V}{(2\pi\hbar c)^3}\ |h(p_\pi)|^2\ |F_{L_A J_A}(k_\pi)|^2 \int d\Omega_\pi\, f_A(\epsilon_\pi,\Omega_\pi)\ |Y_{J_A}^{M_a}(\Omega_\pi)|^2 \,. \quad (9)$$

The pion factor is $h(p_\pi) = \sqrt{\frac{4}{3}}\ [4\pi k_\pi\, f(p_\pi)/\sqrt{2\epsilon_\pi V}]\, F_{\Delta N\pi}$ where the $\Delta N\pi$ vertex form factor of the monopole type is obtained from Machleidt (1989) (Model III), where

$$f(p_\pi) = \left[\frac{\Lambda_{\Delta N\pi}^2 - (m_\pi c^2)^2}{\Lambda_{\Delta N\pi}^2 + (p_\pi c)^2}\right]\ , \quad (10)$$

with cut-off parameter $\Lambda_{\Delta N\pi} = 800$ MeV, and the coupling constant is

$$F_{\Delta N\pi}^2 = \left[\frac{4\pi\alpha^2}{(m_\pi c^2/\hbar c)^2}\right]\ \left(\frac{f_\pi^2}{4\pi\hbar c}\right)(\hbar c)^3\,,$$

where

$$\left(\frac{f_\pi^2}{4\pi\hbar c}\right) = 14{\cdot}6\ \left(\frac{m_\pi c^2}{2m_n c^2}\right)^2 \quad \text{for} \quad \alpha = \sqrt{\tfrac{72}{25}}\,,$$

which was obtained from the quark model. The choice of the monopole type with a cutoff parameter around 800 MeV is guided by recent studies of the πNN form factor from lattice QCD calculations (Liu *et al.* 1995). Those studies indicate that experimental information can be well fitted with a monopole energy-dependent form factor with a cutoff of 750 ± 140 MeV. This choice is an excellent guide for the $\Delta N\pi$ form factor used here.

The important new feature obtained in this decay width is the nuclear form factor $F_{J_A L_A}(k_\pi)$ which depends on the nuclear total and orbital momentum modes J_A and L_A and is given by

$$F_{J_A L_A}(k_\pi) = A\, i^{L_A+1}\, \hat{L}_A \begin{pmatrix} L_A & 1 & J_A \\ 0 & 0 & 0 \end{pmatrix}\ S_{L_A}(k_\pi)\ , \quad (11)$$

where $A = (1/4\pi)\left(8/3\sqrt{3}\right)$, $\hat{L}_A = \sqrt{2L_A+1}$, and the 3-$j$ symbol couples the nuclear orbital mode L_A to unit spin to give the total angular momentum J_A of the nucleus in its intermediate state. The unit spin comes from the difference between the $\frac{3}{2}$-spin Δ isobar and the $\frac{1}{2}$-spin nucleon. Furthermore, we have

$$S_{L_A}(k_\pi) = \left\{ C_A^{-1} \sum_{\substack{n_\Delta l_\Delta \\ n_h l_h}} \left[\hat{l}_\Delta \hat{l}_h \begin{pmatrix} l_\Delta & L_A & l_h \\ 0 & 0 & 0 \end{pmatrix}\ R_{L_A}^{\Delta h}(k_\pi)\right]^2 \right\}^{\frac{1}{2}}\ , \quad (12)$$

which is a *constructive-coherent* sum of Δ-hole states which, because of its 3-j symbol, couples the orbital angular momenta of the Δ particle and hole state l_Δ and l_h in the intermediate state as $\mathbf{l}_\Delta + \mathbf{l}_h = \mathbf{L}_A$ and $C_A = (1/4\pi)\left(\frac{4}{3}\right)^2$. The Δ-hole states are contained in the energy-dependent, radial Δ-hole interaction integrals, which is a function of pion wave number k_π and is given by

$$R_{L_A}^{\Delta h}(k_\pi) = \int_0^\infty d\xi\, u_{n_\Delta l_\Delta}(\xi)\, j_{L_A}(k_\pi \xi)\, u_{n_h l_h}(\xi) \ , \tag{13}$$

where the spherical Bessel functions $j_{L_A}(k_\pi \xi)$ come from the angular momentum expansion of the pion plane waves with harmonic oscillator states used to describe the Δ state and hole state by $u_{n_\Delta l_\Delta}(\xi)$ and $u_{n_h l_h}(\xi)$. The integral in equation (9) cannot be done analytically because the variables $E_A, P_{A'}$ and $\Theta_{\pi A'}$ in the phase-space factor (equation 5) contain further complicated dependencies on the pion solid angle Ω_π. By solving the kinematics of two nuclei in the initial state going to three bodies in the final state, the variables listed above are also functions of the independent variables $p_\pi, \theta_\pi, \phi_\pi$ and Θ_P, Φ_P. The additional dependence upon θ_π, ϕ_π is enough to complicate the problem sufficiently to prevent an analytic integration. In order to obtain an analytic expression, since this is the first attempt to examine the energy dependence of the width, as an approximation, $\cos\Theta_{\pi A'}$ is replaced by its angle averaged value $\overline{\cos\Theta_{\pi A'}} = 0$ which gives the angle-averaged expression

