DIVERGENCE OF RADIO RAYS IN THE IONOSPHERE

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Summary

Travelling disturbance manifestations on ionosonde records usually occur at different times on the "*o*" and "*x*" traces. It is shown that this is due to the divergence of the ordinary and the extraordinary rays in the Earth's magnetic meridian plane and that the sense of this time difference, therefore, gives a direct indication of the sense of the north-south component of movement of the disturbance. Furthermore, where the direction and speed of travel of a disturbance can also be determined from spaced station observations, the actual separation of reflection points of the *o*- and the *x*-rays can be deduced. At Sydney, N.S.W. (33° 52′ S., 151° 11′ E.) this is of the order of 30 km at the height of maximum ionization of the F_2 region. On *h′t* records the corresponding time difference observed includes a component due to the vertical separation of reflection points if the front of the disturbance is not vertical. From records of this type taken at three spaced stations the horizontal component may be determined directly. Examination of some 430 such observations taken over a period of 4 years confirms the variation of the time difference with direction of travel predicted by theory, and also indicates the consistent presence of a forward tilt in the front of disturbances.

The theoretical relation between the horizontal and vertical separation is then used to determine the slope of front of disturbances. It is found to have a mean value varying from 65° to the horizontal for disturbances travelling northward to a minimum of 51° for those travelling 120° E. of N.

I. Introduction

In previous papers by one of the authors (Munro 1949, 1950, 1953), the main characteristics of travelling disturbances in the F_2 region of the ionosphere were deduced, and resulting effects commonly observed in ionospheric records explained. It has been observed on both *h′f* and *h′t* records that these disturbances generally affect the ordinary and the extraordinary rays similarly but at different times. This must be due to an actual spatial separation of the reflection points of *o*- and *x*-rays for reflections observed at the recording site.

Booker (1938), Millington (1949), and more recently Kirkpatrick (personal communication) have shown that in the presence of the Earth's magnetic field, magneto-ionic splitting of a radio ray occurs and the ordinary and the extraordinary rays diverge from the vertical in opposite directions in the Earth's magnetic meridian plane, so that the reflection points of these rays are spaced both horizontally and vertically in the ionosphere. This separation has been

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calculated theoretically by Kirkpatrick for any magnetic latitude and his results are shown in Figure 10.

In general, a difference between arrival times of a disturbance at the ordinary and the extraordinary reflection points may be due to both horizontal and vertical progression of the disturbance front. This progression is mainly horizontal, since observations using a three-point network (Munro 1950) establish definitely an apparent horizontal component of motion, and, therefore, since the extraordinary ray deviates to the north in the southern hemisphere and to the south in the northern hemisphere, order of arrival of a disturbance at the o and the x reflection points indicates immediately the general direction of travel of such disturbances.

It has also been observed that disturbances travelling in a direction normal to the magnetic meridian plane arrive at different times at the o and the x reflection points. This time difference can only be due to an apparent vertical component of motion of the disturbance, and supports an earlier assumption in a previous paper (Munro 1953) that the disturbance has a tilted front.

In the present paper observations of time differences between arrival of disturbances on o and x traces on $h'f$ records are used to determine the horizontal separation between o and x penetration points in the F_2 region. Similar time differences, obtained from three-point fixed frequency observations, together with speeds and directions, are used to check divergence theory and to measure the tilt of the front of the disturbance. Theoretical values for separation deduced by Kirkpatrick as shown in Figure 10 are also used to study travelling disturbances.

II. Variable Frequency Records

Since 1951, at Sydney (33° 52′ S., 151° 11′ E.) regular $h'f$ recordings have been taken at 1 min intervals, with a fast sweeping recorder previously described (Heisler 1955). These records show many examples of cusp type anomalies such as that shown in Figure 1. This is a series of $h'f$ records taken at 1 min intervals, showing a disturbance which first becomes evident in the form of an isolated cusp appearing at or close to the normal penetration frequency and at a relatively great height. In our records, this height is usually of the order of 500–600 km. On successive records the cusp moves steadily downwards and eventually joins the main curve. The cusp generally appears first on the ordinary ray trace and later on the extraordinary ray. The origin of these cusps is explained in an accompanying paper (Munro and Heisler 1956).

The ordinary and extraordinary rays at a given frequency f deviate in the F_2 layer as shown in Figure 2 and are reflected at A and B, two points separated both vertically and horizontally in the magnetic meridian plane. There is a frequency f_{ex} greater than f, the extraordinary ray of which deviates as shown by the dotted line in Figure 2, and is reflected at B', which is at the same height as A. This may be defined as the "equivalent frequency", or the value of frequency at which the extraordinary ray is reflected at the same height as the ordinary ray of a given frequency.

