

## COMMENT ON MULTILAYER DIELECTRIC FILTERS\*

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A recent note by Gascoigne (1959) on transmission in multilayer dielectric filters is interesting as an example of an adoption to a given problem of a result from an entirely different problem, namely, the Kronig-Penney model for metal lattices. However, one may wonder whether, in the long run, it would not be more economical of time, especially for students, to notice that the multilayer filter, the Kronig-Penney lattice, and for that matter many other problems, are all essentially forms of the same thing.

That is to say, they are all cases in which there are several regions of different properties superimposed. The behaviour of a function is specified in each region, and across the boundaries between regions there is the condition that the function and its gradient be continuous (or, instead of the gradient, there may be something analogous; e.g. in electromagnetic problems one has the quantities  $E$  and  $H$ ). But this situation is analogous to the cascading of electrical four-terminal networks, where voltage and current are continuous across the junction between adjacent four-terminals. Just as the problem of cascading networks is handled neatly by multiplying matrices, so can the problem involving superimposed layers be handled also by matrices.

In this case one would have two by two matrices relating the values of the function and of its gradient (or of  $E$  and  $H$  in the electromagnetic problem) at each surface of the layer. There would be one such matrix for each layer, and the superposition of layers would be represented by multiplying the matrices. It has already been shown (Armstrong 1956*a*, 1956*b*) how such a representation can be applied to a great variety of problems. In case the layers happen to be of two kinds placed alternately (and the Kronig-Penny model could be considered as such a situation) one will have results involving powers of matrices, and there are some special relations available to be applied to these (Pease 1952; Armstrong 1953, 1956*b*; Mielenz 1959).

In conclusion, then, the writer would like to suggest that it might be more advantageous for students to become familiar with the general methods of dealing with cascaded or iterated structures by using matrices, rather than by considering every such problem as a special case, for which special methods have to be developed.

It is interesting that, about the same time that Gascoigne's article appeared, there was another, about half-way around the world (Mielenz 1959) dealing with much the same problem. In this latter article matrix analysis is used, as well as some special ways of dealing with powers of matrices.

\* Manuscript received December 22, 1959.

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