$$\bar{f}_A(\epsilon_\pi) = (p_\pi c)\ \frac{\epsilon_\pi E_A}{(\epsilon_\pi + E_A)} \ , \tag{14}$$

so that the width $\Gamma_{A(\Delta)}(\epsilon_\pi)$ after integration over $d\Omega_\pi$ is replaced by the angle averaged width

$$\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi) = \frac{2\pi V}{(2\pi\hbar c)^3}\, |h(p_\pi)|^2\, |F_{L_A J_A}(k_\pi)|^2\, \bar{f}_A(\epsilon_\pi) \ . \tag{15}$$

Finally, rewriting this expression in terms of the width at resonance, where R refers to the resonance value, the angle-averaged width becomes

$$\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi) = \bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R) \left(\frac{p_\pi}{p_\pi^R}\right)^3 \left(\frac{\epsilon_\pi^R + E_A^R}{\epsilon_\pi + E_\pi}\right) \left(\frac{E_A}{E_A^R}\right) \left[\frac{f^2(p_\pi)}{f^2(p_\pi^R)}\right] \left\{\frac{|F_{L_A J_A}(k_\pi)|^2}{|F_{L_A J_A}(k_\pi^R)|^2}\right\} \ , \tag{16}$$

which is the final expression that is used in these calculations.

A number of improvements have been made to arrive at this final result as compared to the width of the free Δ decay which in comparison is given by

$$\Gamma_\Delta(\epsilon_\pi) = \Gamma_\Delta(\epsilon_\pi^R) \left(\frac{p_\pi}{p_\pi^R}\right)^3 \left(\frac{\epsilon_\pi^R}{\epsilon_\pi}\right) \ , \tag{17}$$

as obtained from Guet *et al.* (1989). Because relativistic kinematics is used, the first improvement was to generalise the ratio $(\epsilon_\pi^R/\epsilon_\pi)$ which was obtained using nonrelativistic, low-energy theory in equation (17) by the ratio $(\epsilon_\pi^R+E_A^R)/(\epsilon_\pi+E_A)$ in the new expression. This agrees with the relativistically improved isobar model (RIIM) found in Ericson and Weise (1988). The second improvement was to remove the static limit in the kinematics which for the free Δ-decay case, approximates the total nucleon energy by the nucleon mass $(\epsilon_n \to m_n)$. In the Δ-nucleus case the nucleon energy generalises to the nuclear total energy E_A. This gives rise to the ratio (E_A/E_A^R) in equation (16). The third improvement was the inclusion of the $\Delta N\pi$ vertex form factor $f(p_\pi)$. Energy calculations have been carried out with each of these factors examined one at a time and it turns out that the major improvement to the energy dependence comes from the nuclear form factor $F_{L_AJ_A}(\epsilon_\pi)$. As a theoretical check, a low-energy approximation to the new width (equation 16) was made in order to compare it with the low-energy behaviour of the free width (equation 17). First, the penetrability factor p_π^3 still remains. Secondly, the next two factors in equation (16) approximately cancel out since both ϵ_π^R and ϵ_π are small compared to E_A^R and E_A. Thirdly, the form factor $f(p_\pi)$ is approximately close to unity for small p_π. Lastly, the nuclear form factor $F_{L_AJ_A}(k_\pi)$ is proportional to $R_{L_A}^{\Delta h}(k_\pi)$ which at low momenta is proportional to $p_\pi^{L_A}$, since the spherical Bessel function is $j_{L_A}(k_\pi\xi) \approx (k_\pi\xi)^{L_A}\,(2L_A+1)!!$. Therefore the squared factor $|F_{L_AJ_A}(k_\pi)|^2$ is proportional to $p_\pi^{2L_A}$. Putting this all together, at low energies, the Δ-nuclear width is proportional to $p_\pi^{2L_A+3}$. Therefore, the effect of the nucleus in general is to include an additional factor of $p_\pi^{2L_A}$ to the width at low energies. If the nucleus is removed, then there would be no factor $p_\pi^{2L_A}$ and we would get the low-energy result of equation (17). For the specific modes considered in this paper, $L_A = 0$, so that the dependence on L_A would disappear; however, if one were to consider higher modes, then the dependence on L_A would become more important.

Finally, if all factors relating to the nucleus are removed from the new expression of the nuclear width, then it does lead to the relativistically improved isobar model (RIIM) in the limit of free Δ decay, except with the addition of the $\Delta N\pi$ vertex form factor. It should be remarked that the free Δ-decay width will blow up at large p_π values because the original derivation of Brown and Weise (1975) is done at low energies and the p_π^3 dependence becomes unphysical at higher energies; however, with the inclusion of the $\Delta N\pi$ vertex form factor, the Δ width is protected from the blow up and goes as $1/p_\pi^2$ at high pion momenta. It is curious that for the spin–isospin modes considered here, where $L_A = 0$, the p_π^3 dependence is obtained at low energies; however, at higher energies, the energy dependence of the nuclear form factor begins to play an extremely important role.