The o- and x-ray frequencies which are reflected at any particular height will be equivalent frequencies, but during the passage of a disturbance, because of the horizontal separation of the reflection points, they may be returned from this height at different times.

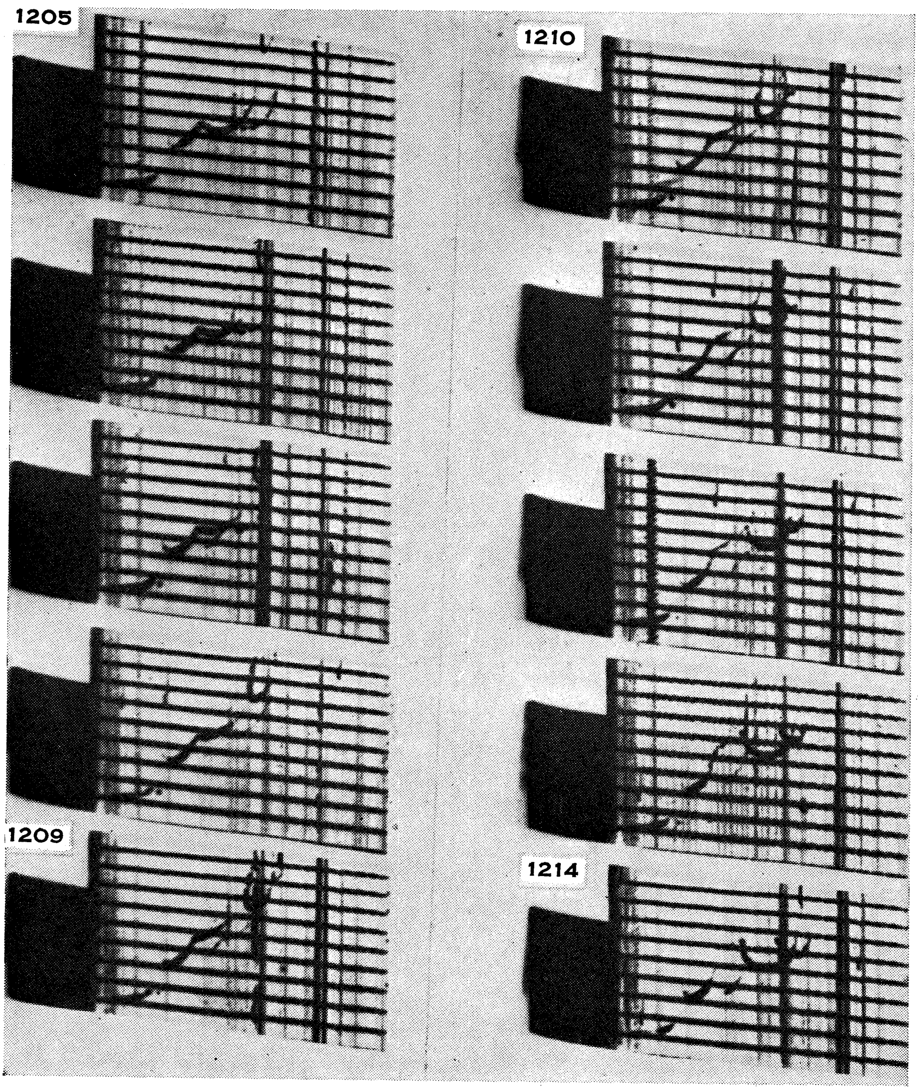


Fig. 1.—A series of $h'f$ records showing a cusp type anomaly.

It follows, therefore, that, if we can detect an indication of a disturbance on the $h'f$ curves at a frequency f on the ordinary ray, and a frequency f_{ex} on the extraordinary ray, then a knowledge of time difference of occurrence at these points, together with the known speed of travel and direction of travel from three-station fixed frequency observations, enables us to estimate the horizontal separation of the two reflection points AB'.

In the absence of disturbances, the o- and x-rays must penetrate the layer at the same height, the height of maximum ionization. For frequencies not far from the penetration frequency, therefore, we may assume that the difference between f and f_{ex} is the difference in penetration frequencies.

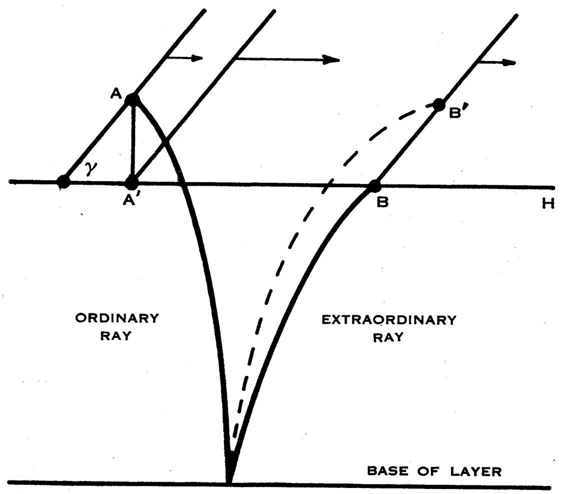


Fig. 2.—Deviation of o- and x-rays in the ionosphere.

