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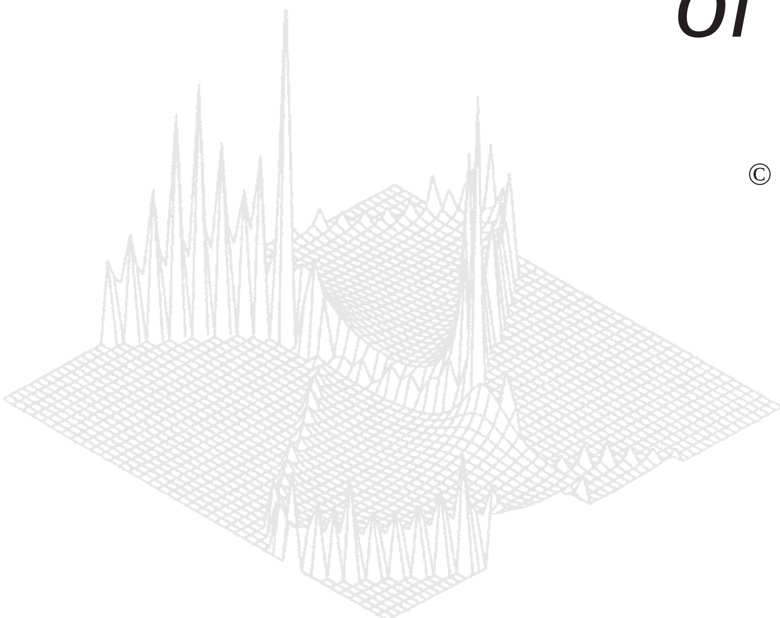

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The Low-energy ^{9}Be(γ, n)^{8}Be Cross Section

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Abstract

Fits are made to low-energy $^9\mathrm{Be}(\gamma, \mathrm{n})^8\mathrm{Be}$ cross-section data using one-level R-matrix formulae including channel contributions. Fits with reasonable parameter values are found for the newer radioactive-isotope data, and also for data obtained from inelastic electron scattering on ^{9}Be, but not for older radioactive-isotope data. This differs from the result of recent fits using a semi-microscopic model, which supported the older data. The difference is attributed to the use in the latter calculation of a single-particle potential description of the continuum wave function.

1. Introduction

Efros *et al.* (1998) have pointed out the discrepancy between older and more recent values of the low-energy $^9\mathrm{Be}(\gamma, \mathrm{n})^8\mathrm{Be}$ cross section that had been obtained using radioactive-isotope gamma rays with energies below about $2{\cdot}2$ MeV. The older values (Hamermesh and Kimball 1953; Gibbons *et al.* 1959; John and Prosser 1962) are appreciably higher at energies above the $1{\cdot}7$ MeV resonance peak than are those of Fujishiro *et al.* (1982), who were aware of the discrepancy and discussed it thoroughly from an experimental viewpoint. Efros *et al.* say that their semi-microscopic model calculations support the older data.

One-level R-matrix fits to the older data, together with the bremsstrahlung data of Berman *et al.* (1967), had previously been made by Barker and Fitzpatrick (1968), and to the newer data by Barker (1983). The same R-matrix formula was used by Kuechler *et al.* (1987) to fit their $^9\mathrm{Be}(\mathrm{e}, \mathrm{e}')^9\mathrm{Be}$ measurements, from which they derived values of the $^9\mathrm{Be}(\gamma, \mathrm{n})^8\mathrm{Be}$ cross section at low energies. It is not obvious, however, that standard R-matrix formulae are justified for reactions involving photons, particularly if they have low energy, because contributions to the collision matrix element can come from large distances. This was partially allowed for in Barker (1984), where contributions to the radiative width coming from the $^8\mathrm{Be}(\mathrm{g.s.}) + \mathrm{n}$ channel were considered. In addition to this resonant contribution there are, however, also nonresonant channel contributions, which change the form of the cross-section formula. This is seen in the R-matrix formulae appropriate for radiative-capture reactions (and the inverse photodisintegration) given by Barker and Kajino (1991). Here we use these formulae to fit the $^9\mathrm{Be}(\gamma, \mathrm{n})^8\mathrm{Be}$ cross-section data.