3. Energy Dependence of the Δ Width

Energy calculations of the angle-averaged width $\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi)$ in equation (16) are shown in Fig. 1 from pion kinetic energies from 10 to 150 MeV at projectile laboratory kinetic energies of 100, 250 and 400 MeV·A for the exclusive reaction $^{12}\mathrm{C} + ^{12}\mathrm{C} \to ^{12}\mathrm{C} + ^{12}\mathrm{C}^*\ (T=1) + \pi^0$. This specific reaction is chosen for several reasons. First the T=1 excited state for $^{12}\mathrm{C}^*$ is a giant resonance state at 15·11 MeV and decays into a well-known photon via an $M1$ electromagnetic

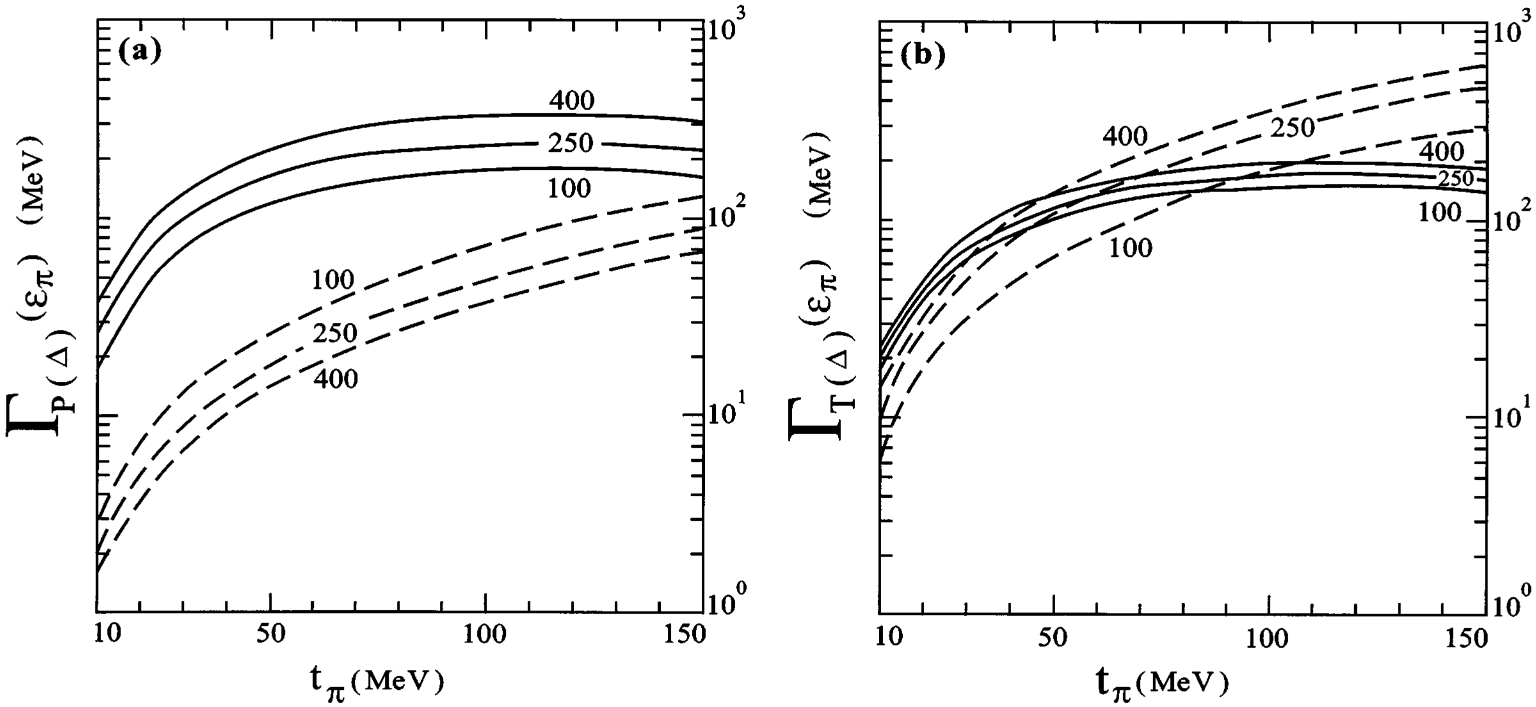

Fig. 1. Nuclear Δ-decay widths as a function of pion kinetic energy t_π in the nucleus–nucleus centre-of-momentum (CM) frame. The bold lines are the angle-averaged widths labelled at laboratory incident energies of 100, 250 and 400 MeV$\cdot A$. The dashed lines are the free Δ-decay widths for the same incident energies. (a) These curves are calculated for $A = P$, and (b) for $A = T$. All curves are calculated under the condition of forward pions ($\theta_\pi = 0°$) and forward ejectiles ($\Theta_P = 0°$).