Referring to Figure 1, at the bottom of the cusp the rate of change of height with frequency is small so that the equivalent frequencies need not be accurately determined. We may simply assume that on the records they will be at the bottom of the respective cusps at the same height.

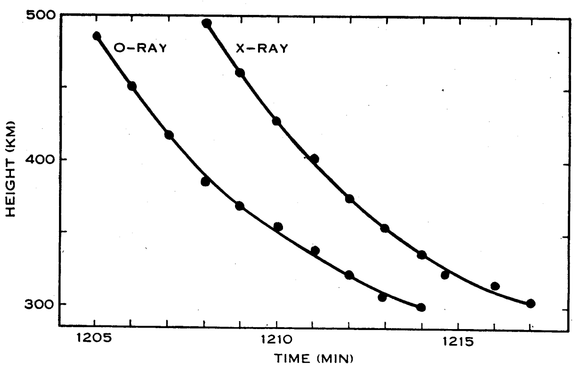


Fig. 3.—Variation with time of height of cusps observed on o- and x-rays.

If, therefore, we plot against time, the heights of the lowest points of the two cusps observed on each record over a period of several minutes, and join the points, we obtain two similar curves with a time displacement. The results obtained in this way for the disturbance of Figure 1 are shown in Figure 3.

It will be seen that the virtual height change is almost linear in the first few minutes and the time displacement is reasonably constant at 3·5 min. It has been shown elsewhere (Munro and Heisler 1956) that, although the virtual height change of a cusp is great (of the order of a hundred kilometres), the actual height change is much less (of the order of tens of kilometres) so that this constancy is not unexpected.

The direction of horizontal movement in this case as obtained from the three-station observations was 8° E. of N., which is almost the declination of the magnetic field, so no correction is necessary. The horizontal speed was 9·8 km/min, the separation of reflection points would, therefore, be $3\cdot5\times9\cdot8=35$ km approximately. This would be the horizontal separation near the maximum of ionization. The results for a number of similar cases are given in Table 1.

TABLE 1

HORIZONTAL SEPARATION BETWEEN THE ORDINARY AND THE EXTRAORDINARY RAY REFLECTION POINTS FOR A NUMBER OF CASES OF TRAVELLING DISTURBANCES

Date	Time	Time Difference, t $(o—x)$ (min)	Direction θ	Speed S (km/min)	Separation (km) $(t\times S\cos\theta)$
18. vi.52	1020	3·6	353°	8·8	31
10. vii.52	1142	3·0	26°	9·4	25
8.viii.52	1342	3·6	325°	10·8	31
26.viii.52	0930	4·3	3°	6·9	30
	1205	3·25	8°	7·8	32
	1310	3·5	355°	10·0	35
27.viii.52	1140	3·6	26°	9·4	30
					Mean : 30·5

It should be noted, that observation of cusps in this way can be used as an approximate indication of the direction of travel of the disturbances. If the disturbance appears on the ordinary ray first then the disturbance must be travelling towards the equator.

The equivalent frequency examination may be applied at frequencies other than the critical if the indications are clear enough on both ray traces; for example, if two frequencies are selected differing by $f^0_x-f^0_0$ and we can observe the time difference of occurrence of disturbances on the x-ray of the higher frequency and the o-ray of the lower frequency, which would be reflected from the same height, this will be the time of travel between the two reflection points. If, then, the horizontal speed and direction are known from three-point observations the separation can be determined. Furthermore, if the time difference of occurrence on the ordinary ray at these two frequencies is observable, the height difference of these two reflection points can be determined by applying analyses derived by Manning (1947) or Kelso (1952) to the undisturbed $h'f$ record, and so the vertical velocity may be deduced. From the vertical and the horizontal velocities the slope of the front can then be calculated. These time differences can be determined more accurately by $h't$ recording on two appropriate fixed frequencies. This method is being used to determine the slope of fronts of disturbances.