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2. Formulae

As is usual, we consider contributions to the ^{9}Be$(\gamma,\mathrm{n})^8$Be cross section $\sigma_{\gamma\mathrm{n}}$ due to E1 transitions from the $\frac{3}{2}^-$ ground state of ^{9}Be to the $\frac{1}{2}^+$ first excited state. We make the one-level approximation in the formulae of Section 2 of Barker and Kajino (1991), with the notation simplified by the omission of unnecessary labels. The choice $B_\ell = S_\ell(E_r)$, where E_r is the resonance energy, makes $E_1 = E_r$. Sums over channels are restricted to the ^{8}Be ground-state channel, labelled g, and the first-excited-state channels, labelled by es, where $s = \frac{3}{2}$ or $\frac{5}{2}$ is the channel spin. Then we have

$$\sigma_{\gamma\mathrm{n}} = \frac{\pi}{4k_\gamma^2}|U|^2\,, \tag{1}$$

with

$$\begin{aligned} U = & -ie^{-i\phi_g}2P_g^{\frac{1}{2}}k_\gamma^{\frac{3}{2}} \\ & \times\left[\frac{\gamma_g\gamma_\gamma}{E_r - E + S_g(E_r)\gamma_g^2 - \{S_e(E) - S_e(E_r)\}(\gamma_{e\frac{3}{2}}^2 + \gamma_{e\frac{5}{2}}^2) - iP_g\gamma_g^2}\right. \\ & \left. - \frac{32\sqrt{2}}{81}\frac{M_n^{\frac{1}{2}}e}{\hbar k}N_f^{\frac{1}{2}}aF_0(a)G_0(a)\theta_g J_1'(0,1)\right]\,, \end{aligned} \tag{2}$$

where

$$\gamma_\gamma = \gamma_\gamma(\mathrm{int}) + \gamma_\gamma(\mathrm{ch}) \tag{3}$$

with

$$\begin{aligned} \gamma_\gamma(\mathrm{ch}) = -\frac{32\sqrt{2}}{81}\frac{M_n^{\frac{1}{2}}e}{\hbar}N_f^{\frac{1}{2}}a^2\left[\gamma_g\theta_g J_{1g}(0,1) + \left(\frac{1}{\sqrt{10}}\gamma_{e\frac{3}{2}}\theta_{e\frac{3}{2}}\right.\right. \\ \left.\left. + \sqrt{\tfrac{3}{5}}\gamma_{e\frac{5}{2}}\theta_{e\frac{5}{2}}\right)J_{1e}(2,1)\right]\,. \end{aligned} \tag{4}$$

Here M_n is the nucleon mass, and

$$N_f^{-1} = 1 + \frac{2}{a}\left\{\theta_g^2\int_a^\infty dr\left[\frac{W_{g1}(r)}{W_{g1}(a)}\right]^2 + \left(\theta_{e\frac{3}{2}}^2 + \theta_{e\frac{5}{2}}^2\right)\int_a^\infty dr\left[\frac{W_{e1}(r)}{W_{e1}(a)}\right]^2\right\}\,. \tag{5}$$

Also, we have

$$J_{1g}(0,1) = J_1''(0,1) + i\frac{F_0(a)G_0(a)}{F_0^2(a) + G_0^2(a)}J_1'(0,1)\,, \tag{6}$$

with

$$J_1'(0,1)=\frac{1}{a^2}\int_a^\infty dr\, r\,\frac{W_{g1}(r)}{W_{g1}(a)}\left[\frac{F_0(r)}{F_0(a)}-\frac{G_0(r)}{G_0(a)}\right], \tag{7}$$

$$J_1''(0,1)=\frac{1}{a^2}\int_a^\infty dr\, r\,\frac{W_{g1}(r)}{W_{g1}(a)}\,\frac{F_0(a)F_0(r)+G_0(a)G_0(r)}{F_0^2(a)+G_0^2(a)}\,, \tag{8}$$

while

$$J_{1e}(2,1)=\frac{1}{a^2}\int_a^\infty dr\, r\,\frac{W_{e1}(r)}{W_{e1}(a)}\frac{W_{e2}(r)}{W_{e2}(a)}\,. \tag{9}$$

All of the integrals in equations (5)–(9) may be evaluated analytically and expressed as functions of the energy E (in the ^{8}Be + n c.m. system) and the channel radius a, as may the penetration factor P_g and shift factors S_g and S_e.

For a given value of a, the formula for $\sigma_{\gamma\mathrm{n}}$ contains eight parameters; these are E_r, γ_g, θ_g and $\gamma_\gamma(\mathrm{int})$, together with four connected with the ^{8}Be excited-state channels: $\gamma_{e\frac{3}{2}}$, $\gamma_{e\frac{5}{2}}$, $\theta_{e\frac{3}{2}}$ and $\theta_{e\frac{5}{2}}$. Both the older and the newer sets of radioactive-isotope data contain only six data points, so that not all of the eight parameters can be determined by fitting them. We therefore use values of the excited-state parameters obtained in other ways described below, and find that the results are not sensitive to changes in these values.