decay (Deutchman and Erazmus 1995). Second, the π^0 decays into two γ-ray photons, which in the rest frame of the π^0 would decay back-to-back at an energy of approximately 70 MeV each (Deutchman and Erazmus 1995). These photons, if measured in coincidence, along with the identification of the ejectile and recoil target could then provide a complete measurement for the tell-tail signature of the quantum, constructive coherent process. Continuing, Fig. 1*a* refers to the width of the projectile excited Δ-isobar $\bar{\Gamma}_{P(\Delta)}(\epsilon_\pi)$ and Fig. 1*b* to the target excited Δ-isobar $\bar{\Gamma}_{T(\Delta)}(\epsilon_\pi)$. The bold lines are the new widths at three different incident energies and the dashed lines are the results from the previous free Δ-decay width in equation (17) for comparison. In general, it is seen that the new widths are much flatter or more constant compared to the free Δ-decay width. (The irony is not lost that it takes quite a bit of work to finally calculate a constant.) The continual rise of the free Δ-decay widths is due to its p_π^3 dependence and these widths would grow without bound at higher t_π, going beyond its low-energy assumption. This is especially noticeable in Fig. 1*b*. Calculations were done with a free Δ-decay width modified by including the $\Delta N\pi$ vertex form factor in equation (10), and it did reduce the slopes and magnitudes of the free curves somewhat, but not appreciably so over the range of pion kinetic energies considered here. Even with the inclusion of the nucleon vertex form factor, the free Δ widths still get unreasonably large at higher t_π.

The first ingredients needed to calculate the angle-averaged widths are the parameters $\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R)$ and $p_\pi^R(A)$ taken at the Δ-resonance where $A = P$ or T. This was discussed in more detail in Deutchman (1996). In short, one has to consider the sequential, relativistic kinematics of two incident nuclei in the nucleus–nucleus rest frame producing a Δ isobar in either nucleus in the intermediate state and then after decay, producing two nuclei and a pion in a three-body final state. By solving the relativistic energy and momentum equations, which is not trivial in detail, the pion momentum and energy at resonance, p_π^R and ϵ_π^R can be solved in terms of initial and final energies and angles that can be measured. As a result, it turns out that the pion resonance parameters p_π^R and ϵ_π^R are themselves complicated functions of the incident energies in the nucleus–nucleus reaction. As to the parameter $\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R)$ in equation (16), a relativistic transformation is needed to take one from the nuclear rest frame, where resonance values are quoted, to the nucleus–nucleus rest frame where the overall problem of pion production from nucleus–nucleus collisions is calculated. This is accomplished by using the time-dilation property that an object moving at a relative speed has a longer lifetime or a narrower energy width. This implies that $\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R) = \Gamma_{A(\Delta)}(\epsilon_\pi^R)/\gamma_A(\Delta)$ in which the γ factor is obtained from $\beta_A(\Delta) = P_{A(\Delta)}c/E_A(\Delta)$ where $P_{A(\Delta)}$ and $E_{A(\Delta)}$ are the momentum magnitude and energy of the $A(\Delta)$ nucleus in the nucleus–nucleus rest frame. In general, for a Δ inside nuclei, there is an energy shift and changes to the Δ-decay width because of interactions of the Δ with the nuclear medium. The energy shift may involve a second-order process where the Δ-hole doorway states are coupled to multiple nucleon–hole states in the nucleus (Ericson and Weise 1988). The width changes may include pion coherent multiple Δ-hole scattering, Pauli quenching corrections, and Δ-absorptive processes, where Δ-hole states make transitions to n-particle, n-hole states. At this stage of the calculation, it is more important to ferret out the basic ingredients in the formalism before higher-order calculations are explicitly carried out. These effects, however, can be introduced approximately by using typical Δ-in-nuclei values for the Δ mass m_Δ and Δ-nucleus width. From Roy-Stephan (1982) and Moniz (1982), the Δ has a mass approximately 30 MeV lower and a decay width 40 MeV larger inside nuclei. The values $m_\Delta c^2 = 1202$ MeV and $\Gamma_{A(\Delta)}(\epsilon_\pi^R) = 155$ MeV were used in these calculations. However, it is noted that in this second quantised calculation, to first order, the Pauli Exclusion principle is included because when the Δ-hole states collapse upon decay of the Δ inside the nucleus, the nucleon from the decay process $\Delta \to n\pi$ can only occupy the hole state when the nucleus returns to its ground state. The Pauli Exclusion principle automatically excludes the nucleon from occupying any nuclear shell state other than the hole state. A glance at Table 1 shows the values of the angle-averaged, Δ-width parameter $\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R)$ and the forward going pion-momentum parameters, where the Δ is produced either in the projectile, $p_\pi^R(P)$, or target, $p_\pi^R(T)$, at resonance for three values of the incident projectile lab energy. The width parameters $\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R)$ are equal for $A = P$ or T in the case of equal mass collisions since the γ-factors are equal. The values of the parameters $p_\pi^R(P)$ and $p_\pi^R(T)$ are also given in Table 1. They vary as a function of incident projectile energy in a complicated way such that $p_\pi^R(P)$ increases, whereas $p_\pi^R(T)$ decreases as the incident energy increases, and they do not equal each other.