III. Fixed Frequency Records

In the study of travelling disturbances the virtual heights of echoes from three transmitters spaced some 40 km apart and operating on the same frequency are all recorded on a single film. It is quite often found that complexities caused by travelling disturbances appear on both ordinary and extraordinary traces for all three transmissions. On most occasions the disturbance occurs first on the ordinary ray, and for the purpose of the present paper the time difference between the ordinary and the extraordinary ray disturbance in this

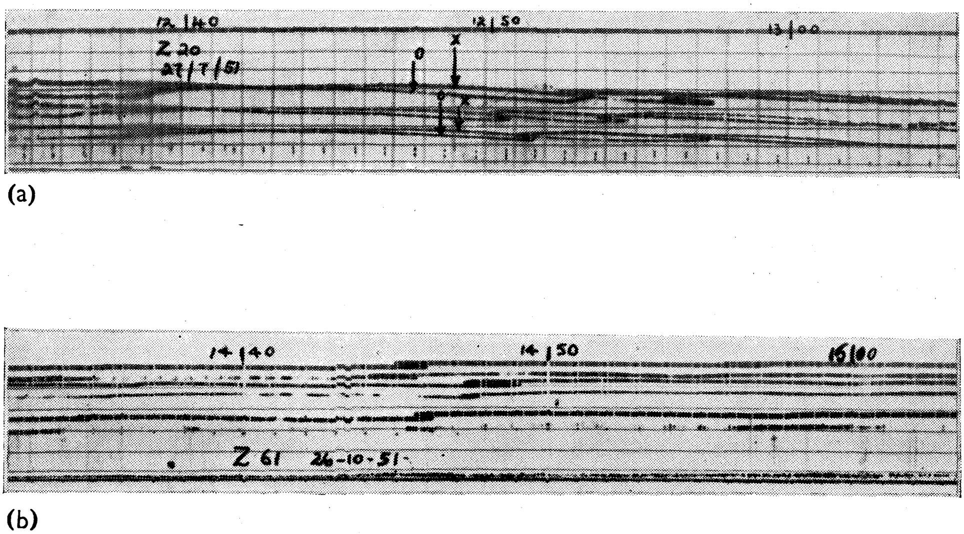


Fig. 4.—Z type disturbances showing time differences between o and x disturbance effects.

case is considered as positive. Two examples of these disturbances are shown in Fig. 4. In Figure 4 (*a*) there is a positive time difference of 4 min between appearances on the o- and x-rays, but in Figure 4 (*b*) appearances are almost simultaneous.

Referring to Figure 2, at any given frequency the o- and the x-rays deviate in the Earth's magnetic meridian plane and are reflected at two points A and B in the F_2 region. It will be seen, therefore, that a horizontally travelling disturbance will in general reach points A and B at different times.

If the disturbance has a vertical front and is travelling in the direction of the Earth's magnetic field, the horizontal separation x is equal to $A'B$, which is equal to Vt_1, where V is the velocity of the disturbance and t_1 is the time difference of occurrence. If the disturbance is travelling in a direction at an angle χ to the Earth's magnetic field, then

$$x = Vt_1/\cos \chi.$$

If the disturbance has a vertical front and is travelling in a direction normal to the magnetic meridian plane, then it should arrive at points A and B simultaneously and t should be zero.

It has been found in practice, however, that disturbances travelling normally to the magnetic meridian plane always show a positive time difference between o- and x-ray disturbance effects. This must be due to the disturbance front possessing a slope γ to the horizontal as shown in Figure 2, and the vertical separation may be expressed as $Vt_2/\cot\gamma$, where t_2 is the time difference contributed by the sloping front of the disturbance. It follows therefore that:

$$x\cos\chi+z\cot\gamma=V(t_1+t_2),$$

or

$$x\cos\chi+z\cot\gamma=Vt,$$

where t is the total time difference.

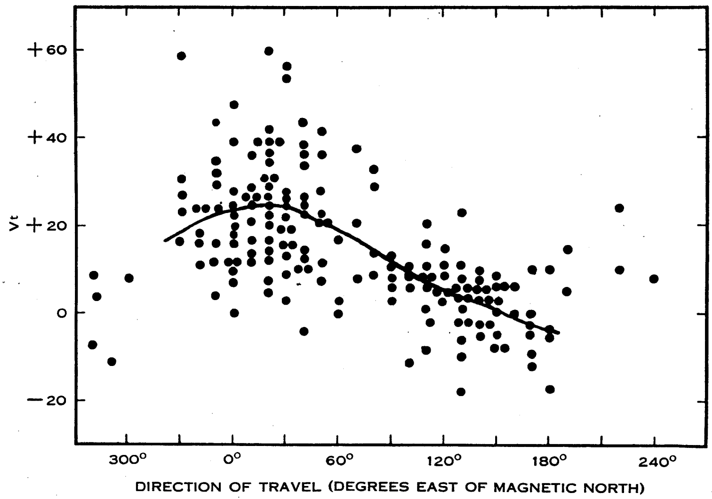


Fig. 5.—Plot of Vt *against* direction of travel for complexities on o- and x-rays. April 1, 1950 to March 31, 1951.