The d-wave reduced-width amplitudes γ_{es} (for the $\frac{1}{2}^+$ excited state of ^{9}Be) and the p-wave dimensionless reduced-width amplitudes θ_{es} (for the $\frac{3}{2}^-$ ground state of ^{9}Be) may be written in terms of spectroscopic amplitudes and single-particle values:

$$\gamma=\theta(\hbar^2/\mu a^2)^{\frac{1}{2}}, \qquad \theta=\mathcal{S}^{\frac{1}{2}}\theta_{\mathrm{sp}}\,, \tag{10}$$

where μ is the reduced mass, and

$$\theta_{\mathrm{sp}}=u(a)\left[a/2\int_0^a dr\, u^2(r)\right]^{\frac{1}{2}}, \tag{11}$$

with $u(r)/r$ the single-particle radial wave function. We calculate $u(r)$ for a central Woods–Saxon potential with conventional values of the radius and diffuseness parameters (1·25 and 0·65 fm), cut off at $r=a$, and with the depth adjusted to fit the appropriate binding energy. For the conventional value of the channel radius $a=1\cdot45(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$ fm $=4\cdot35$ fm, this gives $\theta_{e,\mathrm{sp}}(\frac{1}{2}^+)=0\cdot524$ and $\theta_{e,\mathrm{sp}}(\frac{3}{2}^-)=0\cdot512$ [the corresponding values for the ground-state channel are $\theta_{g,\mathrm{sp}}(\frac{1}{2}^+)=-1\cdot116$ ($\ell=0$)* and $\theta_{g,\mathrm{sp}}(\frac{3}{2}^-)=0\cdot629$ ($\ell=1$)].

Experimental values of the spectroscopic factors $\mathcal{S}_g$ and $\mathcal{S}_e=\sum_s\mathcal{S}_{es}$ are available for the $\frac{3}{2}^-$ ground state of ^{9}Be, obtained from one-neutron pickup reactions. Eleven values of $\mathcal{S}_g$ were given in Table 1 of Barker (1984). Nine of these measurements also give values of $\mathcal{S}_e$ and six of these have the ratio $\mathcal{S}_e/\mathcal{S}_g$ between 1·2 and 1·5. These values are consistent with the ratio given by shell

* We use the usual convention that $u(r)$ is positive for small r; this is opposite to that used for the 2s state in Barker (1984).

model calculations: $1\cdot25$ (Cohen and Kurath 1967), $1\cdot40$ (Barker 1966) and $1\cdot33$ (Kumar 1974). We assume that the calculations also give reliably the absolute values and the separate spectroscopic amplitudes for different channel spins, and take those from Cohen and Kurath: $\mathcal{S}_g^{\frac{1}{2}}(\frac{3}{2}^-) = 0\cdot762$, $\mathcal{S}_{e\frac{3}{2}}^{\frac{1}{2}}(\frac{3}{2}^-) = -0\cdot622$ and $\mathcal{S}_{e\frac{5}{2}}^{\frac{1}{2}}(\frac{3}{2}^-) = -0\cdot582$.

For the $\frac{1}{2}^+$ state of ^{9}Be, experimental values are not available and we have to use shell model values of the spectroscopic amplitudes. Corresponding to the value $\mathcal{S}_g^{\frac{1}{2}}(\frac{1}{2}^+) = 0\cdot781$ given in Barker (1984), one has $\mathcal{S}_{e\frac{3}{2}}^{\frac{1}{2}}(\frac{1}{2}^+) = 0\cdot477$ and $\mathcal{S}_{e\frac{5}{2}}^{\frac{1}{2}}(\frac{1}{2}^+) = 0\cdot363$ (C. L. Woods, personal communication). Thus we take

$$\gamma_{e\frac{3}{2}} = 0\cdot393\ \mathrm{MeV}^{\frac{1}{2}}, \qquad \gamma_{e\frac{5}{2}} = 0\cdot299\ \mathrm{MeV}^{\frac{1}{2}},$$

$$\theta_{e\frac{3}{2}} = -0\cdot318, \qquad \theta_{e\frac{5}{2}} = -0\cdot298, \tag{12}$$

and might expect

$$\gamma_g = -1\cdot369\ \mathrm{MeV}^{\frac{1}{2}}, \qquad \theta_g = 0\cdot479\,. \tag{13}$$

Similarly an expected value of $\gamma_\gamma(\mathrm{int})$ may be obtained. In Barker (1984), a value $a\mathcal{M}_{if} = 0\cdot35$ fm was used as representative of some shell model calculations. From

$$\gamma_\gamma(\mathrm{int}) = \tfrac{8}{27} e N_f^{\frac{1}{2}} a\mathcal{M}_{if}\,, \tag{14}$$

with N_f defined in equation (5), one finds (using $N_f \approx 1$) an expected value

$$\gamma_\gamma(\mathrm{int}) \approx 0\cdot124\ \mathrm{MeV}^{\frac{1}{2}}\mathrm{fm}^{\frac{3}{2}}\,. \tag{15}$$

Standard R-matrix formulae are obtained by taking $\gamma_{es} = 0$ and $\theta_g = 0$.