Table 1. Resonance parameters for the nuclear decay widths

$T_{P_0}^{\mathrm{LAB}}$ (MeV·A)	$\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi^R)$ (MeV)	$p_\pi^R(P)$ (MeV/c)	$p_\pi^R(T)$ (MeV/c)
100	153	269	181
250	147	317	148
400	143	352	130

In order to understand qualitatively why these momentum parameters behave as they do, it is useful to consider a simpler problem analogous to the nuclear decay. First, consider the decay of a free Δ particle that is initially moving forward or backward with momenta $\pm p_\Delta$. This will kinematically mimic the case of a Δ produced in a forward moving projectile or of a Δ produced in a backward moving target as viewed in the nucleus–nucleus rest frame. If the relativistic equations of conservation of energy and momentum are solved for $\Delta \to n\pi$, then there are two values for the forward moving pions depending on whether the pion decayed from a forward moving Δ of momentum $+p_\Delta$, or from a backward moving Δ of momentum $-p_\Delta$. The forward pion momenta at resonance are given by

$$p_\pi^R = \frac{\epsilon_\Delta}{2}\left[\frac{M(-)M(+)}{m_\Delta^2}\right] \pm \frac{p_\Delta}{2}\left(\frac{M}{m_\Delta}\right)^2 , \tag{18}$$

where m_Δ is the Δ mass at resonance (for simplicity the factors of c will be suppressed in these expressions). The other parameters are

$$M(-) = \sqrt{m_\Delta^2 - (m_n - m_\pi)^2} , \tag{19a}$$

$$M(+) = \sqrt{m_\Delta^2 - (m_n + m_\pi)^2} \ , \tag{19b}$$

$$M = \sqrt{m_\Delta^2 - m_n^2 + m_\pi^2} \ . \tag{19c}$$

In order to get a feel for the behaviour of the momentum parameters for simplicity, assume that the Δ decays into two equal hadron masses or $m_n = m_\pi = m$. Under the equal mass assumption, the pion parameters become

$$p_\pi^R = \frac{m_\Delta}{2} \sqrt{\left[1 + \left(\frac{p_\Delta}{m_\Delta}\right)^2\right] \left[1 - \left(\frac{2m}{m_\Delta}\right)^2\right]} \pm \frac{p_\Delta}{2} \ .$$

At this point, the first term contains the effects of p_Δ and the Q-value through $m_\Delta = Q + 2m$, and the second term involves an equal momentum sharing of $\pm p_\Delta$ going to two equal mass decay hadrons. If one assumes high Q-values or $Q \gg 2m$, then

$$p_\pi^R \approx \frac{Q}{2} \sqrt{1 + \frac{p_\Delta^2}{Q^2}} \pm \frac{p_\Delta}{2} \ ,$$

and furthermore, for values of $Q^2 \gg p_\Delta^2$, but only that $Q > p_\Delta$, one obtains the final approximation that $p_\pi^R(\text{forward}) \approx (Q/2) \pm (p_\Delta/2)$. For backward moving pions, it turns out that $p_\pi^R(\text{backward}) \approx -(Q/2) \pm (p_\Delta/2)$. Therefore, for equal mass decays, both the Q value and the original Δ momentum are shared evenly with the decay partners. This simple example shows that a forward moving hadron that came from a forward moving Δ has a momentum that is greater than that of a forward moving hadron that came from a backward moving Δ due to the difference in sign, $\pm p_\Delta/2$. This is analogous to the condition that $p_\pi^R(P) > p_\pi^R(T)$ in Table 1. Furthermore, in the simple problem, as p_Δ increases, $p_\pi^R(\text{forward})$ increases if the momentum bias is in the forward direction, but decreases if the momentum bias is in the backward direction. This trend is also seen in Table 1 for increasing incident energies. Having solved the relativistic, sequential kinematic problem for the parameters $p_\pi^R(P)$ and $p_\pi^R(T)$, the solutions show that for increasing incident energies, both of the intermediate momenta of the projectile in the forward direction and the target in the backward direction increase. This in turn affects the values of the pion momentum parameters as shown in Table 1. The kinematic solutions for the pion momentum parameters for the Δ produced in the projectile and both the pion and ejectile coming out in the forward directions are

$$p_\pi^R(P) = \frac{1}{2M_P^2(\Delta)} \sqrt{M^4(P) P_T^2 + 4M_P^2(\Delta) C_P} + \tfrac{1}{2} \frac{M^2(P)}{M_P^2(\Delta)} P_T \ , \tag{20}$$

where

$$M_P(\Delta) = M_P + (m_\Delta - m_n) \ , \tag{21a}$$

$$M^2(P) = M_P^2(\Delta) - M_P^2 + m_\pi^2 \ , \tag{21b}$$

$$M_P^2(+) = M_P^2(\Delta) - (M_P + m_\pi)^2 \ , \tag{21c}$$

$$M_P^2(-) = M_P^2(\Delta) - (M_P - m_\pi)^2 \ , \tag{21d}$$

$$C_P = \tfrac{1}{4} M_P^2(+)\, M_P^2(-) - m_\pi^2 P_T^2 \ . \tag{21e}$$

The Δ-projectile momentum in the nucleus–nucleus rest frame is $\mathbf{P}_{P(\Delta)} = -\mathbf{P}_T$. Therefore, target quantities which are measurable have been substituted into equations (20) and (21) where $P_{P(\Delta)} = P_T$ and $\Theta_{\pi P(\Delta)} = \pi - \Theta_{\pi T}$. The magnitude P_T is related to the incident energies by