Figure 5 shows a plot of Vt for complexities on o- and x-rays for a period of 12 months from April 1, 1950 to March 31, 1951. These are plotted against directions E. of the magnetic meridian.

In each case considered, the disturbance indication is a complexity (of the loop, Y, or Z type; Munro 1953). The time of occurrence of this disturbance can be recorded to the nearest $\frac{1}{4}$ min and the disturbances are chosen so that their directions of travel as recorded for the o and x traces do not differ by more than 20°. Moreover, the disturbances must be present on both traces for all three of the spaced transmitters to enable this discrimination to be applied.

These criteria ensure a reasonable uniformity in type of disturbances considered. They must have sensibly linear fronts of dimensions greater than the spacing of stations, and be of magnitude great enough to give clear discontinuities, but not great enough to cause serious distortion of the normal ionization gradient. This screening appreciably reduced the scatter of points.

Observational accuracy has been examined and is considered to be sufficient to ensure significance of results. The consistency of results confirms this.

The following features are immediately apparent. There is a definite mean trend from high positive values of Vt at 0° to mainly negative values at 180°. Between 0 and 90°, there are many high points with values from 10 to 40, a few points with values from 0 to 10, and very few negative points; whereas, between 90 and 200° there are very few positive points greater than 10 and many negative ones. This, therefore, is definite confirmation of the general correctness of concepts previously expressed. The scatter of points may be partly due to variations in slope of front and observational limitations as well as to actual differences in separation.

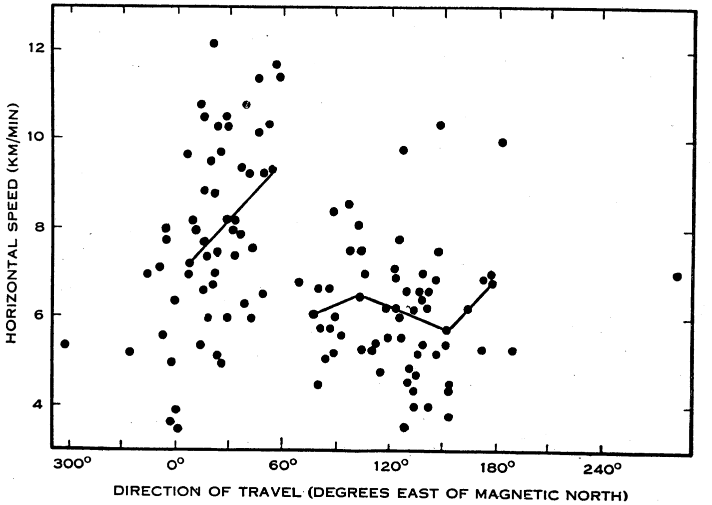
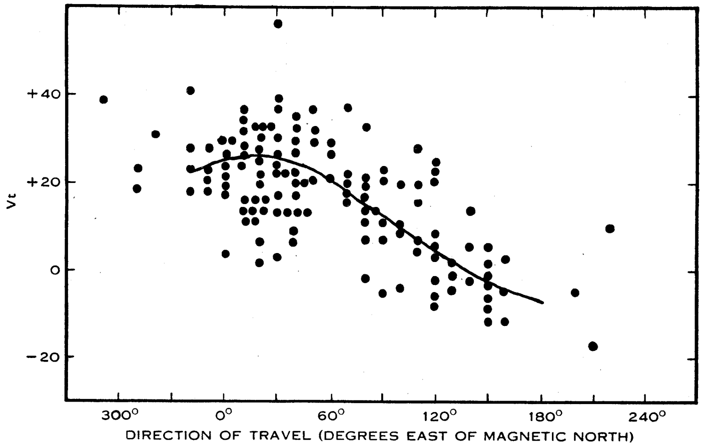


Fig. 6.—Variation of speed with direction of travel for disturbances in Figure 5.