The strength of the E1 gamma-transition is usually expressed as the energy-independent reduced transition strength $B(\mathrm{E1})$, defined by

$$\Gamma_\gamma = \frac{16\pi}{9} e^2 k_\gamma^3 B(E1)\downarrow \qquad (k_\gamma = E_\gamma/\hbar c)\,. \tag{16}$$

We may write

$$\Gamma_\gamma = 2k_\gamma^3 |\gamma_\gamma|^2\,, \tag{17}$$

suggesting

$$B(E1)\downarrow = \frac{9}{8\pi}\frac{1}{e^2}|\gamma_\gamma|^2\,; \tag{18}$$

however, γ_γ is given by equation (3), where $\gamma_\gamma(\mathrm{int})$ is real and energy-independent but $\gamma_\gamma(\mathrm{ch})$ is complex and energy-dependent.

3. Fits and Results

We fit separately the six data points in the older radioactive-isotope set and the six in the newer set, by adjusting the four parameters E_r, γ_g, θ_g and γ_γ(int). We similarly fit the ^{9}Be(γ, n)^{8}Be cross section given by Kuechler *et al.* (1987); they obtained this by point-by-point extrapolation of four measured ^{9}Be(e, e′) spectra to the photon point and taking an error-weighted sum, with a consolidation of data points to the 38 points shown in their Fig. 6. We assume the same energy resolution of 30 keV for this cross section as for the original spectra.

Table 1. Parameter values for fits to ^{9}Be(γ, n)^{8}Be data

Data set	Fit	E_r (keV)	γ_g (MeV$^{\frac{1}{2}}$)	θ_g	γ_γ(int) (MeV$^{\frac{1}{2}}$fm$^{\frac{3}{2}}$)	χ^2	E_p (keV)	$\Gamma_{\frac{1}{2}}$ (keV)	$B(E1)\downarrow$ (mb)
Older	A	104·3	−1·007	0·0[a]	0·952	3·32	17·0	177	2·26
	B	43·4	−0·575	−0·350	0·934	0·577	35·8	163	0·754
	C	43·4	−0·564	−0·336	0·968	0·577			
	D	−21·9	−19·4	0·643	0·124[a]	4·89	20·3	182	1723
	E	43·6	−0·540	−0·333	0·934	0·572			
Newer	A	67·6	−0·694	0·0[a]	0·650	0·935	32·4	140	1·05
	B	49·5	−0·599	−0·120	0·674	0·0967	33·4	117	0·796
	C	49·5	−0·585	−0·116	0·715	0·0967			
	D	248·9	−1·345	0·351	0·124[a]	3·12	25·4	173	3·83
	E	49·5	−0·560	−0·115	0·669	0·0967			
Kuechler	A	66·2	−0·778	0·0[a]	0·743	48·1	21·8[b]	135[b]	1·37
	B	272·0	−1·550	0·411	0·035	43·8	18·8[c]	134[c]	5·68
	C	306·1	−1·607	0·418	0·011	43·8			
	D	236·0	−1·450	0·379	0·124[a]	43·8	18·8[c]	134[c]	4·96
	E	271·6	−1·452	0·394	0·003	43·8			

[A] Best fit for standard formulae ($\gamma_{es} = 0$, $\theta_g = 0$).
[B] Best fit with parameter values (12).
[C] Best fit with $\gamma_{es} = \theta_{es} = 0$.
[D] Best fit with γ_γ(int) fixed at expected value.
[E] As for B, with $a = 5{\cdot}0$ fm.
[a] Fixed value.
[b] Values for unsmeared cross section; for smeared cross section, $E_p = 31{\cdot}5$ keV and $\Gamma_{1/2} =$ 148 keV.
[c] Values for unsmeared cross section; for smeared cross section, $E_p = 29{\cdot}4$ keV and $\Gamma_{1/2} =$ 155 keV.

Table 1 gives values of the parameters and of the total χ^2 for some of these fits. For interest, we also give some values of the peak energy E_p, of the FWHM of the peak $\Gamma_{1/2}$, and of B(E1)↓, evaluated from equation (18) at E_p. Identical fits are obtained if the signs of both γ_g and θ_g are changed, and the other parameter values are unchanged. Approximately the same fits can be obtained if the signs of both θ_g and γ_γ(int) are changed and the parameter values are adjusted slightly. We list fits in which γ_g and γ_γ(int) have the signs expected from the shell model calculations.

The rows labelled A in Table 1 are standard-formulae fits. In each case, γ_g has a magnitude smaller than that expected from the shell model calculation ($-1{\cdot}369$ MeV$^{\frac{1}{2}}$), while γ_γ(int) is larger than the expected value ($0{\cdot}124$ MeV$^{\frac{1}{2}}$ fm$^{\frac{3}{2}}$). Rows B are best fits with the formulae (1)–(9), with the excited-state channel parameters given in equation (12). The fits are not sensitive to these parameter values, and the rows C are for all these parameters set equal to zero.