$$P_T = \frac{1}{2E_0} \sqrt{\{E_0^2 - [M_P^2(\Delta) + M_T^2]\}^2 - 4M_P^2(\Delta) M_T^2} \ , \tag{22}$$

where $E_0 = E_{P_0} + E_{T_0}$, the sum of total projectile and target initial energies. Results for $p_\pi^R(T)$ are obtained by swapping $P \leftrightarrow T$ in the above expressions, except that the last term in equation (22) has a minus sign.

Returning to Fig. 1, the reason the widths are so flat is because of the inclusion of the nuclear form factor $F_{L_A J_A}(k_\pi)$. In these calculations, the valence and core Δ-hole states $(1p_\Delta)(1p)^{-1} + (2p_\Delta)(1p)^{-1} + (1s_\Delta)(1s)^{-1} + (2s_\Delta)(1s)^{-1}$ in ^{12}C are included which gives rise to a Δ-intermediate giant resonance $(L_A = 0, S_A = 1, J_A = 1)$ spin mode. A calculation of $|F_{L_A J_A}(k_\pi)|^2$ alone as a function of k_π starts at a maximum at $k_\pi = 0$ and falls very sharply approaching a negative straight line on a log plot. If re-plotted as a function of t_π, from 10 to 150 MeV, it is essentially a decaying exponential. It is the nuclear form factor that greatly damps out the p_π^3 effect in the Δ width at higher energies. In equation (16), the penetrability factor $(p_\pi/p_\pi^R)^3$ and nuclear form factor $|F_{L_A J_A}(k_\pi)|^2/|F_{L_A J_A}(k_\pi^R)|^2$ work against each other and are the main terms causing the overall effect. The penetrability factor provides the quick rise of the Δ width at low energy which is soon offset by the nuclear form factor at higher energies. Calculations of the Δ width were carried out to $t_\pi = 400$ MeV and the Δ width reaches a flat maximum, drops off and decreases asymptotically as t_π increases. The reason there are different curves for the Δ widths in Fig. 1 is that the resonance parameters $p_\pi^R(A)$ as discussed previously depend on the incident energies of the nucleus–nucleus collisions. Incident energies of 100, 250 and 400 MeV$\cdot A$ below and above the pion threshold were chosen. In Fig. 1a, for fixed p_π, the nucleon Δ width increases as the incident energies increase. This is due to the increase of $p_\pi^R(P)$ as the incident energy increases. As $p_\pi^R(P)$ increases, the penetrability factor $(p_\pi/p_\pi^R)^3$ decreases but is more than offset by the increase of the nuclear form factor $|F_{L_P J_P}(k_\pi)|^2/|F_{L_P J_P}(k_\pi^R)|^2$, where the denominator