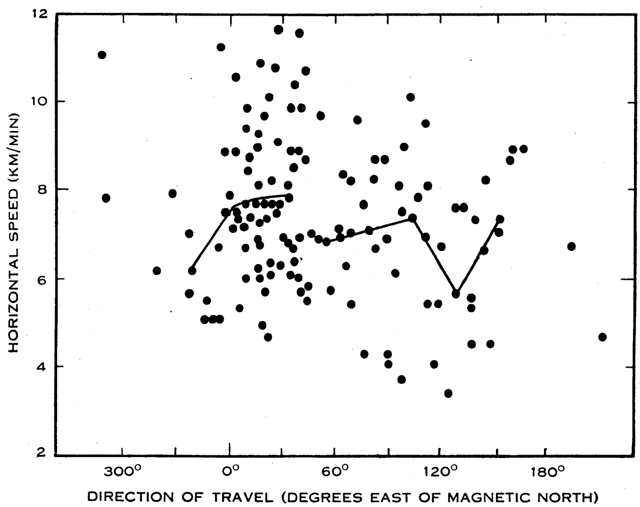
Variations in horizontal speed have been examined and while there is no correlation apparent between speed and time difference on o- and x-rays for a given direction, there is a definite variation of speed with direction. Figure 6 is a plot of speed against direction for the same disturbances as in Figure 5. It will be seen that there is a large change of speed between 60 and 70°. This corresponds closely with the seasonal change in speed, as directions 0 to 50° are recorded mainly in winter when speeds are highest, and other directions at the equinoxes and in summer when speeds are relatively low.

The results for the succeeding year April 1, 1951 to March 31, 1952 were similar and are shown in Figures 7 (a) and 7 (b). The velocity change for this year is not as marked. The similarity of result for the two years firmly establishes the fundamental features confirming general theory. We may therefore make certain deductions on the basis of that theory.

Since the disturbances travelling in directions close to 90° will be moving normally to the Earth's magnetic meridian plane, values of Vt other than zero must be due entirely to the vertical separation of the o and the x reflection points, and if Vt is positive there must be an apparent component of progression



(a)



(b)

Fig. 7 (*a*).—Plot of Vt *against* direction of travel for complexities on o- and x-rays. April 1, 1951 to March 31, 1952.

Fig. 7 (*b*).—Variation of speed with direction of travel for disturbances in Figure 7 (*a*).

vertically downward, which on our assumption would imply a forward tilt of the disturbance.

It will be seen in Figure 5, that all values of Vt between 80 and 100° are positive. This supplies strong evidence that there is a forward slope of front in all cases. This is supported also by the fact that the negative values between 100 and 150° are much smaller than the positive values between 0 and 100°.

To derive a mean curve, all the values of Vt for disturbances occurring from April 1, 1948 to March 31, 1954 have been averaged and plotted in Figure 8 and the best curve drawn through them. Where the data are most reliable this curve suggests a cosine curve and confirms the validity of the expression

$$x \cos \chi + z \cot \gamma = Vt. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

This equation is of the form

$$a \cos \chi + b = c,$$

giving $x=14$ and $z \cot \gamma = 9$ for the curve of Figure 8. In Figure 10 Kirkpatrick shows that in Sydney $x/z = 0 \cdot 8$, therefore, $z = 17 \cdot 5$. At $\chi = 0°$, $\gamma = 65°$, which means that when a disturbance is travelling in the direction of the Earth's

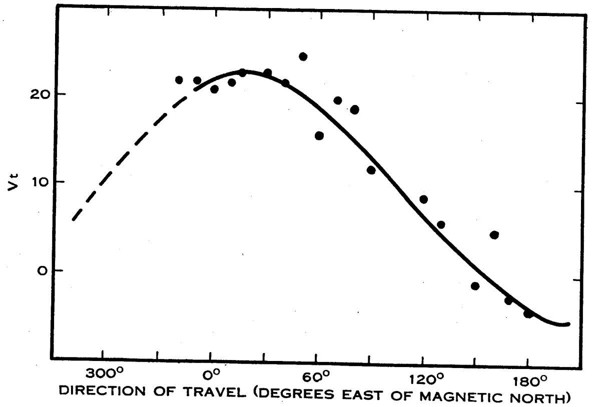


Fig. 8.—Variation of mean Vt with direction of travel. April 1, 1948 to March 31, 1954.

magnetic meridian its front is tilted at an angle of 65° to the horizontal. At $\chi = 90°$, $\gamma = 54°$, so that the disturbance still has a tilted front when travelling at right-angles to the Earth's magnetic meridian. γ has a minimum value of 51° when $\chi = 120°$. In Sydney, the magnetic dip angle is $63 \cdot 9°$ so that a disturbance travelling along the Earth's magnetic meridian travels with its front approximately parallel to the field.

It will be noticed in Figure 8 that the curve is not symmetrical about 90°. The value of Vt at 90° is 13 and at 0° is 23, a difference of 10, whereas, the value at 180° is -5, a difference of 18 from the value at 90°. This asymmetry could result from the mean slope of front, γ, being greater in winter than in summer. This is supported by other evidence which will be discussed in another paper.