In these fits to the older and newer data sets with the revised R-matrix formulae, θ_g has the opposite sign to the expected value $(0\cdot479)$, while γ_g is smaller in magnitude than expected and $\gamma_\gamma(\text{int})$ larger. In contrast to the situation in Table 2 of Barker (1984), where the internal and channel contributions to the radiative width added constructively, with the channel contribution dominating, these present fits have a dominant internal contribution with the channel contribution tending to cancel it. If $\gamma_\gamma(\text{int})$ is restricted to be near its expected value, as in rows D, the fit to the older data gives $E_r < 0$; such a fit is inadmissible, as explained in Barker and Fitzpatrick (1968). The fit D to the newer data gives reasonable parameter values, with a greatly increased but still acceptable value of χ^2.

The fits B and C to the data of Kuechler *et al.* give parameter values not far from those expected, and essentially the same fit is obtained in row D.

The fits in the rows E correspond to those in rows B but with a larger channel radius $a = 5\cdot0$ fm.

The best fits A, B and D are shown in Figs 1*a* and 1*b* for the older and newer radioactive-isotope data, and in Fig. 2 for the data of Kuechler *et al.*

4. Discussion and Summary

While some of the fits listed in Table 1 have unexpectedly small values of χ^2, all the fits may be regarded as acceptable as far as the χ^2 values are concerned. For the fits to the Kuechler *et al.* data, the χ^2 values exceed the number of degrees of freedom, but in each case more than one third of the total χ^2 comes from the four points below threshold (see Fig. 2), and these contributions are sensitive to the energy calibration and resolution.

The standard-formulae fits (A) may be compared with previous fits. Direct comparison with the Barker and Fitzpatrick (1968) fit to the older data is not possible, because their fit was greatly influenced by the inclusion in the fitted data of the bremsstrahlung measurements of Berman *et al.* (1967). With the Berman data modified to allow for neutron energy loss in the target, as explained by Barker and Fitzpatrick, the parameter values obtained there were not dissimilar to those given in Table 1; for example, for $B(\text{E1})\!\downarrow$ equal to $2\cdot0$ mb, they obtained $E_r = 90\cdot1$ keV and $\gamma_g = -0\cdot959$ MeV$^{\frac{1}{2}}$.

Fit A to the newer (Fujishiro *et al.*) data gives parameter and χ^2 values very close to those given in Barker (1983); the slight differences are due to the different Q values used ($1\cdot6654$ MeV here, $1\cdot666$ MeV previously).

The parameter values given in fit A to the Kuechler data differ considerably from those of Kuechler *et al.* (1987), who gave $E_R = 1\cdot684$ MeV, corresponding to $E_r = 19$ keV, $\Gamma = 217$ keV, and $B(E1)\uparrow = 0\cdot27$ mb, corresponding to $B(E1)\downarrow = 0\cdot54$ mb. These values of E_r and $B(E1)\downarrow$ are much less than those in Table 1. Also, it is not clear what their value of Γ represents, as the Γ appearing in their formulae is energy-dependent; the parameter values in row A of Table 1 give $\Gamma(E_r) = 280$ keV and $\Gamma(E_p) = 160$ keV.

It is of some interest that the standard-formulae fits all give $B(E1)\downarrow \geq 1\cdot05$ mb. As the Wigner unit for this transition is $2\cdot79$ mb, this gives $B(E1)\downarrow \geq 0\cdot38$ W.u. This is a very strong transition. Millener *et al.* (1983) found $B(E1)\downarrow$

$= 0 \cdot 36 \pm 0 \cdot 03$ W.u. for the transition in ^{11}Be from the $\frac{1}{2}^-$ first-excited state to the $\frac{1}{2}^+$ ground state, and pointed out that this is the strongest known E1 transition between bound nuclear levels.