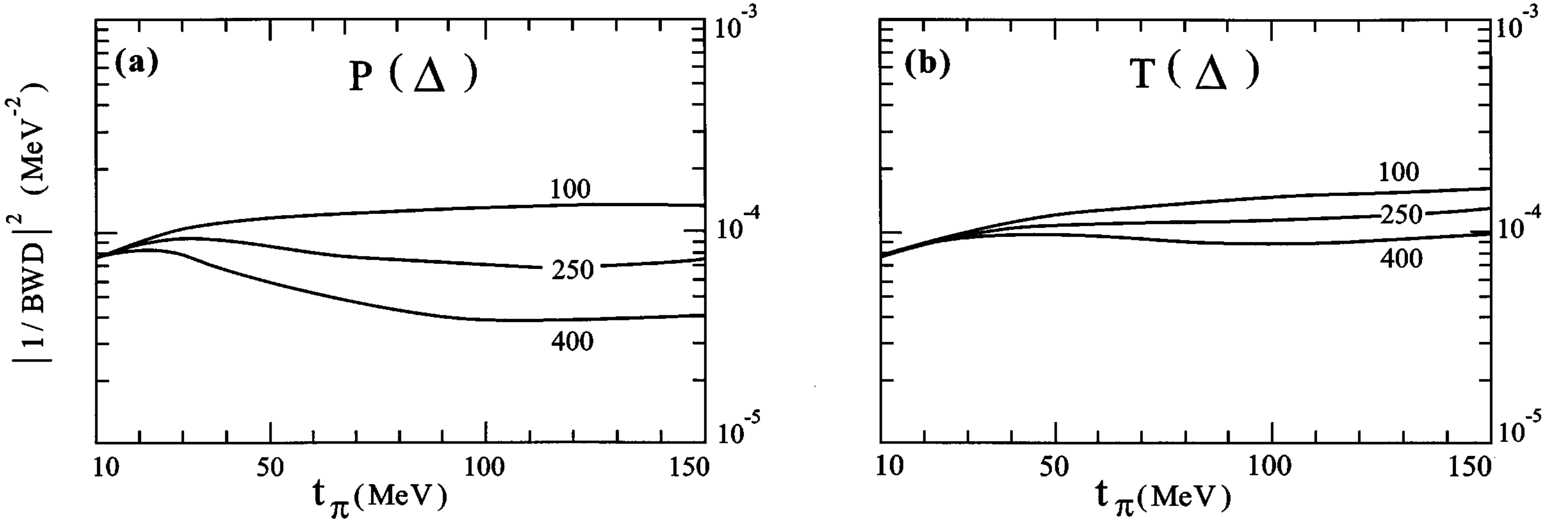

Fig. 2. Absolute magnitude squared of the Breit–Wigner denominator (BWD) as a function of pion kinetic energy in the nucleus–nucleus CM frame taken at laboratory incident energies of 100, 250 and 400 MeV·A: (a) $A = P$ and (b) $A = T$. All curves are calculated under the condition of forward pions ($\theta_\pi = 0°$) and forward ejectiles ($\Theta_P = 0°$).

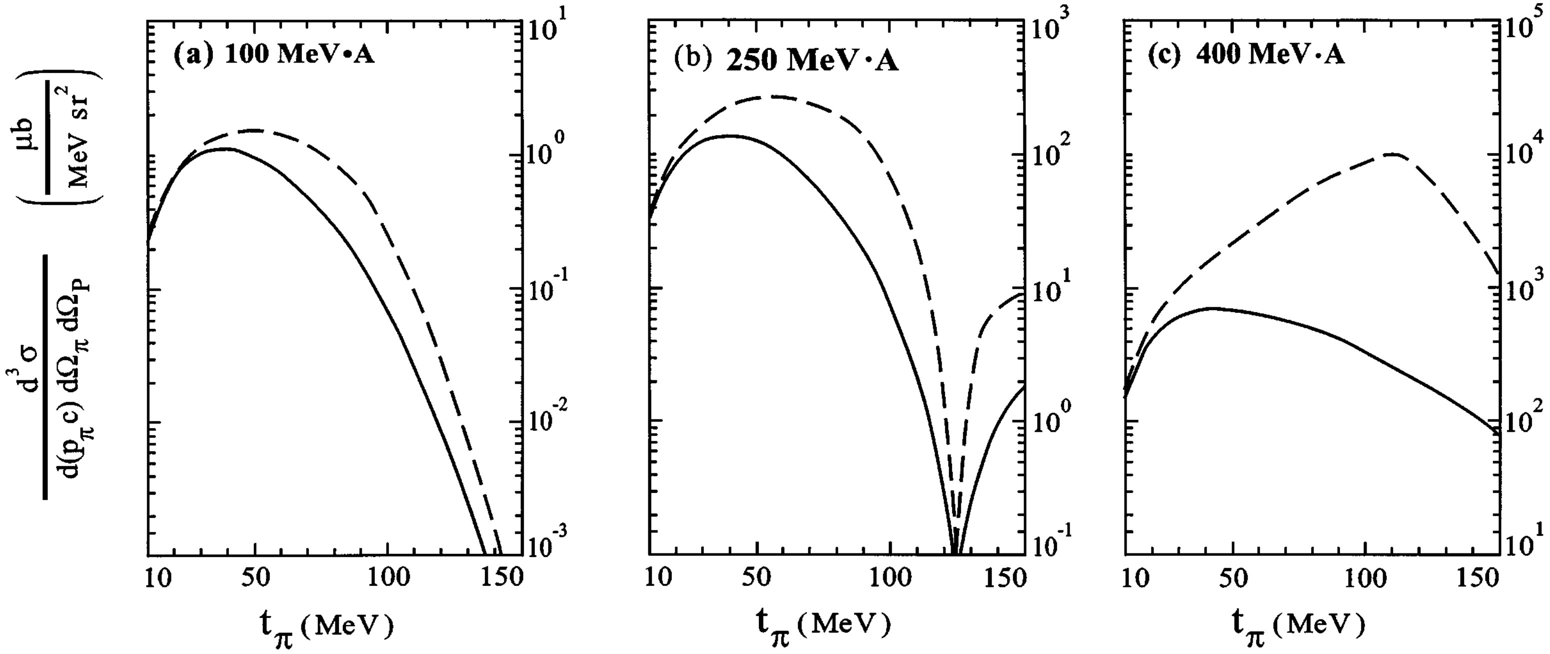

Fig. 3. Triple differential cross sections in the exclusive reaction $^{12}\mathrm{C} + {}^{12}\mathrm{C} \to {}^{12}\mathrm{C} + \pi^0 + {}^{12}\mathrm{C}^*(T = 1)$ as a function of pion kinetic energy t_π in the nucleus–nucleus CM frame at (a) 100 MeV·A, (b) 250 MeV·A and (c) 400 MeV·A incident projectile laboratory energies. The bold lines are the results of the angle-averaged, Δ-nuclear decay widths and the dashed lines are the results with the free Δ-decay widths. All curves are calculated under the conditions of forward pions ($\theta_\pi = 0°$) and forward ejectiles ($\Theta_P = 0°$).

drops quickly as $p_\pi^R(=\hbar k_\pi^R)$ increases. The reverse effect occurs in Fig. 1*b* since $p_\pi^R(T)$ *decreases* as the incident energies increases. In this case $(p_\pi/p_\pi^R)^3$ increases but is not offset by the decrease of $|F_{L_T J_T}(k_\pi)|^2/|F_{L_T J_T}(k_\pi^R)|^2$, so that a mild increase results as the incident energy increases.