Since, as mentioned above, directions vary with the seasons, and the sign of the time difference alone is an indication of general direction, it is of interest to consider these data on a seasonal basis. In Figure 9, all the recorded values of t are plotted for each month. The dots are the data used in Figure 5 selected on a directional consistency basis, and the crosses are the rejected values. It will be seen that their inclusion increases the scatter but has little effect on mean values. The scarcity of negative points in the winter months and their frequency in summer months is very marked.

It will be seen therefore that a seasonal plot of t for disturbances on o and x traces at a single station on a fixed frequency can give quite definite information as to the seasonal trend of movement.

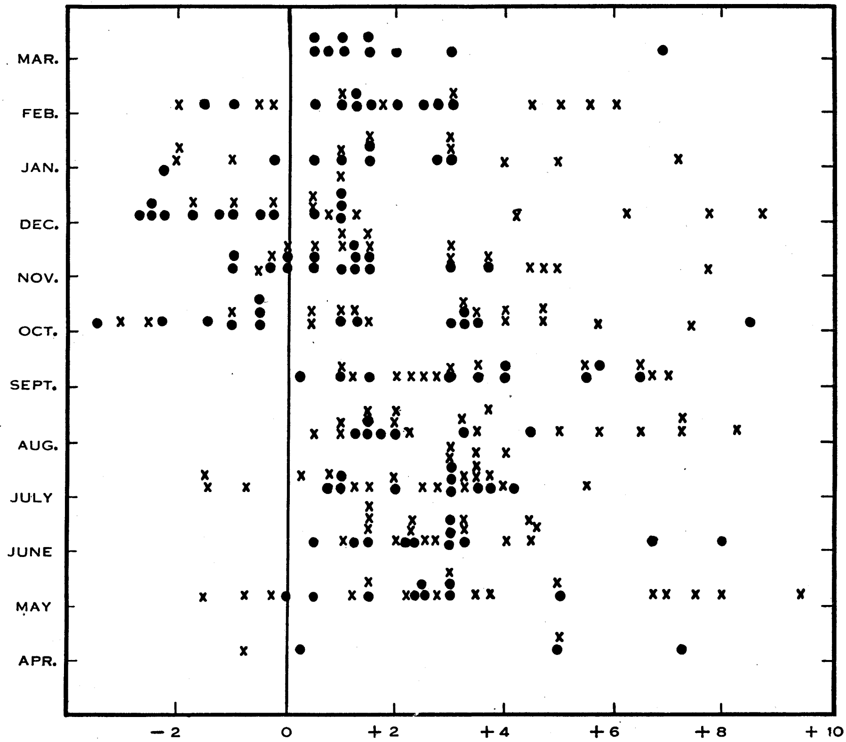


Fig. 9.—Seasonal variation of time-differences on o- and x-rays 1950–51. Dots are selected cases used in Figure 5 and crosses are cases considered insufficiently reliable for use in Figure 5.

IV. Interpretation of Records

It follows from the above results that certain features of disturbance irregularities on ionospheric records will vary according to the direction of travel of the disturbances. When the disturbance is travelling in the direction of the Earth's magnetic field its effect, usually apparent as an increase in virtual height, will appear first on the o-ray and a few minutes later on the x-ray. On a fixed frequency record, this generally produces crossing over of the two traces and the formation of loops as shown in Figure 4 (a).

If the disturbance is travelling in a direction normal to the field, the two rays will be affected almost simultaneously and the traces will move up and down together. This may be seen in Figure 4 (b).

On variable frequency ($h'f$) records the effects are more complex, but the general feature is a perturbation appearing first at the top of the trace for one ray and progressing downward with some progressive change of form. Similar changes will appear on the other trace, generally with a time difference at any given height. As on $h't$ records, when the disturbance is travelling along the field and the time difference is great, there will be considerable alteration in the relative position of the two traces; for example, under steady conditions the F_1 retardation peak on the x-ray usually brings this trace above the o trace in virtual height for part of the frequency range. A disturbance may temporarily reduce this peak and increase the height of the o trace so that the two traces are separated for their full length. However, if the disturbance is travelling across the field any time difference is small and an alteration in the relative position of the two traces may not be evident.

It should be noted that a travelling disturbance can cause a change in penetration frequency and, if this occurs at different times on the two rays, there will be fluctuations in the difference in the o and x critical frequencies at a given time. Therefore, in making deductions (e.g. the value of H) from this difference, care must be taken to avoid, or allow for, disturbance effects. Only if the disturbance is crossing the field will the perturbations affect both traces at the same height at the same time.

V. Conclusions

As the information from the two types of recording are supplementary and in agreement we may summarize the conclusions as follows:

Both fixed frequency and variable frequency observations confirm the theory of divergence of the o- and the x-rays in the F region at Sydney, with an actual separation of the order of 30 km near the maximum of ionization.