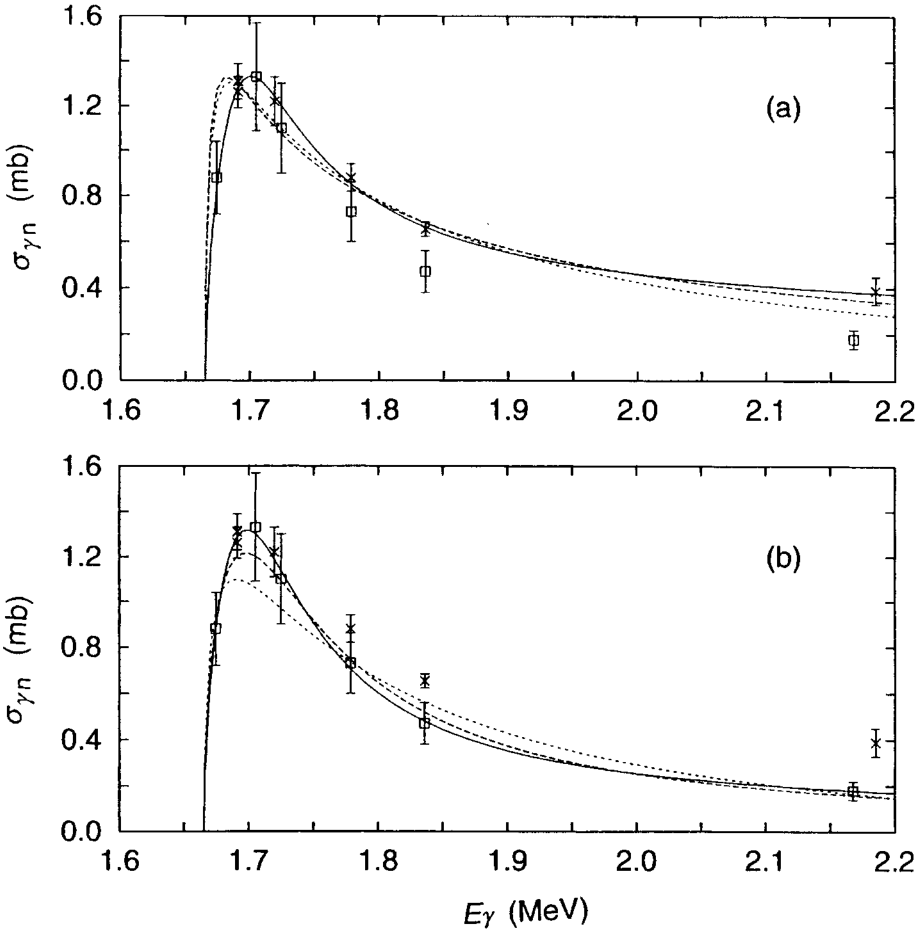

Fig. 1. ^{9}Be(γ, n)^{8}Be cross section $\sigma_{\gamma n}$ as a function of γ energy E_γ. The experimental points are for the older (crosses) and the newer (squares) radioactive-isotope data. The dashed, solid and dotted curves are best fits corresponding to rows A, B and D respectively in Table 1. Fits to the older data are shown in (*a*), and to the newer data in (*b*).

The best fits with R-matrix formulae including channel contributions (rows B, C and E of Table 1) give χ^2 smaller than for the standard formulae but, except for the fits to the Kuechler data, the values of γ_g, θ_g and γ_γ(int) are not close to the expected values (13) and (15). When γ_γ(int) is fixed at the expected value (15) (rows D), the value of $|\gamma_g|$ becomes unreasonably large for the fit to the older data (as well as E_r becoming negative, which makes the fit inadmissible), whereas for the fits to the newer data and to the Kuechler data, both γ_g and θ_g are close to the expected values (13). These fits give $E_r \approx 240$ keV, which is not unreasonable, while $E_p \approx 20$ keV and $\Gamma_{1/2} \approx 150$ keV. They also give large values of $B(E1)\downarrow \approx 1\cdot 5$ W.u.; this is larger than the values calculated in Barker (1984) (column A of Table 2), due mainly to the value of $\mathcal{S}_i^{\frac{1}{2}}$ used there (about $0\cdot 4$ for $a = 4\cdot 35$ fm) being smaller than the value $0\cdot 781$ used here in obtaining (13).

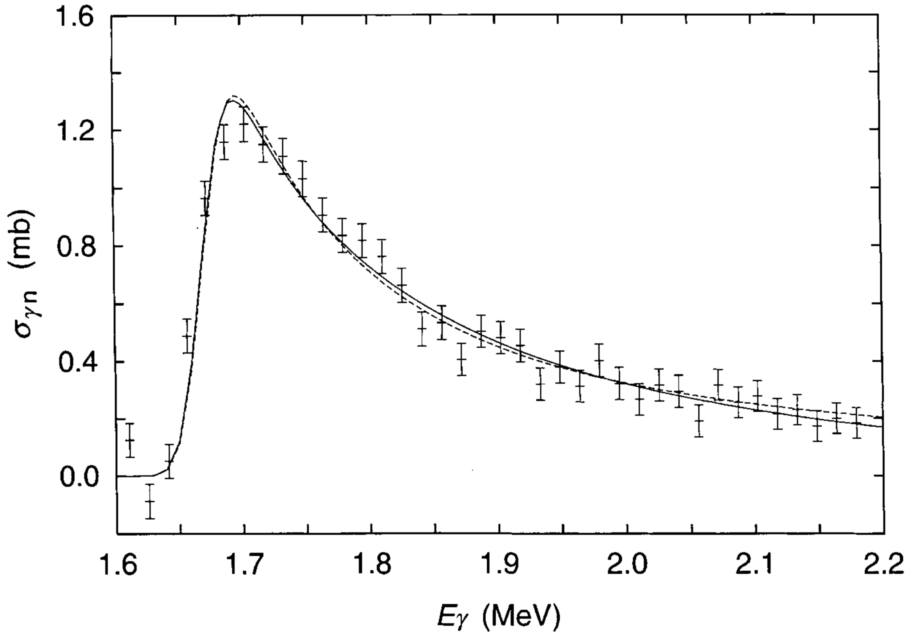

Fig. 2. ^{9}Be(γ, n)^{8}Be cross section $\sigma_{\gamma n}$ as a function of γ energy E_γ. The experimental points are from Kuechler *et al.* (1987), and the dashed and solid curves are fits corresponding to rows A and B respectively in Table 1. The fit for row D is indistinguishable from that for row B.