Since the nuclear Δ widths occur in the denominators of the Δ-formation and decay amplitudes as given in equation (7), Fig. 2 shows the energy dependence of the denominators separately to see how they affect the pion energy differential cross sections. In Fig. 2, we have BWD $= (\epsilon_\pi + m_\pi - m_\Delta) + i\bar{\Gamma}_{A(\Delta)}(\epsilon_\pi)/2$. The effects of the projectile Δ-width $\bar{\Gamma}_{P(\Delta)}(\epsilon_\pi)$ and the target Δ width $\bar{\Gamma}_{T(\Delta)}(\epsilon_\pi)$ are shown in the $P(\Delta)$ and $T(\Delta)$ panels respectively. These effects are just a denominator effect which reverses the magnitudes of the incident energy labels. The wider spread in the $P(\Delta)$ widths in Fig. 1*a* show up more dramatically as a wider difference in Fig. 2*a* as compared to the $T(\Delta)$ widths.

Finally, in Fig. 3, forward pion energy distributions are shown for the three different incident energies, where the bold lines include the new nuclear Δ widths and the dashed lines are those of previous calculations using the free Δ-decay width model. The new calculations are lower in magnitude. At forward pion angles, the pion energy distributions are mainly due to projectile Δ production (Deutchman and Erazmus 1995), so that the shapes are mainly sensitive to $\bar{\Gamma}_{P(\Delta)}(\epsilon_\pi)$. Since the projectile Δ widths are broader than the free model widths at these energies, then the magnitudes are increasingly reduced at higher incident energies. Also, these curves are much smoother and flatter than those using the free model. This is especially noticeable at 400 MeV$\cdot A$, where the old width is too narrow and over emphasises the resonance effect. Also, pion angular distributions were calculated for the same three incident energies; however, the shapes are very similar to previous calculations, with only the magnitudes decreasing slightly.

I would like to set the record straight concerning the history that eventually led to the calculations that are shown in Fig. 3. The original formalism for the overall amplitude was given in Deutchman (1992), where the projectile and target amplitudes were included in the formalism, but initial calculations were done with the target amplitude only. Then, in the following work by Deutchman and Li (1993), triple differential cross sections were calculated which included the completely, coherent model for the first time. It is here that I wish to set the record straight. In Deutchman and Li (1993), the projectile amplitude was *not* included in the calculations as erroneously stated. The projectile amplitude was included in the *formalism*, but not yet included in the *calculations* since computer codes had not yet been written to do so. Later, in the work of Deutchman and Erazmus (1995), both amplitudes were calculated. For example, the cross section (full line) at 100 MeV$\cdot A$ in Fig. 4*a* of Deutchman and Li (1993) is the same as the target-only calculation (dotted line) of Fig. 1*a* in Deutchman and Erazmus (1995). The inclusion of the projectile contribution raises the overall cross section by an order of magnitude. This is also in agreement with the calculation (dashed line) shown in Fig. 3*a* of this work. As first author in the Deutchman and Li (1993) paper, I apologise for the oversight, the error was solely mine, and should not implicate my co-author in any way.

4. Conclusions

An angle-averaged, nuclear Δ-decay width has been derived that now includes the all important nuclear form factor describing the Δ-nucleus decay to a pion and final nucleus for the collision of $^{12}\mathrm{C}+^{12}\mathrm{C}\rightarrow^{12}\mathrm{C}+^{12}\mathrm{C}^{*}(T=1)+\pi^{0}$. Minor improvements involve removal of the static limit in the relativistic kinematics as well as inclusion of the energy-dependent $\Delta N\pi$ vertex form factor. The new nuclear width reduces to the relativistically improved isobar model for free Δ decay when nuclear factors are removed. The energy dependence of the new nuclear Δ widths rise more quickly, are much flatter, and do not increase indefinitely as p_{π}^{3} at higher energies when compared to the free Δ-decay widths that were used previously. It is recommended that the free Δ-decay width include the $\Delta n\pi$ vertex form factor to prevent this blow up from occurring at higher energies. As a result of the flatness of the nuclear widths and because they are broader than the free Δ-decay width, the pion energy distributions are smaller in magnitude but smoother in shape. The new Δ widths depend on the constructive coherence of the Δ-hole states that build up in the nuclear form factor so that the widths become a measure of this coherence. It is concluded that the new widths developed here represent an improvement over our past calculations that describe the Δ decay from nuclei in the problem of coherent pion production in nucleus–nucleus collisions.

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