For a fixed frequency of $5 \cdot 8$ Mc/s over the period of observation March 1950 to April 1954, the average horizontal separation of o and x reflection points at Sydney is 14 km while the average vertical separation is $17 \cdot 5$ km.

The observations also indicate that a travelling disturbance always has an apparent vertical component of progression which is here assumed to be the result of a forward tilt in the wave front of the disturbance.

When the disturbance is travelling in the direction of the Earth's magnetic field, the tilt $\gamma=65°$, so that the front of the disturbance in Sydney is almost parallel to the Earth's magnetic field (dip angle in Sydney is $63 \cdot 9°$). This may be of significance in determining the nature of travelling disturbances. Furthermore, if the disturbance is travelling in a direction normal to the Earth's magnetic field, there is an appreciable tilt of its front. A "cellular wave" theory of disturbances advanced by Martyn (1950) suggests that such disturbances should have no tilt so that some modification seems to be required.

With a knowledge of disturbance effects on records, it is also possible from a set of $h'f$ records to deduce an approximate direction of travel of the disturbances. For this purpose, it is desirable to have records at intervals of 1 to 2 min. This method is being investigated for extending the knowledge of disturbance movements over a wide area.

Conditions in this paper apply particularly to the geomagnetic latitude of Sydney. Corresponding effects to be expected in other places may be deduced from a knowledge of the appropriate magnetic constants and the use of Figure 10 to determine the ratio of horizontal to vertical separation.

From the curves of Figure 10, the horizontal separation per kilometre of vertical penetration of the layer for a frequency of $5 \cdot 8$ Mc/s at Sydney is approximately $0 \cdot 2$ km. Since the mean observed separation is 14 km this implies

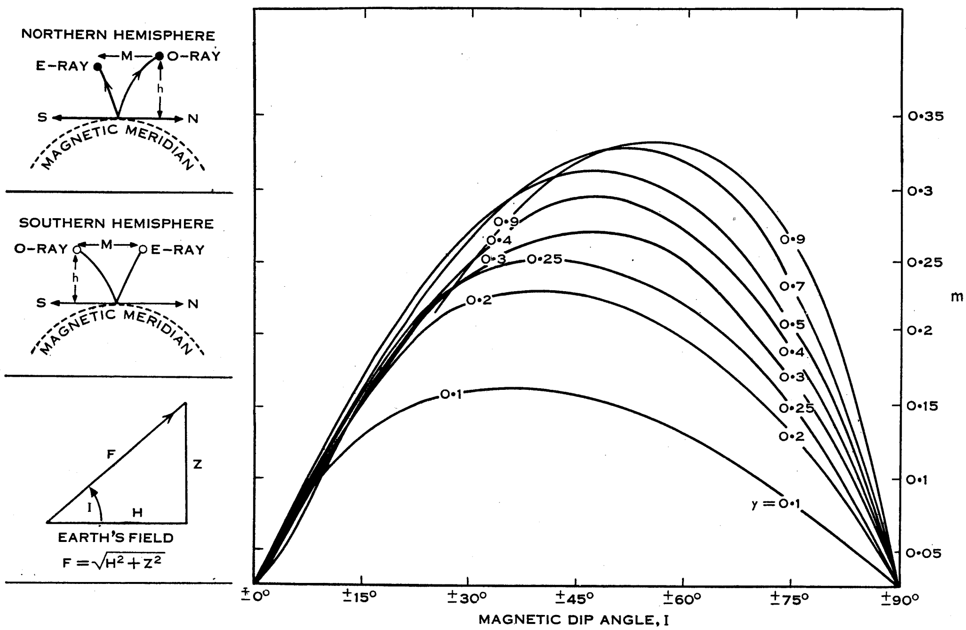


Fig. 10.—Chart giving ratio of horizontal to vertical separation for various values of magnetic dip angle (Kirkpatrick).

m = total horizontal separation in km per km of vertical penetration by the o-ray.

h = height of o-ray reflection point above base of layer.

$(1-y)h$ = height of x-ray reflection point above base of layer.

$M = mh$ = total (horizontal) separation.

yh = vertical separation.

f_H = gyrofrequency $= 2 \cdot 82F$ (Mc/s).

F = total magnetic field strength (gauss or Γ).

f = wave frequency (Mc/s).

$y = f_H/f$.

At Sydney, $F = 0 \cdot 511$, $f_H = 1 \cdot 43$ Mc/s.

that the divergence commences some 70 km below the o-ray reflection height thus giving an indication of the thickness of the ionized region during the day-time.

VI. Acknowledgments

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