The difficulty in fitting the older data, in which $\sigma_{\gamma n}$ at higher energies is appreciably greater than in the newer data, when the main contribution is assumed to come from the channel region (fits D), is due to $J_1''(0,1)$, given by equation (8), decreasing as E increases, so leading to smaller γ_γ(ch) at higher energies and consequently smaller calculated values of $\gamma_{\gamma n}$ and $\sigma_{\gamma n}$ at higher energies.

Our finding that reasonable parameter values can be obtained in fits to the newer radioactive-isotope data, and to the data of Kuechler *et al.* (1987), but not to the older data, is opposite to that of Efros *et al.* (1998). They say that their analysis supports the older data. Although they found fits to the newer data with acceptable χ^2 values, they considered the values of the resultant potential parameters to be unrealistic. Their potentials (12) give a $\frac{1}{2}^+$ state of ^{9}Be about 30 keV below the ^{8}Be + n threshold, and such a state is not observed experimentally; also, because the potentials are shallow, the calculated state is 0s, whereas it is more reasonable that a 1s state should exist near threshold, as found for their potentials (10) and (11) obtained from fitting the older data.

We investigate why the results for the semi-microscopic model of Efros *et al.* (1998) and the one-level R-matrix model used here are different. Efros *et* *al.* represented the ^{8}Be + n s-wave continuum state by a single-particle wave function for a central Woods–Saxon potential. They pointed out that the cross section depends mainly (but not entirely) on the asymptotic wave functions, and so on the low-energy s-wave phase shift, which is determined by the scattering length a_s and effective range r_0. Fig. 3 shows the dependence of a_s and r_0 on the potential depth V_0 for a Woods–Saxon potential with conventional values of the radius (2·50 fm) and diffuseness (0·65 fm). The potentials (10) and (11) of Efros *et al.* fitting the older data and giving an unbound 1s state near threshold, give $a_s \approx -28$ fm and $r_0 \approx 9$ fm. These are close to the values obtained for $V_0 \approx$ 56·5 MeV in Fig. 3. On the other hand, the Efros *et al.* potentials (12), fitting the newer data and giving a weakly-bound 0s state, give $a_s \approx 32$ fm and $r_0 \approx 3$ fm, close to those for $V_0 \approx 9$ MeV in Fig. 3. One could obtain a weakly-bound 1s state with the same value of a_s for $V_0 \approx 60$ MeV, but r_0 would be appreciably larger, leading to a poor fit to the data.

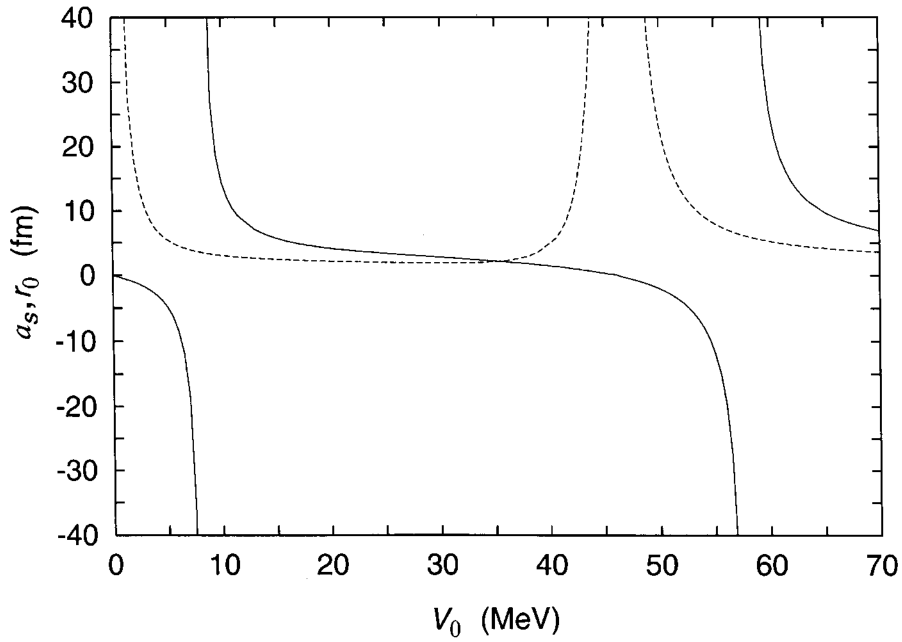

Fig. 3. Scattering length a_s (solid lines) and effective range r_0 (dashed lines) as functions of potential depth V_0, for a ^{8}Be + n Woods–Saxon potential with conventional radius (2·50 fm) and diffuseness (0·65 fm).

Values of a_s and r_0 can also be obtained from the parameter values in Table 1. Neglecting contributions from the ^{8}Be excited-state channels, one has (for $E_r \geq 0$)

$$a_s = (1 - \gamma_g^2/E_r)a\,, \tag{19}$$

$$r_0 = \frac{2}{(\gamma_g^2 - E_r)a}\left[\gamma_g^2 a^2 - \frac{\hbar^2}{2\mu} - E_r\left(\frac{\hbar^2}{2\mu} - \frac{E_r a^2}{3}\right)\Big/(\gamma_g^2 - E_r)\right]. \tag{20}$$

For $E_r < 0$, one replaces E_r in equations (19) and (20) by $E_r + S_g(E_r)\gamma_g^2$. The parameter values from fit D to the newer data give $a_s = -27{\cdot}3$ fm and $r_0 = 2{\cdot}2$ fm. There is no depth V_0 in Fig. 3 that gives values near these. The reason appears to be that γ_g^2 is appreciably less than the single-particle value; the value $\gamma_g = -1{\cdot}345$ MeV$^{\frac{1}{2}}$ is close to the shell model value ($-1{\cdot}369$ MeV$^{\frac{1}{2}}$) given in equation (13), and this corresponds to a spectroscopic factor $\mathcal{S}_g(\frac{1}{2}^+) = 0{\cdot}781^2 = 0{\cdot}610$. With $\mathcal{S}_g(\frac{1}{2}^+) = 1$, as assumed by Efros *et al.*, γ_g^2 would be larger and the first term in the square brackets in equation (20) would become more dominant, leading to a larger value of r_0 as suggested by Fig. 3.

It seems that fits to the older (newer) data require a large (small) value of r_0. With our small value of $\mathcal{S}_g(\frac{1}{2}^+)$ we could not obtain a large value of r_0 to fit the older data without going to a bound 1s state; the parameters from fit D to the older data give $a_s = 37{\cdot}0$ fm and $r_0 = 7{\cdot}7$ fm. In contrast, fit D to the Kuechler data gives $a_s = -34{\cdot}4$ fm and $r_0 = 3{\cdot}4$ fm.

After the present paper was essentially complete, a paper based on the model and results of Efros *et al.* (1998) appeared, in which properties of the first excited states of ^{9}Be and ^{9}B were discussed (Efros and Bang 1999). This paper uses various approximations that require some comment. It uses the single-particle potential model for the s-wave continuum states, with the parameter values found by Efros *et al.* (1998), and so assumes $\mathcal{S}_g(\frac{1}{2}^+) = 1$, in conflict with the results of shell model calculations. For ^{9}Be, it uses the Migdal–Watson form for the lineshape or density-of-states function $\rho(E) \propto \sin^2\delta/k$; this involves several approximations, which were discussed in Barker (1988). The same approach but with fewer approximations led Hamburger and Cameron (1960) to the form $\rho(E) \propto \sin^2\beta/P$, where $\beta = \delta + \phi$, with $-\phi$ the hard-sphere phase shift and P the penetration factor [this form for $\rho(E)$ is identical with the one-level R-matrix approximation]. For ^{9}Be this reduces to the Migdal–Watson form if ϕ is negligible compared with δ, i.e. $a \ll |a_s|$, which is valid. For ^{9}B, Efros and Bang use $\rho(E) \propto \sin^2\delta/kC^2$, where kC^2 is just the K-matrix penetration factor, or the R-matrix penetration factor calculated for zero channel radius. In this case it is not at all obvious that ϕ can be neglected in comparison with δ. Also kC^2 increases with energy more rapidly than does P calculated for a non-zero channel radius. Thus Efros and Bang, using $\rho(E) \propto \sin^2\delta/kC^2$, found a peak with a maximum at $E_{\max} = 1{\cdot}13$ MeV and FWHM $= 1{\cdot}64$ MeV; for the same δ, and with ϕ and P calculated for a channel radius of $4{\cdot}35$ fm, the form $\rho(E) \propto \sin^2\beta/P$ gives $E_{\max} \approx 1{\cdot}5$ MeV and FWHM $\approx 4{\cdot}8$ MeV.

In summary, the R-matrix model used here for the low-energy ^{9}Be(γ, n)^{8}Be cross section finds acceptable fits with reasonable parameter values to the newer radioactive-isotope data, as well as to the data of Kuechler *et al.* (1987), but not to the older radioactive-isotope data. This is opposite to the finding of Efros *et al.* (1998), whose calculations supported the older data. The difference may be due to the assumption by Efros *et al.* of a single-particle description of the ^{8}Be + n s-wave scattering system, whereas we have used a one-level approximation for the $\frac{1}{2}^+$ state of ^{9}Be, with a shell model value for its spectroscopic factor that is appreciably smaller than unity.

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